STUDIES AND EXERCISES

IN

FORMAL LOGIC

STUDIES AND EXERCISES

IN

FORMAL LOGIC

INCLUDING

A GENERALIZATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX INFERENCES

BY

JOHN NEVILLE KEYNES M.A.

UNIVERSITY LECTURER IN MORAL SCIENCE AND LATE FELLOW OF PEMBROKE COLLEGE IN THE UNIVERSITY OF CAMBRIDGE

SECOND EDITION REVISED AND ENLARGED

London

MACMILLAN AND CO.

AND NEW YORK

1887

Cambridge:
PRINTED BY C. J. CLAY, M.A. AND SONS
AT THE UNIVERSITY PRESS.

PREFACE TO THE FIRST EDITION[1].

In addition to a somewhat detailed exposition of certain portions of what may be called the book-work of Formal Logic, the following pages contain a number of problems worked out in detail and unsolved problems, by means of which the student may test his command over logical processes. While certain topics, however, are dealt with in considerable detail, others are altogether omitted; *e.g.*, the doctrines of Definition and Division and the Predicables are not touched upon, and no systematic discussion of first principles has been introduced. This volume must therefore be regarded, not as superseding the study of an elementary text-book of Formal Logic, but rather as supplementing it.

In the expository portions of Parts I. II. and III., dealing respectively with Terms, Propositions, and Syllogisms, the traditional lines are in the main

[1] With slight omissions and alterations.

followed, though with certain modifications; *e.g.*, in the systematization of immediate inferences, and in several points of detail in connexion with the syllogism. For purposes of illustration Euler's diagrams are employed to a greater extent than is usual in English manuals.

In Part IV., which contains a generalization of logical processes in their application to complex inferences, a somewhat new departure is taken. So far as I am aware this part constitutes the first systematic attempt that has been made to deal with formal reasonings of the most complicated character without the aid of mathematical or other symbols of operation and without abandoning the ordinary non-equational or predicative form of proposition. This attempt has on the whole met with greater success than I had anticipated; and I believe that the methods formulated will be found to be both as easy and as effective as the symbolical methods of Boole and his followers. The book concludes with a general and sure method of solution of what Professor Jevons called the Inverse Problem, and which he himself seemed to regard as soluble only by a series of guesses.

Of the Questions and Problems more than half are my own composition. Of the remainder, about a hundred have been taken from various examination papers, and about sixty are from the published writings of Boole, De Morgan, Jevons, Solly, Venn, and Whately. In the latter case the name of the

author is appended, generally with a reference to the work from which the example is taken. In the case of problems selected from examination papers, a letter is added indicating their source, as follows:— C. = University of Cambridge; J. = W. E. Johnson, King's College, Cambridge; L. = University of London; N. = J. S. Nicholson, Professor of Political Economy in the University of Edinburgh; O. = University of Oxford; R. = G. Croom Robertson, Professor of Mental Philosophy and Logic in University College, London; V. = J. Venn, Fellow and Lecturer of Gonville and Caius College, Cambridge; W. = J. Ward, Fellow and Assistant Tutor of Trinity College, Cambridge.

The logicians to whom I have been chiefly indebted are De Morgan, Jevons, and Venn. De Morgan's various logical writings are rendered somewhat formidable and uninviting by reason of the symbols and formulae which he multiplies in the most perplexing way, and this is probably the reason why his works are now little read; they nevertheless constitute a mine of wealth for all who are interested in the developments of Formal Logic. With Jevons I have continually found myself in disagreement on points of detail, and it is possible that I may appear to have taken up a specially antagonistic position towards him. This is far from being really the case. I believe that since Mill no other English writer has given such an impetus to the study of Logic, and I hold that in more directions than one he has led

the way in new developments of the science which are of great importance.

To Mr Venn I am peculiarly indebted, not merely by reason of his published writings, especially his Symbolic Logic, but also for most valuable suggestions and criticisms while this book was in progress. I am glad to have this opportunity of expressing to him my thanks for the ungrudging help he has afforded me. I am also under great obligation to Miss Martin of Newnham College and to Mr Caldecott of St John's College for criticisms which I have found extremely helpful.

6, Harvey Road, Cambridge,
19 *January*, 1884.

PREFACE TO THE SECOND EDITION.

THIS edition has been carefully revised, and numerous sections have been almost entirely re-written.

In addition to the introduction of some brief prefatory sections, the following are among the more important modifications. In Part I. an attempt has been made to differentiate the meaning of the three terms connotation, intension, comprehension, with the hope that such differentiation of meaning may help to remove an ambiguity which is the source of much of the current controversy on the subject of connotation. In Part II. a distinction between conditional and hypothetical propositions is adopted for which I am indebted to Mr W. E. Johnson; and the treatment of the existential import of propositions has been both expanded and systematized. In Part IV. particular propositions, which in the first edition were practically neglected, are treated in detail; and, while the number of mere exercises has been diminished, many

points of theory have received considerable development. Throughout the book the unanswered exercises are now separated from the expository matter and placed together at the end of the several chapters in which they occur. An index has been added.

I have to thank several friends and correspondents, amongst whom I must especially mention Mr Henry Laurie of the University of Melbourne and Mr W. E. Johnson of King's College, Cambridge, for suggestions and criticisms from which I have derived the greatest assistance. Mr Johnson has kindly read the proof sheets throughout; and I am particularly indebted to him for the generous manner in which he has placed at my disposal not only his time but also the results of his own work on various points of Formal Logic.

CAMBRIDGE,
22 *June*, 1887.

CONTENTS.

PART III.

SYLLOGISMS.

PART IV.

A GENERALIZATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX PROPOSITIONS.

STUDIES AND EXERCISES IN FORMAL LOGIC.

INTRODUCTION.

1. Definition of Formal Logic.

Formal Logic may be defined as the science which investigates those regulative principles of thought that have universal validity whatever may be the particular objects about which we are thinking. It is a science which is concerned with the *form* as distinguished from the *matter* of thought.

In a proposition we affirm some kind of relation between certain objects of thought. By the *matter* of the proposition we mean the particular things thus related; by its *form* we mean the mode of their relation. For example, in the propositions "All men are mortal," "All crystals are solid," the form is the same while the matter is different; in the propositions "All crystals are solid," "Some rich men are not happy," the form as well as the matter is different. By using symbols to which any signification we please may be assigned, attention is concentrated on the form of propositions; *e.g.*, *All S is P*, *Some S is not P*. The employment

of non-significant symbols of this kind is accordingly advisable in dealing with most of the problems which fall within the scope of Formal Logic.

When we speak of a reasoning as being *formally* valid, we mean that its validity is determined solely by its form and is in no way dependent upon the particular subject-matter to which it relates. The cogency of a formally valid argument will therefore be unaffected, if for the particular terms involved are substituted others. The following is an example: All whales are mammals; Some water animals are whales; therefore, Some water animals are mammals.

In formal inference the conclusion is always implicitly contained in the premisses; and mere consistency compels assent to the conclusion if the premisses are once admitted. Formal Logic is accordingly sometimes spoken of as the Logic of mere consistency; and it follows that the observance of the laws which Formal Logic investigates will not do more than secure freedom from self-contradiction and inconsistency. Absolute truth Formal Logic cannot guarantee. At the same time, to draw out correctly all that is essentially involved in any given statement or set of statements is a function of fundamental importance; and the performance of this function alone may in many cases lead to knowledge that is to all intents and purposes new.

Whether Formal Logic properly constitutes the whole of Logic is a disputed question that we need not attempt here to decide. Accepting the above definition of Formal Logic, it is at any rate open to us to recognise also another branch of the science, in which we take account of the matter of thought and are concerned with all the methods of reasoning and research by the aid of which we advance in the attainment of truth.

It should be added that in the traditional treatment of Formal Logic various topics find a place that cannot strictly speaking be brought under the above definition.

2. Logic and Language.

Some logicians, in their treatment of the problems of Formal Logic, endeavour to abstract not merely from the matter of thought but also from the language which is the instrument of thought; they seek to deal exclusively with the thought-products as they exist in the mind, not with these same products as expressed in language. This method of treatment is not adopted in the following pages. In opposition to it, however, we do not go so far as to maintain that language is essential for thought in its very lowest manifestations. It is enough that all thought-processes of any degree of complexity are as a matter of fact carried on by the aid of language. That language is in this sense the universal instrument of thought is admitted by all. And we hold accordingly that the principles by which valid thought is regulated, and more especially the application of these principles, cannot be adequately discussed, unless some account is taken of the way in which this instrument actually performs its functions.

Language is full of ambiguities, and we cannot proceed far in Logic until we have placed a precise interpretation upon certain forms of words as representing thought. It frequently happens that in everyday discourse the same propositional form may admit of different interpretations, according to the context or to the subject-matter of the statement. But of context and subject-matter Formal Logic has no cognizance. It is therefore necessary to determine definitely which of these interpretations is in our further investigations to be adopted. In ordinary discourse,

to take a simple example, the word *some* may mean merely *some at least*, not exclusive of all, or it may mean *some at most;* the logician must determine at the outset in which sense he will employ the word. Again, a disjunctive statement in ordinary speech may be understood to imply that the different alternatives are mutually exclusive, or it may not; the logician must fix his meaning. Now if we considered exclusively thought in itself, such questions as these could not arise; they have to do with the expression of thought in language. The fact that they do arise and cannot help arising shews that to eliminate all consideration of language from Logic is an impossibility.

Moreover, all the thoughts with which Logic is concerned are expressed in language, reasonings being combinations of propositions, and propositions combinations of terms. To analyse the import of terms and propositions ought therefore to be recognised as included amongst the functions of the science. To investigate the form of conclusions obtainable from premisses of a given form is indeed the main object of Formal Logic. For this alone a discussion of the import of propositions is indispensable.

The method of treating Logic here advocated is sometimes called *nominalist*, and the opposed method *conceptualist*. Such an application of these terms is however in danger of leading to misunderstanding. Nominalism and Conceptualism usually denote certain doctrines concerning the nature of general notions. Nominalism is understood to involve the denial that there is anything general in thought itself, and even the refusal to recognise that the concept has any mental existence. But a so-called nominalist treatment of Logic does not involve this. It involves no more than a clear recognition of the importance of language as the instrument of thought; and this is a

circumstance upon which moderate advocates of conceptualism have themselves insisted.

It is perhaps necessary to add that on the view here taken Logic in no way becomes a mere branch of grammar, nor does it cease to be a mental science. Whatever may be the aid derived from language, it remains true that the validity of formal reasonings depends ultimately on laws of thought. Formal Logic therefore is still primarily concerned with thought, and only secondarily with language as the instrument of thought.

3. Logic and Psychology.

Since the laws regulating the processes of formal reasoning are laws depending upon the constitution of our minds, they fall within the cognizance of Psychology as well as of Logic. But they are regarded from different points of view by these two sciences. Psychology deals with them as laws in the sense of uniformities, that is, as laws in accordance with which men are found by experience normally to think and reason; Psychology investigates also their genesis and origin. Logic, on the other hand, deals with them purely as regulative and authoritative, as affording criteria by the aid of which valid and invalid reasonings may be discriminated, and as determining the formal relations in which different products of thought stand to one another.

Looking at the relations between Psychology and Logic from a slightly different standpoint, we may say that the former is concerned with the actual, the latter with the ideal. Logic does not, like Psychology, treat of all the ways in which men actually reach conclusions, or of all the various modes in which, through the association of ideas or otherwise, one belief actually generates another. It is concerned with reasonings only in respect of their cogency,

and with the dependence of one judgment upon another only in so far as it is a dependence in respect of proof.

Logic has thus a unique character of its own, and is not a mere branch of Psychology. At the same time it must be admitted that at certain points, in connexion with theories of conception and judgment for example, psychological and logical discussions are apt to overlap one another. I should add, however, that in the following pages the more psychological side of Logic is purposely but little touched upon. The metaphysical questions also to which Logic tends to give rise are as far as possible avoided.

PART I.

TERMS.

CHAPTER I.

GENERAL AND SINGULAR NAMES. CONCRETE AND ABSTRACT NAMES.

4. The Logic of Terms.

A *name* is defined by Hobbes as "a word taken at pleasure to serve for a mark which may raise in our minds a thought like to some thought we had before, and which, being disposed in speech and pronounced to others, may be to them a sign of what thought the speaker had or had not before in his mind."

In this definition the words "taken at pleasure" have rightly been criticised on the ground that they suggest an arbitrary and capricious selection, which is not in accordance with any generally accepted theory of the origin and growth of language. It is true that when an astronomer discovers a new planet and names it, or when a florist raises a seedling dahlia and gives it a distinctive title, or when an inventor names a new invention, the choice made is purely voluntary. But no one would now affirm that there are many names in

common use which took their rise in this way. It is agreed that their formation was in general a natural and spontaneous process, involving nothing of the nature of deliberate invention and selection.

Hobbes's definition is further criticised on the ground that it is not wide enough to cover a many-worded name. Not all names consist of a single word, *e.g.*, Prime Minister, Lord Chief Justice of England.

Accepting these criticisms, we may substitute for Hobbes's definition the following: A *name* is a word or set of words serving as a mark to raise in our minds a given idea, and also to indicate to others what idea is before the mind of the speaker.

A *term* is a name regarded as the possible subject or predicate of a proposition.

It has been already indicated that to discuss up to a certain point the import of names is properly regarded as one of the functions of the logician. Particularly important from his standpoint is the distinction between the denotation and the connotation of terms. In this part of his subject, however, it is particularly difficult for the logician who follows at all on the traditional lines to avoid discussing problems that belong more appropriately to psychology, metaphysics, or grammar. And it must be confessed with regard to some of the distinctions which we go on to discuss, that we are not ultimately able to give a very satisfactory account of them. The above remark applies especially to the distinction between *abstract* and *concrete* terms, a distinction, moreover, which is of little further logical utility or significance. It is introduced in the following pages in accordance with custom; but adequately to discriminate between things and their attributes is the function of Metaphysics rather than of Logic.

A *concept* is defined by Sir William Hamilton as "the cognition or idea of the general character or characters, point or points, in which a plurality of objects coincide." In other words, a concept is the mental equivalent of a general name.

With those logicians who seek to exclude from their science all consideration of language a Logic of Concepts takes the place of a Logic of Terms. The discussions however which they introduce in this place,—for example, the mode of formation of concepts and the controversy between conceptualism and nominalism,—are apt to be of a markedly psychological character. The distinction between the extension and intension of concepts is undoubtedly of fundamental importance; but practically the same points are raised whether we discuss the extension and intension of concepts or the denotation and connotation of names.

5. Categorematic and Syncategorematic Words.

A *categorematic* word is one which can by itself be used as a term, *i.e.*, which can stand alone as the subject or the predicate of a proposition.

A *syncategorematic* word is one which cannot by itself be used as a term, but only in combination with one or more other words.

Any noun substantive in the nominative case, or any other part of speech employed as equivalent to a noun substantive, may be used categorematically.

When adjectives stand alone as subjects or predicates, they may be regarded as qualifying some substantive understood. Adjectives are therefore said to be used categorematically by a *grammatical ellipsis*. In the examples, "The rich are happy," "Blue is an agreeable colour," we may consider that we have elliptical expressions for "Rich

men are happy men," "A blue colour is an agreeable colour."

Any part of speech, or the inflected cases of nouns substantive, may be used categorematically by a *suppositio materialis*, that is, by speaking of the mere word itself as a thing; for example, "*John's* is a possessive case," "*Rich* is an adjective," "*With* is an English word."

Using the word *term* in the sense in which it was defined in the preceding section, it is clear that we ought not to speak of syncategorematic *terms*.

6. General, Singular, and Proper Names.

A *general* name is a name which is capable of being correctly affirmed, in the same sense, of each of an indefinite number of things, real or imaginary. A *singular* or *individual* name is a name which is capable of being correctly affirmed, in the same sense, of only one thing, real or imaginary. A *proper* name is a singular name given merely to distinguish an individual person or thing from others, its application after it has been once given being independent of any special attributes that the individual may possess[1].

Thus, *prime minister of England* is a general name, since at different times it may be applied to different individuals. We may, for example, speak of "the prime ministers of England of the present century." The name may however be transformed into a singular name by means of an individualising prefix; *e.g.*, "the present prime minister," or "the prime minister at the time to which we are referring."

[1] A proper name might perhaps be defined as "a non-connotative singular name." But this definition presupposes a distinction which is best given subsequently, and it would give rise to a controversy, that also had better be postponed. Compare section 14.

Similarly, any general name may form part of a singular name; for example, man, *the first man;* star, *the pole star.*

The name *God* is singular to a monotheist as the name of the Deity, general to a polytheist, or as the name of any object of worship. *Universe* is general in so far as we distinguish different kinds of universes, *e.g.*, the material universe, the terrestrial universe, &c.; it is singular if we mean *the* universe. *Space* is general if we mean a particular portion of space, singular if we mean space in the aggregate. *Water* is general. Professor Bain takes a different view here; he says, "Names of Material,—earth, stone, salt, mercury, water, flame,—are singular. They each denote the entire collection of one species of material" (*Logic, Deduction*, pp. 48, 49). But when we predicate anything of these terms it is generally of *any portion* (or of some particular portion) of the material in question, and not of the entire collection of it *considered as one aggregate;* thus, if we say "Water is composed of oxygen and hydrogen," we mean any and every particle of water, and the name has all the distinctive characters of the general name. Again, we can distinguish *this* water from *that* water, and we can say, "*some* water is not fit to drink;" but the word *some* cannot be attached to a really singular name. Similarly with regard to the other terms mentioned in the above quotation. It is also to be observed that we distinguish different kinds of stone, salt, &c.[1]

A name is to be regarded as general if it may be *potentially* affirmed of more than one, although it accidentally happens that as a matter of fact it can be actually affirmed

[1] Terms of the kind here under discussion are called by Jevons *substantial terms*. Their peculiarity is that, although they are concrete, the things denoted by them possess a peculiar homogeneity or uniformity of structure. Cf. *Principles of Science*, chap. 2, § 4.

of only one, *e.g.*, *King of England and Spain*. We must also note the case in which we are dealing with a name that actually is not applicable to any individual at all; *e.g.*, *President of the British Republic*. A really singular name is distinguished from these by not being even potentially applicable to more than one individual; *e.g.*, *the last of the Mohicans*, *the eldest son of King Edward the First*.

Victoria is the name of more than one individual, and can therefore be correctly affirmed of more than one individual. Is it therefore general? Mill answers this question in the negative, and rightly, on the ground that the name is not here affirmed of the different individuals *in the same sense;* it is not in reality affirmed of them in any sense at all, as we shall presently try to shew. Professor Bain brings out this distinction very clearly in his definition of a general name: "A general name is applicable to a number of things in virtue of their being similar, or having something in common." Victoria is then not general but singular; and it belongs to the sub-class of proper names[1].

7. Collective Names.

A *collective* name is one which is the name of a group of similar things considered as constituting one whole; *e.g.*, regiment, nation, army.

A collective name may be singular or general. It is the name of a group or collection of things, and so far as it

[1] It may be said that proper names become general when we speak of the class composed of those who bear the name, and who are constituted a distinct class by this common feature alone; *e.g.*, all Victorias are honoured or happy in their name. In this case, however, we may regard the subject as elliptical; written out more fully, it becomes "all who are called Victoria." Compare further the concluding paragraph of section 14.

is capable of being correctly affirmed in the same sense of only one such group, it is singular; *e.g.*, the 29th regiment of foot, the English nation, the Bodleian Library. But if it is capable of being correctly affirmed in the same sense of each of several such groups it is to be regarded as general; *e.g.*, regiment, nation, library. Professor Bain writes as if a name could be general and singular at the same time,—"Collective names as nation, army, multitude, assembly, universe, are singular; they are plurality combined into unity. But, inasmuch as there are many nations, armies, assemblies, the names are also general. There being but one 'universe,' that term is collective and singular." I should rather say that as the above stand, with the possible exception of universe, they are not singular at all. Some logicians seem to imply an antithesis between collective and general names. There is, properly speaking, no such antithesis; since these two classes of names overlap. The correct and really important logical antithesis is between the *collective* and *distributive* use of names. A collective name such as nation, or any name in the plural number, is the name of a collection or group of similar things. These we may regard as one whole, and something may be predicated of them that is true of them only as a whole; in this case the name is used *collectively*. On the other hand, the group may be regarded as a series of units, and something may be predicated of these which is true of them taken individually; in this case the name is used *distributively*. Also, when anything is predicated of a series of such groups the name is used distributively.

The above distinction may be illustrated by the propositions,—All the angles of a triangle are equal to two right angles, All the angles of a triangle are less than two right angles. In the first case the predication is true only

of the angles all taken together, while in the second it is true only of each of them taken separately; in the first case therefore the term is used collectively, in the second distributively. Compare again the propositions,—The people filled the church, The people all fell on their knees.

When, apart from its use in any given proposition, we speak of a name as being collective, we mean that it can be used collectively in the singular number[1]. Other names can be used collectively only in the plural number. We have already seen that since collective names may themselves in the plural number be used distributively, it is not correct to say that all collective names are singular. More plausibly it may be maintained that, while this is true, still when a name is used collectively, it is equivalent to a singular name. For example, The whole army was annihilated, The mob filled the square. But I am doubtful whether even this is true in such a case as the following,—In some cases two sides of a triangle (taken together) are just double the third side.

8. Concrete and Abstract Names.

A name given to an individual thing in virtue of special qualities which it possesses, or to a class of things in virtue of some quality or set of qualities which they have in common, is called a *concrete* name[2]. The name given to the quality or qualities themselves apart from the indi-

[1] To *collective name* thus understood there is I think no distinctive antithetical term.

[2] Names given to concrete objects as mere distinguishing marks, *i.e.*, proper names, also fall into the category of concrete names. In this case however there are no corresponding abstracts; and we do not get the antithesis which it is here desired to bring out.

viduals to which they belong is the corresponding *abstract* name[1].

This distinction is in most cases of easy application; for example, *triangle* is the name of all figures that possess the attribute of being bounded by three straight lines, and is a concrete name; *triangularity* is the name of this distinctive attribute of triangles, and is an abstract name. Similarly, *man*, *living being*, *generous* are concretes; *humanity*, *life*, *generosity* are the corresponding abstracts. It follows from our definitions that in the case of every concrete name there is or may be constructed a corresponding abstract[2]; and that in the case of every abstract name there is or may be constructed a corresponding concrete.

[1] Mill defines abstract and concrete names as follows:—"A concrete name is a name which stands for a thing; an abstract name is a name which stands for an attribute of a thing" (*Logic*, I. ch. 2, § 4). These definitions correspond with those given in the text, but are not quite so explicit. In a later chapter Mill remarks,—"An abstract name is the name of an attribute or combination of attributes. The corresponding concrete is a name given to things, because of, and in order to express, their possessing that attribute or combination of attributes" (*Logic*, I. ch. 5, § 7).

The student should observe that we do not call a name abstract, simply because the corresponding idea is the result of abstraction, *i.e.*, attending to some qualities of a thing or class of things to the exclusion as far as possible of others. In this sense all general names would be abstract.

[2] Strictly speaking, we ought here to except proper names. It is true that we have such abstracts as Cæsarism and Bismarckism. These names however do not denote all the differentiating attributes of Cæsar and Bismarck respectively, but only certain qualities supposed to be specially characteristic of these individuals. In forming the above abstracts we generalise, and contemplate a certain type of character and conduct that may possibly be common to a whole class. Compare the concluding paragraph of section 14.

Names of definite states of consciousness are concrete[1]. For states of consciousness are phenomena which we arrange and classify according to the varying characteristics which belong to them; they are not themselves to be described as attributes or qualities. Thus, while *sensibility*, the faculty of experiencing sensation, is an abstract name corresponding to the concrete *sensible*, the name of any class of sensations or feelings is to be regarded as concrete. By a *pleasure*, for example, we mean any presentation that possesses the attribute of being pleasant.

But here a difficulty arises, since, as pointed out by Mill, in many cases the same name is the name both of a feeling and of an attribute[2]. For example, by *colour* we may mean sensations of blue, red, green, &c., or we may mean the attribute which all coloured objects possess in common. In the former case, colour is a concrete name, in the latter an abstract name. *Sound*, again, is concrete, in so far as it is the name of a sensation, *e.g.*, "the same sound is in my ears which in those days I heard;" but in the following cases, it should rather be regarded as abstract,

[1] Professor Ray (*Deductive Logic*, Part 1, ch. 1. § 4) expressly defines a concrete name as the name of a *substance*, an abstract name being defined as the name of an attribute apart from the substance in which it exists. If, however, we accept these definitions, it is clear that some names are neither abstract nor concrete. For example, besides names of states of consciousness, take such as the following: *astronomy*, *proposition*, *point*. These are not substances, but neither are they attributes.

[2] On Mill's view that attributes may be analysed into sensations,—"the distinction which we verbally make between the properties of things and the sensations we receive from them, originating in the convenience of discourse rather than in the nature of what is signified by the terms,"—the question becomes still further complicated. For logical purposes however the analysis certainly need not be pursued so far as this.

—"a tale full of sound and fury," "a name harsh in sound." To take one more illustration, *pleasure* signifying the property common to all pleasures, *i.e.*, to all definite pleasurable feelings, is abstract.

Many names therefore are either abstract or concrete according to the precise signification attached to them, and this naturally tends to obscure the distinction itself. It is to be observed further that many names originally abstract are very liable to come to be used as concrete[1]. There are also cases in which idiomatically we use abstract terms, but where in precisely analysing the meaning intended to be conveyed we should have to substitute concretes[2]. It is chiefly to this that I attribute the fact that in working out the distinction between abstract and concrete names it is so difficult to avoid somewhat fruitless subtleties.

Logically the only important point is the relation between concretes and the corresponding abstracts. Given two

[1] "*Relation* properly is the abstract name for the position of two people or things to each other, and those people are properly called *relatives*. But we constantly speak now of *relations*, meaning the persons themselves; and when we want to indicate the abstract relation they have to each other we have to invent a new abstract name *relationship*. *Nation* has long been a concrete term, though from its form it was probably abstract at first; but so far does the abuse of language now go, especially in newspaper writing, that we hear of *a nationality*, meaning a nation, although of course if nation is the concrete, nationality ought to be the abstract, meaning the quality of being a nation." Jevons, *Elementary Lessons in Logic*, pp. 21, 22.

[2] When for instance we say that oriental civilizations are stationary, we are not making an assertion about the attribute civilization considered independently of special states of society to which this attribute belongs. Strictly speaking it is not of any *attribute* at all that we are predicating stationariness. What we really mean is that in the East political usages and social customs are little liable to progressive changes. It is of these usages and customs that stationariness is predicated.

terms which are thus related, we shall I think never find any difficulty in determining which is concrete and which is abstract in relation to the other.

9. Can the distinction between Generals and Singulars be applied to Abstract Names?

An attribute, considered entirely apart from the things possessing it, is one and indivisible and does not admit of numerical distinction[1]. The following answer to the above question is therefore clearly wrong. "Most abstract names," it is sometimes said, "are general, since they are names of attributes which are found in different objects; *Deity*, however, to the monotheist may be given as an example of a singular abstract, for it is the name of an attribute which can be affirmed of God only." This criterion would make the corresponding abstract of every general concrete name,

[1] Hence Jevons argues that all abstract names are singular. "Abstract terms are strongly distinguished from general terms by possessing only one kind of meaning; for as they denote qualities there is nothing which they can in addition imply. The adjective 'red' is the name of red objects, but it implies the possession by them of the quality *redness*; but this latter term has one single meaning—the quality alone. Thus it arises that abstract terms are incapable of number or plurality. Red objects are numerically distinct each from each, and there are a multitude of such objects; but redness is a single existence which runs through all those objects, and is the same in one as it is in another. It is true that we may speak of *rednesses*, meaning different kinds or tints of redness, just as we may speak of *colours*, meaning different kinds of colours. But in distinguishing kinds, degrees, or other differences, we render the terms so far concrete. In that they are merely red there is but one single nature in red objects, and so far as things are merely coloured, colour is a single indivisible quality. Redness, so far as it is redness merely, is one and the same everywhere, and possesses absolute oneness or unity" (*Principles of Science*, ch. 2, § 3).

general, and of every singular concrete name, singular; but it is evidently based on a misapprehension. By an abstract name we mean the name of an attribute considered apart from the things possessing it; and, as indicated above, the attribute itself is to be regarded as one and the same, whether it is possessed by one thing only or by an indefinite number of things.

It remains however to be discussed whether some abstracts are not to be considered general on the ground that they are names of attributes of which there are various kinds or subdivisions. It certainly is a fact that we frequently write abstracts in the plural number, as when we say "Redness and yellowness are colours"; and Mill accordingly answers the above question in the affirmative. He remarks,—"Some abstract names are certainly general. I mean those which are names not of one single and definite attribute, but of a class of attributes. Such is the word *colour*, which is a name common to whiteness, redness, &c. Such is even the word *whiteness*, in respect of the various shades of whiteness to which it is applied in common; the word *magnitude*, in respect of the various degrees of magnitude and the various dimensions of space; the word *weight*, in respect of the various degrees of weight. Such also is the word *attribute* itself, the common name of all particular attributes. But when only one attribute, neither variable in degree nor in kind, is designated by the name; as visibleness; tangibleness; equality; squareness; milk-whiteness; then the name can hardly be considered general; for though it denotes an attribute of many different objects, the attribute itself is always conceived as one, not many" (*Logic*, I. ch. 2, § 4). In answer to this it may, I think, be plausibly maintained that when we begin to distinguish kinds and differences, and hence use abstract names in the plural number, we

have expressions in which the true generality belongs to the concrete names for which idiomatically abstract names are substituted. For example, "Temperance and justice are virtues," "Redness and yellowness are colours," are only different ways of expressing the propositions "Temperate and just acts are virtuous acts," "Red and yellow things are coloured things." Possibly this explanation may by most readers be regarded as far-fetched and unnecessary; and on the whole it may be best to accept Mill's solution of the question as on the face of it correct and at the same time the simplest that can be offered.

10. All Adjectives as such are Concrete General Names.

This follows immediately from our definitions of concrete and general names. For an adjective is essentially a name which can be applied to whatever possesses a certain attribute; *great*, for instance, is the name of whatever possesses the attribute of greatness[1]. It is true that an adjective may qualify an abstract term, *e.g.*, we may speak of "great beauty" or of "great strength"; it may also qualify a singular term, *e.g.*, we may speak of "Alexander the Great" or of "the great Goliath." In cases like these, combinations of names containing adjectives may certainly be abstract or singular. But it does not follow that the adjective itself becomes abstract or singular, any more than that such a term as *man* is itself singular because it forms part of the singular term "the first man."

[1] Using terms which will be explained in the following chapter, *great* connotes greatness, but denotes all great things.

CHAPTER II.

CONNOTATION AND DENOTATION.

11. The Connotation and Denotation of Names.

Every concrete general name is the name of a class, real or imaginary: by its *connotation* we mean the attributes on account of which any individual is placed in the class or called by the name; by its *denotation* we mean the individuals which possess these attributes, and which are therefore placed in the class and called by the name. The terms *intension* (or *comprehension*) and *extension* are also used as equivalent to connotation and denotation respectively. These terms were however originally applied to concepts rather than to names.

Thus, the connotation of "plane triangle" is given when it is defined as a plane figure contained by three straight lines; under its denotation are included all plane figures fulfilling this condition. The connotation of "man" consists of those attributes, whatever they may be, which are regarded as essential to the class man, *i.e.*, in the absence of any one of which we should refuse to call any individual by the name; its denotation is made up of all the individuals actually possessing these attributes.

12. Connotative Names.

(i) Mill's use of the word *connotative* is that now generally accepted. "A non-connotative term is one which signifies a subject only, or an attribute only. A connotative term is one which denotes a subject, and implies an attribute" (Mill, *Logic*, I. p. 31). According to this definition, a connotative name must possess *both* connotation and denotation.

The following kinds of names are connotative in Mill's sense:—(1) All concrete general names. (2) Some singular names. For example, "city" is a general name, and as such no one would deny it to be connotative. Now if we say "the largest city in the world," we have individualised the name, but it does not thereby cease to be connotative. Proper names are, however, according to Mill, not connotative, since they merely denote a subject and do not imply any attributes. To this point, which is a disputed one, we must return. (3) While admitting that most abstract names are non-connotative, since they merely signify an attribute and do not denote a subject, Mill maintains that some abstracts may justly be "considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes" (*Logic*, I. ch. 2, § 5). To this point also we will return in a later section.

(ii) The use of the word *connotative* does not seem to have been quite fixed with the Schoolmen. Mansel (*Aldrich*, p. 17) while admitting that there was some license in the use of the word, gives the following account on the authority of Occam. With the Schoolmen, a connotative term was one that "primarily signified an attribute, secondarily a subject"; and it was said to *connote* or *signify*

secondarily the subject. Thus "white" was regarded as connotative, whilst the original substances or attributes, as "man" or "whiteness" were called *absolute*; the former signifying primarily a subject, the latter not signifying a subject at all. Only adjectives and participles therefore (words called by Professor Fowler "attributives") are connotative in this sense.

Mill (*Logic*, I. p. 42, note) says that the Schoolmen used the term in his own sense, though some of their expressions are vague. He quotes James Mill as using it more nearly in the sense ascribed by Mansel to the Schoolmen.

(iii) Professor Fowler uses the term connotative in a sense different from that of Mill. "A term may be said to *denote* or designate individuals or groups of individuals, to *connote* or mean attributes or groups of attributes." In this sense, general names are both connotative and denotative; abstract names are connotative but not denotative[1], (whereas, according to Mill, they are generally speaking denotative but not connotative). This use of the term avoids some difficulties; but if it is adopted an explicit statement should be made to that effect. Careful attention to the divergence of usage will, at any rate, enable us to grasp Mill's meaning the more clearly.

13. Is every property possessed by a class connoted by the class-name?

This question suggests a possible ambiguity in the meaning of "connotation"; and there is in fact an ambiguity, inattention to which has given rise to misunderstanding and needless controversy.

Taking any general name, there are three points of view

[1] Fowler, *Deductive Logic*, p. 19.

from which we may regard the properties of the corresponding class :—

(1) There are those properties which are essential to the class in the sense that the name implies them. Were any of this set of properties absent the name would not be applicable. Any individual thing lacking them would therefore not be regarded as a member of the class. The standpoint here taken is in a sense *conventional*.

(2) There are those properties which in the mind of any given individual are associated with the name in such a way that they are normally called up in idea when the name is used. These properties will probably not exhaust the essential qualities of the class, while on the other hand they may include some that are not essential to it. The standpoint here taken is *subjective* and relative. Even when there is agreement as to the actual meaning of a name, the qualities that we naturally think of in connexion with it may vary both from individual to individual, and, in the case of any given individual, from time to time.

(3) There is the sum-total of properties actually possessed in common by every member of the class. These include all the essential qualities, and often many others not essential. The standpoint here taken is *objective*.

This classification suggests three possible meanings of connotation, and each of these different meanings has as a matter of fact been given to the term, sometimes without any clear recognition of divergence from the usage of other writers. It is of importance that we should be quite clear in our own minds in which sense we intend to employ the term.

(1) Mill would answer the question given at the head of this section in the negative, and in that answer I should myself concur.

By the connotation of a class-name then I do not mean all the properties that may be possessed in common by the class; nor do I mean those particular properties which are suggested to my mind by the name; but I mean just those properties on account of the possession of which any individual is placed in the class, or called by the name. In other words, I include in the connotation of a class-name only those attributes upon which the classification is founded, and in the absence of any of which the name would not be regarded as applicable. For example, although all equilateral triangles are equiangular I should not include equiangularity in the connotation of equilateral triangle; although all kangaroos may happen to be *Australian* kangaroos, this is not part of what we mean to *imply* when we use the name,—an animal subsequently found in the interior of New Guinea, but otherwise possessing all the properties of kangaroos would not have the name kangaroo denied to it; although all ruminant animals are cloven-hoofed, we cannot regard cloven-hoofed as part of the *meaning* of ruminant, and we may say with Mill that were an animal to be discovered which chews the cud, but has its feet undivided, it would certainly still be called ruminant.

(2) Some writers who regard proper names as connotative appear to include in the connotation of a name all those attributes which the name suggests to the mind, whether or not they are implied by the name. And it is here to be observed that a name may in the mind of any given individual be closely associated with properties which even the same individual would in no way regard as implied in the meaning of the name, as for instance, "Trinity undergraduate" with a blue gown. This interpretation of connotation therefore is clearly to be distinguished from that given in the preceding paragraph.

(3) Other writers use the term in still another sense and would answer the above question in the affirmative, including in the connotation of a class-name *all* the properties possessed in common by all members of the class. It is used in this sense by a writer in a recent number of *Mind*. "Just as the word 'man' denotes every creature, or class of creatures having the attributes of humanity, whether we know him or not, so does the word properly connote the *whole* of the properties common to the class, whether we know them or not. Many of the facts, known to physiologists and anatomists about the constitution of man's brain, for example, are not involved in most men's idea of the brain: the possession of a brain precisely so constituted does not, therefore, form any part of their meaning of the word 'man.' Yet surely this is properly connoted by the word....We have thus the denotation of the concrete name on the one side and its connotation on the other, occupying perfectly analogous positions. Given the connotation,—the denotation is all the objects that possess the whole of the properties so connoted. Given the denotation,—the connotation is the whole of the properties possessed in common by all the objects so denoted." (E. C. Benecke, in *Mind*, 1881, p. 532). Professor Jevons also uses the term in the same sense. "A term taken in intent (connotation) has for its meaning the whole infinite series of qualities and circumstances which a thing possesses. Of these qualities or circumstances some may be known and form the description or definition of the meaning; the infinite remainder are unknown" (*Pure Logic*, p. 4). Professor Bain appears to use the term in an intermediate sense, including in the connotation of a class-name not *all* the attributes common to the class but all the *independent* attributes, that is, all that cannot be derived or inferred from others.

Since we have three terms,—connotation, intension, and comprehension,—which are usually employed almost synonymously, I venture to suggest that they may be differentiated by having attached to them the above three meanings respectively. *Connotation* will then include only those attributes which are implied or signified by a name; *intension* will include those that are mentally associated with it, whether or not they are actually implied by it[1]; *comprehension* will include all the attributes possessed in common by all members of the class denoted by the name[2].

14. Are proper names connotative or non-connotative?

To this question absolutely contradictory answers are given by ordinarily clear thinkers as being obviously correct. To some extent however the divergence is merely verbal, the term connotation being used in different senses.

Mill speaks decisively,—"The only names of objects which connote nothing are *proper* names; and these have, strictly speaking, no signification" (*Logic*, I. p. 36). The opposite view is taken by Jevons, F. H. Bradley, and others.

In one or two places I am inclined to think that Jevons tends somewhat to obscure the point at issue. Thus with reference to Mill he says,—"Logicians have erroneously

[1] It is clear that in this sense intension may vary with each individual. How far connotation also may vary with each individual is discussed in section 16.

[2] These distinctions of meaning will I think be found useful in connexion with the questions whether proper names are connotative, and whether connotation and denotation necessarily vary inversely. Compare sections 14, 15, 18.

asserted, as it seems to me, that singular terms are devoid of meaning in intension, the fact being that they exceed all other terms in that kind of meaning" (*Principles of Science*, I. pp. 32, 33, with a reference to Mill in the foot-note). But Mill distinctly says that some singular names are connotative, *e.g.*, the sun, the first emperor of Rome (*Logic*, I. pp. 34, 35). Again, Jevons says,—"There would be an impossible breach of continuity in supposing that after narrowing the extension of 'thing' successively down to animal, vertebrate, mammalian, man, Englishman, educated at Cambridge, mathematician, great logician, and so forth, thus increasing the intension all the time, the single remaining step of adding Augustus de Morgan, Professor in University College, London, could remove all the connotation, instead of increasing it to the utmost point" (*Studies in Deductive Logic*, pp. 2, 3). But every one would allow that we may narrow down the extension of a term till it becomes individualised without destroying its connotation; "the present Professor of Pure Mathematics in University College, London" is a singular term,—we cannot diminish the extension any further,—but it is certainly connotative.

We must then clearly understand that the only controversy is with regard to what are strictly *proper* names. Even yet there is a source of ambiguity to be cleared up, namely, that which we have discussed in the preceding section. If by the connotation of a name we mean all the attributes possessed by the individuals denoted by the name, then Professor Jevons's view is correct[1]. This does appear to be what Jevons himself means, but it is distinctly *not* what Mill means,—he means only those attributes which are signified by the name. Jevons puts his case as follows:—

[1] It is also correct if by the connotation of a name we mean the attributes suggested by it. Compare the following section.

"Any proper name, such as John Smith, is almost without meaning until we know the John Smith in question. It is true that the name alone connotes the fact that he is a Teuton, and is a male; but, so soon as we know the exact individual it denotes, the name surely implies, also, the peculiar features, form, and character, of that individual. In fact, as it is only by the peculiar qualities, features, or circumstances of a thing, that we can ever recognise it, no name could have any fixed meaning unless we attached to it, mentally at least, such a definition of the kind of thing denoted by it, that we should know whether any given thing was denoted by it or not. If the name John Smith does not suggest to my mind the qualities of John Smith, how shall I know him when I meet him? For he certainly does not bear his name written upon his brow" (*Elementary Lessons in Logic*, p. 43). A wrong criterion of connotation in Mill's sense is here taken. The connotation of a name is not the quality or qualities by which I or any one else may happen to recognise the class which it denotes. For example, I may recognise an Englishman abroad by the cut of his clothes, or a Frenchman by his pronunciation, or a proctor by his bands, or a barrister by his wig; but I do not *mean* any of these things by these names, nor do they (in Mill's sense) form any part of the connotation of the names. Compare two such names as "John Duke Coleridge" and "the Lord Chief Justice of England." They denote the same individual, and I should recognise John Duke Coleridge, and the Lord Chief Justice of England by the same attributes; but the names are not equivalent,—the one is given as a mere mark of a certain individual to distinguish him from others, and it has no further signification; the other is given on account of the performance of certain functions, on the cessation of which the name would cease

to apply. Surely there is a distinction here, and one which it is important that we should not overlook.

Nor is it true that such a name as "John Smith" connotes "Teuton, male, &c." John Smith might be a race-horse, or a negro, or the pseudonym of a woman, as in the case of George Eliot. In none of these cases could a name be said to be misapplied as it would be if a horse were called a man, or a negro a Teuton, or a woman a male.

Still, it may fairly be said that in a certain sense many proper names do signify something, that at any rate they were chosen in the first instance for a special reason. For example, Strongi'th'arm, Smith, Jungfrau. But such names even if in a certain sense connotative when first imposed soon cease to be connotative in the way in which other names are connotative. Their application is in no way dependent on the continuance of the attribute with reference to which they were originally given. As Mill puts it, "*the name once given is independent of the reason.*" Thus, a man may in his youth have been strong, but we should not continue to call him strong when he is in his dotage; whilst the name Strongi'th'arm once given would not be taken from him. The name "Smith" may in the first instance have been given because a man plied a certain handicraft, but he would still be called by the same name if he changed his trade, and his descendants continue to be called Smith whatever their occupations may be. It cannot however be said that the name necessarily implies ancestors of the same name.

Proper names of course become connotative when they are used to designate a certain type of person; for example, a Diogenes, a Thomas, a Don Quixote, a Paul Pry, a Benedick, a Socrates. But, when so used, such names

have really ceased to be proper names at all; they have come to possess all the characteristics of general names.

15. Do proper names possess intension or comprehension?

If intension and comprehension are used as synonymous with connotation, then this question has been answered in the preceding section. But if we differentiate these terms and use them in the senses indicated in section 13, then we must say that while proper names have no connotation nevertheless they have both intension and comprehension. An individual object can be recognised only through its attributes; and a proper name when understood by me to be a mark of a certain individual undoubtedly suggests to my mind certain qualities. The qualities thus suggested by the name, (though not as I should still maintain signified or implied by it), constitute its intension. The comprehension of the name will include a good deal more than its intension, namely the whole of the properties which belong to the individual denoted.

16. Formal and Material treatment of Connotation.

When we speak of the connotation of a name we may have in view either the signification that the name bears in common acceptation, or some special meaning that a given individual may choose to assign to it. It has to be borne in mind that as a matter of fact different people may by the same name mean to imply different things, that is, the attributes they would include in the connotation of the name would be different; and not unfrequently some of us may be unable to say precisely what is the meaning that we ourselves attach to the words we use. In

Formal Logic, however, we work on the assumption that every name has a fixed and definite connotation. In other words, we assume that every name employed is either used in its ordinary sense and that this is precisely determined, or else that being used with a special meaning, this meaning is adhered to consistently and without equivocation. Formal Logic is indifferent to what particular connotation is attached to any given term; but it prescribes absolute consistency.

Mill in his treatment of connotation goes beyond this. He discusses the principles in accordance with which the connotation of names should be determined[1]. This is the treatment of the subject proper to Material or Applied Logic. When we define names already in use our object is to give them, that which Formal Logic assumes them to have, a fixed and definite connotation. It may be observed that in the case of an ideal language properly employed every name would have the same fixed and precise meaning for everyone.

17. Denotation and Extension.

In the application of these terms also there is a possible ambiguity. When we speak of the denotation or extension of a term we may be referring either (1) to the actually existing things of which the term can be predicated, or (2) to the sub-classes real or imaginary of which it can be predicated. In the first sense, the name of a non-existent class will have no denotation, but since every class can be formally divided into sub-classes, in this case also non-existent[2], it will have denotation in the second sense.

[1] *Logic*, Book i. chapter 8; Book iv. chapter 4.

[2] For example, the imaginary class X may be subdivided into the two imaginary classes, the X's that are A's and the X's that are not A's.

This distinction is of no great importance from the point of view of Formal Logic; still I think it might conduce to clearness if we differentiated the meanings of the terms denotation and extension, using the former in the first of the above senses and the latter in the second sense. For example, by the *denotation* of "Anglican bishop" we should then mean Dr Lightfoot, Dr Temple, and so on; under the *extension* of the term we should include such classes as "bishops who are members of the University of Cambridge," "bishops who are members of the University of Oxford," "bishops who are members of the University of Edinburgh," the last-named being possibly a non-existent class.

18. Relation between Connotation and Denotation.

In general, as connotation is increased denotation is diminished, and *vice versa*. If, for example, to the connotation of "substance" we add "corporeal," we clearly have a term with diminished denotation since all incorporeal substances are now excluded while they were formerly included. Hence the following law has been formulated: "In a series of common terms standing to one another in a relation of subordination[1] *the denotation and connotation vary inversely*." Is this law to be accepted? In the first place it is to be observed that the notion of inverse variation is at any rate not to be interpreted in any strict mathematical sense. It is certainly not true that whenever the number

[1] As in the *Tree of Porphyry*: Substance, Corporeal Substance (Body), Animate Body (Living Being), Sensitive Living Being (Animal), Rational Animal (Man). In this series of terms the connotation is at each step increased, and the denotation diminished.

of attributes taken into the connotation is doubled (or halved), then the number of individuals included in the denotation will be exactly halved (or doubled)[1]. There is in short no *regular* law of variation. What then we must understand the statement to mean is simply that the connotation of a name cannot be increased or diminished without the denotation being diminished or increased accordingly, and *vice versa*. We will discuss the statement in this form.

(1) It is not always true if we use *connotation* in Mill's sense as explained in section 13. Let the connotation of a name, *i.e.*, the attributes signified by it, be *a*, *b*, *c*. Now add *d* to the connotation. But it may happen that in fact wherever the attributes *a*, *b*, *c* are present, the attribute *d* is also present. If so, the alteration in the connotation of the name will have left its denotation unaffected[2]. As examples we may suppose "equiangularity" added to the connotation of "equilateral triangle," or "cloven-hoofed" added to the connotation of "ruminant." The point here indicated turns no doubt on material, and not on purely formal, considerations. It should be added that, apart from ma-

[1] Again, if to the connotation of a name we add different single attributes, the denotation will be affected in different degrees in different cases. If, for example, to the connotation of "member of the Senate of the University of Cambridge" we add "resident" its denotation will be reduced in a much greater degree than will be the case if we add "non-resident."

[2] Another statement sometimes made is that "of the denotation and connotation of a term one may, both cannot, be arbitrary." But again using "connotation" in Mill's sense this is not in all cases strictly true. There are certain cases in which although the denotation is fixed, the connotation is still arbitrary within certain limits. For example, given the denotation of "parallel straight lines," we nevertheless have a choice between two or three different definitions of such lines.

terial considerations, any alteration in the connotation of a term affects potentially its *extension*[1].

(2) Nor, using *intension* in the sense indicated in section 13, is it true that a change in intension necessitates a change in denotation; for the qualities which a name suggests to my mind will vary with my knowledge and experience, and yet the things denoted by the name may remain precisely what they were[2].

(3) Using *comprehension* however in the sense indicated in section 13, and *denotation* in the sense indicated in section 17, it is true that if comprehension be arbitrarily increased or diminished, denotation will be diminished or increased accordingly, and *vice versa*. Let the comprehension of a name be $p_1, p_2, \ldots p_n$; and the denotation of the same name $A_1, A_2, \ldots A_m$. Then p_1, &c. are all the attributes common to A_1, &c.; and A_1, &c. are all the individuals possessing p_1, &c. The truth of the above statement follows therefore immediately.

Of course we may discover fresh things possessing the comprehension in question, and the comprehension of the name will not be thereby affected. But in this case the denotation itself has not actually varied; only our knowledge of it has varied. Or we may discover fresh attributes previously overlooked; in which case similar remarks will apply. Again, new things may be brought into existence coming under the denotation of the name, and still its com-

[1] Using *extension* in the special sense indicated in the preceding section. In this sense we may regard the law of inverse variation as true of connotation (or intension) and extension. If, for example, the connotation (or intension) of a term X is a, b, c, and we add d; then the (real or imaginary) class of X's that are not d is necessarily excluded, while it was previously included, in the extension of the term X.

[2] But see preceding note.

prehension may remain the same. Or possibly new qualities may be developed by the whole of the class. In these cases however we have no "arbitrary" increase of denotation or connotation, and hence no real exception to the law as stated above.

19. Are any abstract names connotative?

A connotative name (in Mill's sense) is one which denotes a subject and implies an attribute; and (according to the definition in section 8) every name given to a class of things in virtue of some quality or set of qualities which they have in common is concrete. It follows that all connotative names are concrete, and that the above question must be answered in the negative[1]. Abstracts and concretes go in pairs. What is connoted by a concrete name is denoted by the corresponding abstract name, and there is nothing left for the latter to connote.

Mill takes a different view here. He holds that while most abstract names are non-connotative, still "even abstract names, though the names only of attributes, may in some instances be justly considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes" (*Logic*, I. p. 33). I have some difficulty in interpreting this passage, and I do not see how it is to be reconciled with Mill's further statement, —"the real signification of a concrete general name is its

[1] It should however be observed that in Professor Fowler's use of the term connotative, all abstract names are connotative, since they mean attributes, while none are denotative, that is, they do not denote individuals or groups of individuals. Professor Fowler himself admits that it sounds paradoxical to say that abstract names are not denotative, but he is of opinion that the employment of the terms connotative and denotative in his sense simplifies the statement and explanation of many logical difficulties.

connotation; and what the concrete term connotes, forms the entire meaning of the abstract name" (*Logic*, I. p. 118).

No doubt we may affirm that some particular attribute is always accompanied by some other attribute, as for example humanity by mortality; but a truly abstract name cannot be regarded as ever implying in its signification the possession of another attribute by the attribute that it denotes. The only example that Mill gives of a connotative abstract[1] is *fault*, which he says is a name common to many attributes, and which connotes hurtfulness, an attribute of those various attributes. But it may be doubted whether "fault" is really abstract at all. The true abstract appears to be *faultiness*[2]. "Fault" is at any rate not generally used as the name of a quality considered apart from the individuals possessing it; the name is rather that of some particular act or habit, as when we talk of committing a fault, or of excusing a fault and thereby making it worse[3].

[1] Mr Killick in his *Handbook of Mill's Logic* makes Mill include in the class of connotative names all those abstracts that are the names of groups of attributes (*e.g.*, *humanity*). I do not think that Mill himself intended this, nor do I think the view correct. If an abstract name has both denotation and connotation because it is the name of a group of attributes, on what principle shall we distinguish between the attributes that it denotes and those that it connotes?

[2] Compare Jevons, *Elementary Lessons in Logic*, p. 44.

[3] The question is perhaps rendered less difficult if we explicitly admit, as I think we ought, that particular qualities of particular objects may sometimes be included under the denotation of a concrete name. *Cause* and *source*, for example, are certainly concrete names; but we may say that a man's perseverance is the cause of his success or that his ability is a source of income to him. Similarly we may say that a man's unpunctuality is his only fault; but *fault* is not therefore abstract. It is only abstract if used to denote that attribute which is common to all faulty actions and objects; and, as already suggested, the term *faultiness* rather than *fault* is used in this sense.

20. Verbal and Real Propositions.

A *Verbal Proposition* is one which states only what is implied in the meaning of the words involved, or which gives information only with regard to the application of names; a proposition, for example, in which the connotation of the predicate is a part or the whole of the connotation of the subject[1]. Definitions constitute the most important class of verbal propositions, their essential function being to analyse the connotation of names[2]. Besides propositions giving such an analysis more or less complete, the following classes of propositions are to be included under the head of verbal propositions: where the subject and predicate are both proper names, *e.g.*, Tully is Cicero; where they are dictionary synonyms, *e.g.*, wealth is riches, a story is a tale, charity is love.

A *Real Proposition* is one which gives information of something more than the meaning or application of names; a proposition, for example, which predicates of a connotative subject some attribute not included in its connotation[3].

[1] Dr Bain describes the verbal proposition as "the notion under the guise of the proposition." A "contradiction in terms" is the antithesis of a verbal proposition; it is a statement that denies what is actually implied in the very meaning of the words involved.

[2] The importance of definitions is of course very considerable, and we ought not to speak of verbal propositions as being in all cases trivial. In general they are trivial only in so far as their true nature is misunderstood; when, for example, people waste time in pretending to prove what has already been assumed in the meaning they have assigned to the terms they employ.

[3] Very nearly the same distinction as that between verbal and real propositions is also expressed by the pairs of terms,—*analytic* and *synthetic*, *explicative* and *ampliative*, *essential* and *accidental*. These terms however apply only to propositions with connotative subjects. We should not say that such a proposition as "Tully is Cicero" is either analytic or synthetic, explicative or ampliative, essential or accidental.

Whether any given proposition is verbal or real will depend on the meaning which we attach to our terms; and since it is not the function of Formal Logic to discuss definitions, this science cannot attempt to determine under which category any given proposition should be placed. Still, while we cannot with certainty distinguish a verbal proposition by its form, it may be observed that the use of the words *some*, *all*, &c., in general indicate that in the view of the person laying down the proposition a fact is being stated and not merely a term explained. For example, in order to give a partially correct idea of the meaning of such a name as *square*, we should not say "all squares are four-sided figures," but "a square is a four-sided figure."

21. Are the propositions "Homer wrote the Iliad," "Milton wrote Paradise Lost," real or verbal?

This is a question that cannot be answered from the purely formal standpoint; but a brief consideration of it may help to illustrate the distinction between real and verbal propositions.

"Homer wrote the Iliad" is regarded by Dr Bain as a verbal predication. "We know nothing about Homer except the authorship of the Iliad. We have not a meaning to attach to the subject of the proposition, 'Homer', apart from the predicate, 'wrote the Iliad.' The affirmation is nothing more than that the author of the Iliad was called Homer" (*Logic*, *Deduction*, p. 67). But is it true that we attach nothing more to "Homer" than "wrote the Iliad"? Do we not, for example, attach to "Homer" the authorship of other poems, and also an individuality[1]? If it is the fact

[1] I do not of course mean that this is the connotation of "Homer," for I hold that no proper names are connotative. I mean that *Homer* denotes for me a certain individual who was a Greek, who lived prior

that the Iliad was the work of various authors, as has been asserted, would not the proposition become false? Some light may perhaps be thrown on the question here raised by an answer once given in an examination: "The accepted opinion is that the Iliad was not written by Homer, but by another man of the same name." The point at issue is really this: Does the proposition in question merely inform us that the author of the Iliad was named Homer, or does it tell us that a certain individual of whom we are able also to predicate other facts wrote this poem. "Milton wrote Paradise Lost" is undoubtedly a real proposition.

22. Formal Propositions.

There are propositions usually classed with verbal propositions which are really worth putting in a class by themselves, namely, those which are true whatever may be the meaning of the terms involved; *e.g.*, all A is A, No A is not-A, All A is either B or not-B, If all A is B then no not-B is A, If all A is B and all B is C then all A is C. These we may call *formal propositions*, since their validity is determined by their bare form[1].

We shall then have three classes of propositions,—formal, verbal, and real,—the validity or invalidity of which is determined respectively by their bare form, by the mere meaning or application of the terms involved, by questions of fact concerning the things denoted by these terms.

to a certain date, and who was the author of certain poems other than the Iliad.

[1] These are the only propositions whose truth is examined and guaranteed by Formal Logic itself irrespective of other sources of knowledge. In a sense they seem almost to coincide with the scope of Formal Logic; for any formally valid reasoning can be expressed by a formal hypothetical proposition as in the last two examples given above.

EXERCISES.

23. Enquire whether the following names are respectively connotative or non-connotative:—Caesar, Czar, Lord Beaconsfield, the highest mountain in Europe, Mont Blanc, the Weisshorn, Greenland, the Claimant, the pole star, Homer, a Daniel come to judgment.

24. If all x is y, and some x is z, and p is the name of those z's which are x; is it a verbal proposition to say that all p is y? [V.]

CHAPTER III.

POSITIVE AND NEGATIVE NAMES. RELATIVE NAMES.

25. Positive and Negative Terms.

The essential distinction between positive and negative names as ordinarily understood may be expressed as follows:—a *positive* name implies the *presence* of certain definite attributes, or, if non-connotative, denotes a particular person or thing,—*e.g.*, man, Socrates; a *negative* name implies the *absence* of one or other of certain definite attributes, or denotes everything with the exception of some particular person or thing,—*e.g.*, not-man, not-Socrates.

"Every name," as remarked by De Morgan, "applies to everything positively or negatively"; for example, everything either *is* or *is not* a horse. Every name then divides all things in the universe into two classes. Of one of these it is itself the name; and a corresponding name can be framed to denote the other. This pair of names, which between them denote the whole universe, are respectively positive and negative. But which is which? Which is the negative name, since each positively denotes a certain class of objects? The distinction lies in the manner in which the class is determined. A strictly negative name has its denotation determined indirectly. It denotes an indefinite

and unknown class outside a definite and limited class. In other words, we first mark off the class denoted by the positive name, and then the negative name denotes what is left. The fact that its denotation is thus determined is the distinctive characteristic of the negative name.

We have here supposed that between them the positive name and the corresponding negative name exhaust the whole universe. But something different from this is often meant by a negative name. Thus De Morgan considers that *parallel* and *alien* are negative names. "In the formation of language, a great many names are, as to their original signification, of a purely negative character: thus, parallels are only lines which do *not* meet, aliens are men who are *not* Britons (*i.e.*, in our country)" (*Formal Logic*, p. 37). But these names clearly have not the thorough-going negative character that I have just been ascribing to negative names. The difference will be found to consist in this, that in the sense in which *alien* is a negative name, the positive and negative names (Briton and alien) do not between them exhaust the entire universe, but only a limited universe, namely, in the given case, that constituted by the inhabitants of Great Britain. We may perhaps distinguish between names *absolutely negative*, where the reference is to the entire universe; and names *relatively negative*, where the reference is only to some restricted universe.

Now it will be seen that in the use of such a term as *not-white* there is a possible ambiguity; we must decide whether in any given instance the name is to be regarded as absolutely or only as relatively negative. Mill chooses the former alternative; "not-white," he says, "denotes all things whatever except white things." De Morgan and Bain, on the other hand, consider that in such a case the reference is not to the whole universe but to some particular

universe only. Thus, in contrasting white and not-white we are referring solely to the universe of colour; *not-white* does not include everything in nature except white things, but only things that are black, red, green, yellow, &c., that is, all *coloured* things except such as are white[1]. Whately and Jevons agree with Mill; and from a logical point of view I think they are right. Or rather I would say that two such terms as S and not-S must between them exhaust the *universe of discourse*, whatever that may be; and we must not be precluded from making this, if we care to do so, the entire universe of existence. That is, not-S *may be* called upon to assume the absolutely negative character.[2] For if we are unable to denote by not-S all things whatsoever except S, it is difficult to see in what way we shall be able to denote these when we have occasion to refer to them. On the other hand, we must also be empowered to indicate a limitation to a particular universe where that is intended. By not-S then, referred to without qualification expressed or implied by the context, I would understand the absolute negative of S; but I should be quite prepared to find a limitation to some more restricted universe in any particular instance.

It should be noted that in the case of a limited universe it is sometimes difficult to say which of the pair of contrasted names is really to be regarded as the negative name. For example, De Morgan says that *parallel* is a negative name, since parallel lines are simply lines in the

[1] On Bain's view, therefore, it would be incorrect to say that an immaterial entity such as honesty was not-white.

[2] On this view, "not-white" might be used to denote not merely coloured things that are not white, but also things that are not coloured at all. It would for example be correct to say that honesty was not-white.

same plane that do not meet. But we might also define them as lines such that if another line be drawn cutting them both, the alternate angles are equal to one another; and then the name appears as a positive name. Similarly in the universe of property, as pointed out by De Morgan, *personal* and *real* are respectively the negatives of each other; but if we are to call one positive and the other negative, it is not quite clear which should be which.

26. Privative Names.

To the distinction between positive and negative names, Mill adds a class of names called *privative*. "A privative name is equivalent in its signification to a positive and a negative name taken together; being the name of something which has once had a particular attribute, or for some other reason might have been expected to have it, but which has it not. Such is the word *blind*, which is not equivalent to *not seeing*, or to *not capable of seeing*, for it would not, except by a poetical or rhetorical figure, be applied to stocks and stones" (*Logic*, I. p. 44). Perhaps also *idle*, which Mill gives as a negative, should rather be regarded as a privative term. It does not mean merely "not-working," but "not-working where there is the capacity to work." We should hardly speak of a stone as being "idle."

The distinction here indicated does not appear to be of logical importance[1].

27. Contradictory Terms.

A positive term and the corresponding negative term are called *contradictories*. A pair of contradictory terms

[1] It may be added that by some logicians the term *privative* is used as simply equivalent to *negative*.

are so related that between them they exhaust the entire universe to which reference is made, whilst in that universe there is no individual of which both can be at the same time affirmed. The nature of this relation is expressed in the two laws of Contradiction and Excluded Middle. Nothing is at the same time both X and not-X; Everything is X or not-X.

28. Contrary Terms.

The *contrary*[1] of a term is usually defined as the term denoting that which is furthest removed from the original term in some particular universe; *e.g.*, black and white, wise and foolish. Two contraries may in some cases happen to make up between them the whole of the universe in question, *e.g.*, Briton and alien; but this is not necessary, *e.g.*, black and white. It follows that although two contraries cannot both be true of the same thing at the same time, they may both be false. The contrary of a positive term will in general be another positive term.

The above may be called the *material* contrary. In the case of complex terms, we may also assign a *formal* contrary, as will be shewn later on.

29. Names positive in form but negative in reality, and *vice versa*.

Mill points out that "names which are positive in form are often negative in reality, and others are really positive though their form is negative." The fact that a really

[1] De Morgan uses the terms contrary and contradictory as equivalent, his definition of them corresponding to that given in the preceding section. Hamilton identifies contrariety with simple incompatibility.

positive term may be negative in form results from the circumstance that the negative prefix is sometimes given to the contrary of a term. But we have seen that a term and its contrary may both be positive. For example, pleasant and unpleasant; "the word *unpleasant*, notwithstanding its negative form, does not connote the mere absence of pleasantness, but a less degree of what is signified by the word *painful*, which, it is hardly necessary to say, is positive." That "pleasant" and "unpleasant" are not contradictories follows from the fact that they admit of a mean, namely that which is "indifferent." On the other hand, some names positive in form may be regarded as relatively negative, *e.g.*, parallel, alien. But I do not think that an absolutely negative name can be found that is positive in form.

From the standpoint of Formal Logic, however, it does not much matter whether any given term is positive or negative. What the formal logician is really concerned with is the relation between contradictory terms. Not-S is the contradictory of S, and S is the contradictory of not-S, whichever of the terms may be more strictly the positive and the negative respectively.

Mr Monck, in his valuable *Introduction to Logic*, p. 104, suggests that it might be "better to define a Negative term as a term negative in form, (*i.e.*, a term in which 'non,' 'un,' 'in,' 'mis,' or some other negative particle occurs)." A possible drawback to this definition is that it might lead to a confusion between contradictory and contrary terms.

30. Infinite or Indefinite Terms.

Infinite and *indefinite* are designations applied to terms having a thoroughgoing negative character; to such a term for example as "not-white," understood as denoting not

merely coloured things other than white, but the whole infinite or indefinite class of things of which "white" cannot truly be affirmed, including such entities as Mill's *Logic*, a dream, Time, a soliloquy, New Guinea, the Seven Ages of Man.

It is however to be observed that if symbols are used, it is impossible to say which of the terms S or not-S really partakes of this indefinite character, since, for example, there is nothing to prevent our having originally written S for "not-white," in which case "white" becomes not-S, and S is the really *indefinite* or *infinite* term. The distinction indicated depends therefore on *material* rather than on *formal* considerations.

31. Relative Names.

A name is *relative*, when, over and above the object which it denotes, it implies in its signification another object, to which in explaining its meaning reference must explicitly or implicitly be made. The name of this other object is called the *correlative* of the first. Non-relative names are sometimes called *absolute*.

Jevons considers that all terms are in one sense relative. By the law of relativity, consciousness is possible only under circumstances of change. Every term therefore implies its negative as an object of thought. Take the term *man*. It is an ambiguous term, and in many of its meanings is clearly relative,—for example, as opposed to master, to officer, to wife, to boy. If in any sense it is absolute it is when opposed to not-man; but even in this case it may be said to be relative to not-man. To avoid this difficulty, Jevons remarks, "Logicians have been content to consider as relative terms those only which imply some peculiar and striking kind of relation arising from position in time or

space, from connexion of cause and effect, &c.; and it is in this special sense therefore that the student must use the distinction."

I doubt, however, whether every name can be said to imply its negative *in its signification*. Granted that all *notions* are relative, it hardly seems to follow that all *terms* are relative in the sense defined above. The matter is of no great importance, and at any rate the difficulty might be avoided by defining a relative term as one which implies in its signification the existence of another object, *other than its mere negation*.

The fact or facts constituting the ground of both correlative names is called the *fundamentum relationis*. For example, in the case of partner, the fact of partnership; in the case of husband and wife, the facts which constitute the marriage tie; in the case of shepherd and sheep, the acts of tending and watching which the former exercises over the latter.

Sometimes the relation which each correlative bears to the other is the same; for example, in the case of partner, where the correlative name is the same name over again. Sometimes it is not the same; for example, father and son, husband and wife.

EXERCISES.

32. Discuss the logical characteristics of the following terms:—beauty, immortal, slave, England, feeling of pride, friendship, law, sovereign, the Times, the Arabian Nights, George Eliot, Mrs Grundy, Vanity Fair, sleep, truth, attitude, ungenerous, nobility, treason, fault.

[In discussing the character of any term it is necessary first of all to determine whether it is *univocal*, that is, used in one definite sense only, or *equivocal* (or *ambiguous*), that

is, used in more senses than one. In the latter case, its logical characteristics may of course vary according to the sense in which it is used.]

33. Give one example of each of the following,—(i) a collective general name, (ii) a singular abstract name, (iii) a connotative abstract name, (iv) a connotative singular name; or, if you deny the possibility of any of these combinations, state clearly your reasons.

PART II.

PROPOSITIONS.

CHAPTER I.

KINDS OF PROPOSITIONS. THE QUANTITY AND QUALITY OF PROPOSITIONS.

34. Categorical, Hypothetical, and Disjunctive Propositions.

A *Proposition* may be defined as a sentence indicative or declaratory (as distinguished, for example, from sentences imperative or exclamatory); in other words, a proposition is a sentence making an affirmation or denial, as—All *S* is *P*, No vicious man is happy. It is the verbal expression of a judgment.

A proposition is *Categorical* if the affirmation or denial is absolute, as in the above examples. It is *Hypothetical* or *Conditional* if made under a condition, as—If *A* is *B*, *C* is *D*; Where ignorance is bliss, 'tis folly to be wise. It is *Disjunctive* if made with an alternative, as—Either *P* is *Q*, or *X* is *Y*; He is either a knave or a fool[1].

[1] Whether or not disjunctives can be reduced to the hypothetical form, or hypotheticals to the categorical form, will be discussed later on.

[The above threefold division is called by Kant the division of judgments according to *Relation*. Some logicians commence with a twofold division, the second member of which is again subdivided, the term *hypothetical* being employed sometimes in a wider and sometimes in a narrower sense. To prevent confusion, it may be helpful to give the following table of the usage of one or two modern logicians with regard to this division.

Whately, Mill and Bain:—

1. Categorical.
2. Hypothetical, or Compound, or Complex. { (1) Conditional. (2) Disjunctive.

Hamilton and Thomson:—

1. Categorical.
2. Conditional. { (1) Hypothetical. (2) Disjunctive.

Fowler (following Boethius):—

1. Categorical.
2. Conditional or Hypothetical. { (1) Conjunctive. (2) Disjunctive.

Mansel gives at once the threefold division:—

1. Categorical.
2. Hypothetical or Conditional.
3. Disjunctive.

He states his reasons for his own choice of terms as follows:—"Nothing can be more clumsy than the employment of the word *conditional* in a specific sense, while its Greek equivalent, *hypothetical*, is used generically. In Boethius, both terms are properly used as synonymous, and generic; the two species being called *conjunctivi*, *conjuncti*,

or *connexi*, and *disjunctivi* or *disjuncti*. With reference to modern usage, however, it will be better to contract the Greek word than to extend the Latin one. *Hypothetical* in the following notes, will be used as synonymous with *conditional*" (Mansel's edition of *Aldrich*, p. 103). A distinction between *Conditionals* and *Hypotheticals*, differing from all the above, will be suggested in a later section.]

35. An analysis of the Categorical Proposition.

The categorical proposition consists of two terms united by a copula.

The *subject* is that term about which affirmation or denial is made.

The *predicate* is that term which is affirmed or denied of the subject.

The *copula* is the link of connexion between the subject and the predicate, and consists of the words *is* or *is not* according as we affirm or deny the latter of the former.

In attempting to apply the above analysis to such a proposition as "All that love virtue love angling," we find that, as it stands, the copula is not separately expressed. It may however be written,—

subj.	cop.	pred.
All lovers of virtue	are	lovers of angling;

and in this form the three different elements of the proposition are made distinct. An analysis of this kind is useful in the case of any proposition that may at first present itself in an abnormal form. A difficulty that may sometimes arise in discriminating the subject and the predicate is dealt with subsequently,—see section 48.

The older logicians distinguished propositions *secundi adjacentis*, and propositions *tertii adjacentis*. In the former,

the copula and the predicate are not separated; *e.g.*, The man runs, All that love virtue love angling. In the latter, the copula and the predicate are made distinct; *e.g.*, The man is running, All lovers of virtue are lovers of angling.

To complete our analysis we ought to note a fourth element in the categorical proposition, namely, the sign of quantity attached to the subject. In the proposition *All S is P* that sign of quantity is *all*, and we accordingly understand the affirmation to be made of each and every individual denoted by the term *S*.

36. The Quantity and Quality of Propositions[1].

The *Quality* of a proposition is determined by the copula, being *affirmative* or *negative* according as the copula is of the form "is" or "is not."

Propositions are also divided into *universal* and *particular*[2], according as the affirmation or denial is made of the whole or of a part of the subject. This division of Propositions is said to be according to their *Quantity*.

Combining the two principles of division, we get four fundamental forms of propositions:—

(1) the *universal affirmative*, All *S* is *P*, (or Every *S* is *P*, or Any *S* is *P*, or All *S*'s are *P*'s), usually denoted by the symbol **A**;

(2) the *particular affirmative*, Some *S* is *P*, (or Some *S*'s are *P*'s), usually denoted by the symbol **I**;

(3) the *universal negative*, No *S* is *P*, (or No *S*'s are *P*'s), usually denoted by the symbol **E**;

[1] We may say that the subject and predicate of a proposition constitute its *matter*; while its quantity and quality constitute its *form*.

[2] Instead of these terms Professor Bain suggests the terms *total* and *partial*.

(4) the *particular negative*, Some S is not P, (or Not all S is P, or Some S's are not P's, or Not all S's are P's), usually denoted by the symbol **O**.

These symbols A, I, and E, O, are taken from the Latin words *affirmo* and *nego*, the affirmative symbols being the first two vowels of the former, and the negative symbols the two vowels of the latter.

Besides these symbols, it will also be found convenient sometimes to use the following,—

$SaP =$ All S is P;
$SiP =$ Some S is P;
$SeP =$ No S is P;
$SoP =$ Some S is not P.

The above are useful when we wish that the symbol which is used to denote the proposition as a whole should also indicate what symbols have been chosen for the subject and the predicate respectively. Thus,

$MaP =$ All M is P;
$PoQ =$ Some P is not Q.

The universal negative should not be written in the form *All S is not P*; for this form is ambiguous and would usually be understood to be merely particular. Thus, "All that glitters is not gold" is really an **O** proposition, and is equivalent to "Some things that glitter are not gold."

37. Exponible, Copulative, Remotive, and Exceptive Propositions.

Propositions that are resolvable into more propositions than one have been called *exponible*, in consequence of their susceptibility of analysis. *Copulative* propositions are formed by a direct combination of simple affirmative propositions;

e.g., All P is both Q and R, (*i. e.*, All P is Q, All P is R). *Remotive* propositions are formed by a similar combination of negatives; *e.g.*, No P is either Q or R, (*i.e.*, No P is Q, No P is R). Copulatives and remotives fall within the class of exponibles. *Exceptive* propositions limit the subject by such a word as "unless" or "except"; *e.g.*, All P is Q, unless it happens to be R. Exceptives may be regarded as forming another class of exponibles[1].

38. Exclusive Propositions.

Cont with C- who reis[?] them to E

Exclusive propositions contain some such word as *only* or *alone* thereby limiting the predicate to the subject; *e.g.*, *Only S is P.* Propositions of this kind may be written in the form *Some S is all P*[2]; but this is not one of the forms recognised in the traditional scheme as given in section 36[3]. In order to deal with exclusives under the traditional scheme it is necessary to replace them by one of the equivalent forms,—*All P is S, No not-S is P.* But it has to be observed that this is not very satisfactory. We have not kept the original subject and predicate, and have in truth performed upon the given proposition a process of immediate inference.

[1] Only on the supposition however that the above proposition implies not merely that if P is not R then it is Q, but also that if P is R then it is not Q. Thus interpreted it is equivalent to the following,—All P is either Q or R, but no P is both of these.

[2] It is to be observed that the exclusive proposition "Only S is P" does not necessarily imply that *all* S is P, though it does imply that all P is S; *e.g.*, "Only graduates are eligible" does not imply that all graduates are so, for some may be disqualified for other reasons.

[3] For Sir William Hamilton's scheme of propositions in which "Some S is all P" does receive distinctive recognition, see Part III., Chapter 9.

39. Indefinite Propositions.

According to Quantity, Propositions have sometimes been divided into (1) Universal, (2) Particular, (3) Singular, (4) Indefinite. Singular propositions are discussed in the following section.

By an *indefinite* proposition is meant one "in which the Quantity is not explicitly declared by one of the designatory terms *all*, *every*, *some*, *many*, &c."; *e.g.*, S is P, Cretans are liars. We may perhaps say with Hamilton that *indesignate* or *preindesignate* would be a better term to employ. There can be no doubt that, as Mansel remarks, "the true indefinite proposition is in fact the particular; the statement 'some *A* is *B*' being applicable to an uncertain number of instances, from the whole class down to any portion of it. For this reason particular propositions were called indefinite by Theophrastus" (*Aldrich*, p. 49).

When a proposition is given in the indesignate form, we can generally tell from our knowledge of the subject matter or from the context whether it is meant to be universal or particular. Probably indesignate propositions are in general intended to be understood as universals[1], *e.g.*, Comets are subject to the law of gravitation; but if we are really in doubt with regard to the quantity of the proposition it must logically be regarded as particular.

Other designations of quantity besides *all* and *some*, *e.g.*, *most*, are discussed in section 41.

40. Singular Propositions.

By a *singular* or *individual* proposition is meant a proposition in which the affirmation or denial is made of a

[1] And I do not think that any confusion would result from the understanding that this should be their logical interpretation.

single individual only; *e.g.*, Brutus is an honourable man; Much Ado about Nothing is a play of Shakespeare's; My boat is on the shore.

Singular propositions may usually be regarded as forming a sub-class of universals, since in every singular proposition the affirmation or denial is of the *whole* of the subject. Such propositions have however certain peculiarities of their own, as we shall note subsequently; *e.g.*, they have not like other universal propositions a contrary distinct from their contradictory[1].

Hamilton distinguishes between Universal and Singular Propositions, the predication being in the former case of a *Whole Undivided*, and in the latter case of a *Unit Indivisible.* This separation is sometimes useful; but I think it better not to make it absolute. A singular proposition may generally without risk of confusion be denoted by one of the symbols **A** or **E**; and in syllogistic inferences, a singular may ordinarily be treated as equivalent to a universal proposition. The use of independent symbols for affirmative and negative singular propositions would introduce considerable additional complexity into the treatment of the Syllogism; and for this reason it seems desirable as a rule to include singulars under universals. We may however divide universal propositions into *general*[2] and *singular*, and we

[1] It may also be held that they imply the existence of their subjects while this is not the case with ordinary universal propositions. Cf. section 107.

[2] Lotze (*Logic*, § 68) distinguishes between *general* and *universal* judgments. In the former the predication is of the whole of an indefinite class, including both examined and unexamined cases. In the latter we have merely a summation of what is found to be true in every individual instance of the subject. "The universal judgment is only a collection of many singular judgments, the sum of whose subjects does as a matter of fact fill up the whole extent of the universal concept;...the universal

shall then have terms whereby to call attention to the distinction wherever it may be necessary or useful to do so.

There is also a certain class of propositions which, while *singular* inasmuch as they relate but to a single individual, possess also the indefinite character which belongs to the *particular* proposition: for example, A certain man had two sons; A great statesman was present; An English officer was killed. Having two such propositions in the same discourse we cannot, apart from the context, be sure that the same individual is referred to in both cases. Carrying the distinction indicated in the preceding paragraph a little further, Mr W. E. Johnson suggests a fourfold division of propositions:—*general universal*, "All *S* is *P*"; *general particular*, "Some *S* is *P*"; *singular universal*, "This *S* is *P*"; *singular particular*, "A certain *S* is *P*." This classification admits of our working with the ordinary two-fold distinction into universal and particular wherever this is adequate, as in the traditional doctrine of the syllogism; at the same time it introduces a further distinction which in certain connexions is of considerable importance.

proposition, 'all men are mortal,' leaves it still an open question whether, strictly speaking, they *might* not all live for ever, and whether it is not merely a remarkable concatenation of circumstances, different in every different case, which finally results in the fact that no one remains alive. The general judgment on the other hand, 'man is mortal,' asserts by its form that it lies in the character of mankind that mortality is inseparable from every one who partakes in it." In Applied Logic the distinction here indicated may be of importance; a somewhat similar distinction is indicated by Mill in his treatment of "inductions improperly so-called." But it cannot be regarded as a *formal* distinction; it depends not so much on the propositions themselves as on the manner in which they are obtained. I cannot agree with Lotze's implication that propositions of the form "all *S* is *P*" are always in his sense universal, while those of the form "*S* is *P*" are always in his sense general.

There are propositions of another kind with a singular term for subject which call for one or two remarks, namely, such propositions as, "Browning is sometimes obscure," "That boy is sometimes first in his class." These propositions may be treated as universal with a somewhat complex predicate, (and it should be noted that in bringing propositions into the usually recognised logical forms very complex predicates are sometimes necessary); thus, Browning is a poet who is sometimes obscure, That boy is a boy who is sometimes first in his class. By a certain transformation the propositions may also be dealt with as particulars, and such transformation may in certain cases be convenient; thus, Some of Browning's writings are obscure, Some of the boy's places in his class are the first places. But when the proposition is thus modified, the subject is no longer a singular term.

41. The logical signification of the words *some, most, few, all, any.*

Some may mean merely "some at least," *i.e.*, not none, or it may carry the further implication, "some at most," *i.e.*, not all. In ordinary speech the latter meaning is probably the more usual. But this is generally regarded as one of the implications or suggestions of ordinary speech that Logic cannot recognise; and accordingly with most modern logicians the logical interpretation of *some* is limited to *some at least.* Using the word in this sense, if we want to express "some, but not all, S is P," we must make use of two propositions,

Some S is P,
Some S is not P.

The particular proposition, thus interpreted, is not ex-

clusive of the universal. As already suggested, it is indefinite, though with a certain limit; that is, it is indefinite so far that it may apply to any number from a single one up to all, but on the other hand it is definite in so far as it excludes "none."

It may be added that in regarding "some" as implying no more than *at least one*, we are probably again departing from the ordinary usage of language, which would regard it as implying *at least two*[1].

[It should perhaps be noted that on rare occasions "some" may have a slightly different implication. For example, the proposition "Some truth is better kept to oneself" may be so emphasized as to make it perfectly clear to what particular kind of truth reference is made. This is however extra-logical. Logically the proposition must be treated as particular, or it must be written in another form, "All truth of a certain specified kind is better kept to oneself." Thus, Spalding remarks (*Logic*, p. 63), "The logical 'some' is totally indeterminate in its reference to the constitutive objects. It is always *aliqui*, never *quidam;* it designates some objects or other of the class, not some certain objects definitely pointed out."]

Most is to be interpreted "at least one more than half." *Few* has a negative force; and "Few S are P" may be regarded as equivalent to "Most S are not P"; (with perhaps the further implication "although *some* S are P"; thus Few S are P is given by Kant as an example of the *exponible* proposition, on the ground that it contains both an affirmation and a negation, though one of them in a concealed way)[2]. Formal logicians (excepting De Morgan and

[1] Cf. Mansel's *Aldrich*, p. 59.

[2] The proposition "Few S are P" may also be interpreted in a slightly different way, as meaning that an absolutely small number of S's are P's. It can then be written in the form, "The number of S's which are P's

Hamilton) have not as a rule recognised these additional signs of quantity; and it is true that in many logical combinations we are unable to regard them as more than particular propositions, Most S are P being reduced to Some S are P, and Few S are P to Some S are not P. Sometimes however we are able to make use of the extra knowledge given us; *e.g.*, from Most M are P, Most M are S, we can infer Some S are P, although from Some M are P, Some M are S, we can infer nothing.

Propositions of the forms "Most S are P" and "Few S are P" are called *plurative* propositions. *Numerically definite* propositions are those in which a predication is made of some definite proportion of a class; *e.g.*, "Two thirds of S are P."

It is to be observed that *a few* has not the same signification as *few*, but must be regarded as affirmative, and, generally, as simply equivalent to *some*; *e.g.*, A few S are P = Some S are P. Sometimes, however, it means "a small number," and in this case the proposition is perhaps best regarded as singular, the subject being collective. Thus, "a few peasants successfully defended the citadel" may be rendered "a small band of peasants successfully defended the citadel," rather than "some peasants successfully defended the citadel," since the stress is intended to be laid at least as much on the paucity of their numbers as on the fact that they were peasants[1].

It may here be remarked that in all cases, where we are dealing with propositions which as originally stated are not

is small." If however we recognise *few* as a logical sign of quantity, it is necessary to give it a fixed interpretation, and on the whole I prefer the interpretation indicated in the text.

[1] Whilst the proposition is singular, not general, it is to be regarded as a singular particular, not a singular universal; for what small band is alluded to is left indeterminate.

in a form ordinarily recognised by Logic, the first problem in reducing them to logical form is one of interpretation, and we must not be surprised to find that in many cases different methods of interpretation lead to different results. No confusion will ensue if we make it perfectly clear what we do regard as the logical form of the proposition, and also how we have arrived at our result.

All is ambiguous, so far as it may be used either distributively or collectively. In the proposition "All the angles of a triangle are less than two right angles" it is used distributively, the predicate applying to each and every angle of a triangle taken separately. In the proposition "All the angles of a triangle are equal to two right angles" it is used collectively, the predicate applying to all the angles taken together, and not to each separately[1].

Any as the sign of quantity of the subject of a categorical proposition, (*e.g.*, any S is P), is logically equivalent to "all" in its distributive sense. Whatever is true of any member of a class taken at random is necessarily true of the whole of that class. When not the subject of a categorical proposition, *any* may have a different signification. For example, in the hypothetical proposition, "If any A is B, C is D," it has the same indefinite character which we logically ascribe to *some*[2]; since the antecedent condition

[1] This ambiguity attaches to the symbolic form *All S is P*, but not to the form *All S's are P's*. In general, *all* is interpreted distributively, unless by the context or in some other way an indication is given to the contrary.

[2] It appears to have this meaning (*a*) in the principal clause of an interrogative sentence, *e.g.*, Are any subscribers dissatisfied because some non-subscribers were admitted? (*b*) in the subordinate clause of a negative sentence, *e.g.*, Some people do not think that any men are perfect; (*c*) in the antecedent clause of a hypothetical sentence, *e.g.*, If any men are perfect, some men are mistaken.

is satisfied if a single *A* is *B*. The proposition might indeed be written,—"If one or more *A* is *B*, *C* is *D*."

42. Infinite or Limitative Propositions.

Following out the idea indicated in section 30, judgments were divided by Kant into three classes in respect of Quality, namely, *affirmative*—*S* is *P*, *negative*—*S* is not *P*, and *infinite* (or *limitative*)—*S* is not-*P*[1]. Logically, however, the last judgment (which is equivalent to the second in meaning) must be regarded as simply affirmative. As shewn in section 30, it is impossible to say which of the terms *P* or not-*P* is really infinite; and it is therefore also impossible to say which of the propositions "*S* is *P*" or "*S* is not-*P*" is really infinite or limitative. Hence they must be regarded as belonging to the same type of proposition, and we have to fall back upon the two-fold division into affirmative and negative.

43. Conditional Propositions and Hypothetical Propositions.

Propositions commonly written in the form "If *A* is *B*, *C* is *D*," belong to two very different types. *A* being *B* and *C* being *D* may be two events or two combinations of properties, concerning which it is affirmed that whenever the first occurs the second will occur also. For example,—If the barometer falls, rain is sure to follow; If a child is spoilt, his parents suffer; If a straight line falling upon two other straight lines makes the alternate angles equal to one another, these two straight lines shall be parallel; If an import duty is a source of revenue, it does

[1] An infinite judgment may be described as the affirmative predication of a negative.

not afford protection; Where the carcase is, there shall the eagles be gathered together. It is proposed to call propositions of this kind *conditional*. But again, "*A* is *B*" and "*C* is *D*" may be two propositions the relation between which cannot be resolved into any time or space relation or into an affirmation of the co-inherence of attributes in a common subject. What is affirmed may be purely a relation of dependence between two truths. For example,—If the general line of argument in the *Wealth of Nations* is sound, the Mercantile Theory is exploded; If there is a righteous God, the wicked will not escape their just punishment; If virtue is involuntary, so is vice. It is proposed to call propositions of this kind *hypothetical* as distinguished from conditional. Wherever there is danger of misapprehension we may speak of them as *true hypotheticals* or *pure hypotheticals*[1].

We cannot formally distinguish between conditionals and hypotheticals so long as we keep to the expression "If *A* is *B*, *C* is *D*," since this may be either the one or the other. Distinctively, the conditional proposition may be written,—*If ever A is B, then in all such cases C is D*, or *Whenever A is B, C is D*, or *In all cases in which A is B, C is D*. These forms are not ambiguous. The distinctive form for the pure hypothetical will be,—*If S is true, P is true*.

The parts of the conditional and also of the true hypothetical proposition are called the *antecedent* and the *consequent*. Thus in the proposition "If *A* is *B*, *C* is *D*," the antecedent is "*A* is *B*," the consequent is "*C* is *D*."

[1] For the above distinction I am indebted to an essay (at present unpublished) by Mr W. E. Johnson. Compare, further, Part II. chapter 9.

44. Can distinctions of Quality or Quantity be applied to Conditional, Hypothetical, or Disjunctive Propositions?

(1) *Conditional Propositions.*

In this case the ordinary distinctions both of Quality and Quantity can be applied; and we may regard the Quality of the Conditional Proposition as depending upon the Quality of the Consequent. Thus the proposition "If *A* is *B*, *C* is not *D*," may be considered negative[1]. Conditional propositions may also be regarded as Universal or Particular, according as the consequent is affirmed to accompany the antecedent in all or merely in some cases. We have then the four fundamental types of propositions:—

If *A* is *B*, *C* is *D*.	**A.**
Sometimes if *A* is *B*, *C* is *D*.	**I.**
If *A* is *B*, *C* is not *D*.	**E.**
Sometimes if *A* is *B*, *C* is not *D*.	**O.**

As in the case of categoricals, we may give a separate recognition to *singular* conditionals. We have a singular conditional where the event referred to in the antecedent is one which necessarily can happen but once; *e.g.*, When the Emperor William dies, the peace of Europe will be endangered.

(2) *Hypothetical Propositions.*

Here again we have a distinction corresponding to that between affirmation and negation in the case of categoricals.

[1] But in order fully to bring out the negative force of this proposition it should be written in the form, "If ever *A* is *B*, then it is not also the case that *C* is *D*."

The following may be regarded as respectively an affirmative and a negative hypothetical: If *S* is true, *P* is true; If *S* is true, *P* is false.

But we cannot now recognise any distinctions of Quantity. The antecedent of a true hypothetical is not an event which may recur an indefinite number of times, it is a proposition which is simply true or false. We may therefore say that all true hypotheticals are *singular*.

The student must be warned against treating such a proposition as "If any *A* is *B*, some *C* is *D*" as particular[1]. Regarded separately the antecedent and the consequent in this example are both particular; but the connexion between them is not one that is affirmed to hold good merely in some cases[2].

[1] I cannot agree with Hamilton (*Logic*, i. p. 248), in regarding the following as a particular hypothetical,—If some Dodo is, then some animal is. The proposition is a little hard to interpret, but it seems to mean that if there is such a thing as a Dodo, then there is such a thing as an animal; and we must consider that a universal connexion is here affirmed.

[2] It is however to be observed that in the given proposition there is a certain ambiguity. We may say definitely that it is not particular; but it may be either a universal conditional, or a true hypothetical and therefore singular. Compare, for example, the following: If any phenomenon is out of the common, some wiseacre will proclaim it to be ominous; If any phenomena are supernatural, some philosophers are mistaken. The first of these might be written,—Whenever any phenomenon is out of the common, some wiseacre will proclaim it to be ominous. This proposition is obviously universal as distinguished from particular, and general as distinguished from singular. The second of the above propositions might be written,—If it is true that any phenomena are supernatural, then it is true that some philosophers are mistaken. This proposition is obviously singular. There is no reference to distinctions of time, and the antecedent is simply either true or false. We cannot with any meaning talk of its being true *in all cases* or *in some cases* that some phenomena are supernatural.

(3) *Disjunctive Propositions.*

To disjunctive propositions as such we are unable to apply distinctions of Quality. The proposition, *P* is neither *Q* nor *R*, states no alternative, and is therefore not disjunctive at all. Distinctions of Quantity are however still applicable in cases in which it is *a priori* possible with regard to each alternative that it may sometimes hold good and sometimes not[1]. Thus,

Universal,—In all cases either *P* is *Q* or *X* is *Y*;
Particular,—In some cases either *P* is *Q* or *X* is *Y*.

45. The Distribution of Terms in a Proposition.

A term is said to be distributed when reference is made to *all* the individuals denoted by it; it is said to be undistributed when they are only referred to *partially*, *i.e.*, information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class. It follows immediately from this definition that the subject is distributed in a universal, and undistributed in a particular[2], proposition. It can further be shewn that the predicate is distributed in

[1] It is to be observed that in the case of disjunctives we may again draw a distinction similar to that which has been drawn above between conditionals and true hypotheticals. For the alternatives may be events or combinations of properties one or other of which it is affirmed will (always or sometimes) occur, *e.g.*, "He is always either right or he has some good excuse for being wrong"; or they may be propositions which must be simply true or false, *i.e.*, cannot be one or the other according to varying conditions of time or space, *e.g.*, "Either there is a future life or many cruelties go unpunished." It is only in the former case that distinctions of quantity can be applied.

[2] *Some* being used in the sense of "some, it may be all." If by *some* we understand "some, but not all," then we are not really left in ignorance with regard to the remainder of the class which forms the subject of our proposition.

a negative, and undistributed in an affirmative proposition. Thus, if I say, All *S* is *P*, I imply that at any rate *some P* is *S*, but I make no implication with regard to the whole of *P*. I leave it an open question whether there is or is not any *P* outside the class *S*. Similarly if I say, Some *S* is *P*. But if I say, No *S* is *P*, in excluding the whole of *S* from *P*, I am also excluding the whole of *P* from *S*, and therefore *P* as well as *S* is distributed. Again, if I say, Some *S* is not *P*, although I make an assertion with regard to a part only of *S*, I exclude this part from the whole of *P*, and therefore the whole of *P* from it. In this case, then, the predicate is distributed, although the subject is not.

Summing up our results we find that

A distributes its subject only,
I distributes neither its subject nor its predicate,
E distributes both its subject and its predicate,
O distributes its predicate only.

46. Propositions in Extension.

We have seen that general concrete terms have both extension and connotation. Any proposition therefore which couples two such terms may be regarded as expressing a relation either between the extension of these terms, or between the attributes connoted by them, or between the extension of one and the connotation of the other. This gives four possible modes of interpreting the proposition. (1) The *class* mode. Both subject and predicate may be read in extension. Taking the proposition "All men are mortal," we have "The class man is included within the class mortal." (2) The *connotative* mode. Both subject and predicate may be read in connotation[1]. "The attributes

[1] We may recognise still another mode of reading propositions, namely, *in comprehension*, (using the term comprehension in the sense

connoted by 'man' are always accompanied by the attribute mortality." (3) The ordinary *predicative* mode. The subject may be read in extension and the predicate in connotation. Generally speaking this probably represents the course of ordinary thought. We think of the attribute 'mortality' as predicated of all the individuals contained in the class 'man.' (4) We might read the subject in connotation and the predicate in extension. "Wherever the attributes connoted by 'man' are present we have an individual belonging to the class 'mortal'."

This classification is of some importance in connexion with the question discussed in the preceding section. It has just been observed that while we naturally read the

indicated in section 13). This will yield the following:—All *S* is *P*, The comprehension of *S* includes the comprehension of *P*; All men are mortal, The attributes common to all mankind include those common to all mortals. The reading in comprehension should be carefully distinguished from the reading in connotation. In a good deal that is currently said about the intensive rendering of propositions the two are more or less confused. (It may be observed that using the term *intension* in the special sense that we have assigned to it, there is a possible reading in intension distinct both from the reading in connotation and the reading in comprehension. This point does not however seem to be of sufficient importance to make it worth while to work it out in any detail.)

Hamilton's reading of universal judgments in intension, (unless he is using the term intension in the objective sense here given to comprehension), cannot be applied to synthetical judgments. His intensive rendering of "all men are mortal" would be "mortality is part of humanity." But if by humanity we mean only what may be called the distinctive or essential attributes of man, then in order that this reading may be correct, the given proposition must be regarded as analytical.

We may add that Hamilton erroneously speaks of the distinction between judgments in extension and judgments in intension as a *division* of judgments (*Logic*, i., p. 231). It is clear that the distinction is really between two different points of view from which the same judgment may be regarded.

subject of an ordinary categorical proposition in extension, *i.e.*, regard it as the name of a class, we naturally read the predicate in connotation, *i.e.*, regard it as an attribute which is predicated of a class. But when we ask whether the predicate is or is not distributed, we must read that in extension also. This is always possible. Still we must not overlook the fact that in general it involves a passage of thought from connotation to extension. In discussing the distribution of terms then, we pass from the *predicative* to the *class* reading of propositions; and it may be added that the mode of interpreting propositions which the formal logician generally speaking finds it most convenient to adopt is *in extension*. But it is not maintained that this mode of regarding propositions is the only possible one, or even that it is psychologically the most primitive and natural mode.

47. How does the Quality of a Proposition affect its Quantity? Is the relation a necessary one? [L.]

By the Quantity of a Proposition must here be meant the Quantity of its Predicate, and it has been shewn in section 45 that this is determined by its Quality. The predicate is distributed in negative, undistributed in affirmative, propositions.

The latter part of the above question refers to Hamilton's doctrine of the Quantification of the Predicate. According to this doctrine, the predicate of an affirmative proposition is sometimes expressly distributed, while the predicate of a negative proposition is sometimes given undistributed. For example, the following forms are introduced:—

Some S is all P,
No S is some P.

This doctrine is discussed and illustrated in Part III., chapter 9.

48. In doubtful cases how should you decide which is the subject and which the predicate of a proposition? [V.]

The nature of the distinction ordinarily drawn between the subject and the predicate of a proposition may be expressed by saying that the subject is that of which something is affirmed or denied, the predicate is that which is affirmed or denied of the subject; or we may say that the subject is that which we regard as the determined or qualified notion, while the predicate is that which we regard as the determining or qualifying notion.

Now, can we say that the subject always precedes the copula, and that the predicate always follows it? In other words, can we consider the order of the terms to suffice as a criterion? If the proposition is practically reduced to an equation, as in the doctrine of the quantification of the predicate, I do not see what other criterion can be taken; or we may rather say that in this case the distinction between subject and predicate loses all importance. The two are placed on an equality, and we have nothing left by which to distinguish them except the order in which they are stated. This view is indicated by Professor Baynes in his *Essay on the New Analytic of Logical Forms*. In such a proposition, for example, as "Great is Diana of the Ephesians," he would call "great" the subject, reading the proposition, however, "(Some) great is (all) Diana of the Ephesians."

But leaving this view on one side, we cannot say that the order of terms is always a sufficient criterion. In the proposition just quoted, "Diana of the Ephesians" would generally be accepted as the subject. What further criterion

then can be given? In the case of **E** and **I** propositions, (propositions, as will be shewn, which can be simply converted), we must appeal to the context or to the question to which the proposition is an answer. If one term clearly conveys information regarding the other term, it is the predicate. We have seen also that it is more usual that the subject should be read in extension and the predicate in connotation[1]. If none of these considerations are decisive, then the order of the terms must suffice. In the case of **A** and **O** propositions, (propositions, as will be shewn, which cannot be simply converted), a further criterion may be added. From the rules relating to the distribution of terms in a proposition it follows that in affirmative propositions the distributed term, (if either term is distributed), is the subject; whilst in negative propositions, if only one term is distributed, it is the predicate. I am not sure that the inversion of terms ever occurs in the case of an **O** proposition; but in **A** propositions it is not infrequent. Applying the above to such a proposition as "Workers of miracles were the apostles," it is clear that the latter term is distributed while the former is not. The latter term is therefore the subject. A corollary from the rule is that in an affirmative proposition if one and only one term is singular that is the subject, since a singular is equivalent to a distributed term. This decides such a case as "Great is Diana of the Ephesians."

49. Analyse the following propositions, *i.e.*, express them in one or more of the strict categorical forms admitted in Logic:—

(i) No one can be rich and happy unless he is also temperate and prudent, and not always then.

[1] The subject is often a substantive and the predicate an adjective.

(ii) No child ever fails to be troublesome if ill taught and spoilt.

(iii) It would be equally false to assert that the rich alone are happy, or that they alone are not. [V.]

(i) contains *two* statements which may be reduced to the following forms,—

All who are rich and happy are temperate and prudent. **A.**

Some who are temperate and prudent are not rich and happy. **O.**

(ii) may be written,—All ill-taught and spoilt children are troublesome. **A.**

(iii) Here two statements are given *false*, namely,—the rich alone are happy; the rich alone are not happy.

We may reduce these false statements to the following,—all who are happy are rich; all who are not happy are rich. And this gives us these true statements,—

Some who are happy are not rich. **O.**

Some who are not happy are not rich. **O.**

The original proposition is expressed therefore by means of these two particular negative propositions.

EXERCISES.

50. Examine the logical signification of the italicised words in the following propositions:—

Some are born great.
Few are chosen.
All is not lost.
All men are created equal.
All that a man hath will he give for his life.
If *some A* is *B*, *some C* is *D*.

If *any* A is B, *any* C is D.
If *all* A is B, *all* C is D.

51. Determine the Quantity and Quality of the following propositions :—

(1) All men think all men mortal but themselves.
(2) Not to know me argues thyself unknown.
(3) To bear is to conquer our fate.
(4) Berkeley, a great philosopher, denied the existence of Matter.
(5) A great philosopher has denied the existence of Matter.
(6) The virtuous alone are happy.
(7) None but Irish were in the artillery.
(8) Not every tale we hear is to be believed.
(9) Great is Diana of the Ephesians !
(10) All sentences are not propositions.
(11) Where there's a will there's a way.
(12) Some men are always in the wrong. Read
(13) Facts are stubborn things.
(14) He that increaseth knowledge increaseth sorrow.
(15) None think the great unhappy, but the great.
(16) He can't be wrong, whose life is in the right.
(17) Nothing is expedient which is unjust.
(18) Who spareth the rod, hateth his child.

52. What do you consider to be respectively the subject and the predicate of the following sentences, and why ?

(1) Few men attain celebrity.
(2) Blessed are the peacemakers.
(3) It is mostly the boastful who fail.
(4) Clematis is Traveller's Joy. [V.]

53. Everything is either X or Y;

X and Y are coextensive;

Only X is Y;

The class X comprises the class Y and something more.

Express each of the above statements by means of ordinary A, I, E, O categorical propositions. [C.]

CHAPTER II.

THE OPPOSITION OF PROPOSITIONS.

54. The Opposition of Categorical Propositions.

Two propositions are technically said to be *opposed* to each other when they have the same subject and predicate respectively, but differ in quantity or quality or both[1].

Taking the propositions *SaP*, *SiP*, *SeP*, *SoP*, in pairs we find that there are four possible kinds of relation between them.

(1) The pair of propositions may be such that they cannot both be true, nor can they both be false. This is called *contradictory* opposition, and subsists between *SaP* and *SoP*, and between *SeP* and *SiP*[2].

[1] This definition is given by Aldrich (p. 53 in Mansel's edition). Ueberweg (*Logic*, § 97) defines Opposition in such a way as to include only contradiction and contrariety; and Mansel remarks that "subalterns are improperly classed as *opposed* propositions" (*Aldrich*, p. 59). Professor Fowler follows Aldrich's definition (*Deductive Logic*, p. 74), and I think wisely. We want some term to signify this general relation between propositions; and though it might be possible to find a more convenient term, I do not think that any confusion is likely to result from the use of the term *opposition* if the student is careful to notice that it is here used in a technical sense.

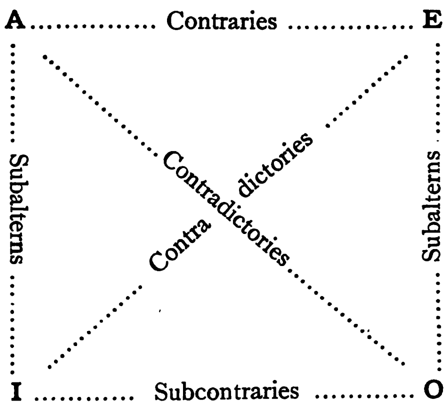
[2] In the present chapter I avoid complications that arise in connexion with the implication or non-implication of existence in propositions. For a further discussion of the doctrine of opposition, see sections 104, 107, 109.

(2) They may be such that they cannot both be true, but may both be false. This is called *contrary* opposition. *SaP* and *SeP*.

(3) They may be such that they cannot both be false, but may both be true. *Subcontrary* opposition. *SiP* and *SoP*.

(4) From a given universal proposition, the truth of the particular having the same quality follows, but not *vice versa*. This is *subaltern opposition*, the universal being called the *subalternant*, and the particular the *subalternate* or the *subaltern*. *SaP* and *SiP*. *SeP* and *SoP*.

All these relations are indicated clearly in the ancient square of opposition.



Propositions must of course be brought to such a form that they have the same subject and the same predicate before we can apply the terms of opposition to them; for example, *All S is P* and *Some P is not S* are not contradictories.

55. Give the contradictory and the contrary of the following propositions:—

(1) A stitch in time saves nine.

(2) None but the brave deserve the fair.

(3) He can't be wrong whose life is in the right.

(1) A stitch in time saves nine. This is to be regarded as a universal affirmative proposition, and we therefore have

Contradictory, Some stitches in time do not save nine. **O.**

Contrary, No stitch in time saves nine. **E.**

(2) None but the brave deserve the fair, = None who are not brave deserve the fair. **E.**

Contradictory, Some who are not brave deserve the fair. **I.**

Contrary, All who are not brave deserve the fair. **A.**

(3) He can't be wrong whose life is in the right. **E.**

Contradictory, Some may be wrong whose lives are in the right. **I.**

Contrary, All are wrong whose lives are in the right. **A.**

56. Contradictory and Contrary Opposition.

To deny the truth of a proposition is equivalent to affirming the truth of its *contradictory*; and *vice versa*. The criterion of contradictory opposition is that *of the two propositions, one must be true and the other must be false*; they cannot be true together, but on the other hand no mean is possible between them. The relation between two contradictories is mutual; it does not matter which is given true or false, we know that the other is false or true accordingly. Every proposition has its contradictory, which may however be more or less complex in form.

The *contrary* of a given proposition goes beyond mere denial, and sets up a further assertion as far as possible removed from the original assertion. It declares not merely the falsity of the original proposition taken as a whole, but the falsity of every part of it. Thus we may say that the contradictory of a proposition denies its entire truth, while its contrary asserts its entire falsehood.

It follows that if we cannot go beyond the simple denial of the truth of a proposition, then it has no contrary distinct from its contradictory. For example, in order simply to deny the truth of "some S is P," it is necessary to affirm that "no S is P," and it is impossible to go further than this in opposition to the given proposition. "Some S is P" has therefore no contrary as distinguished from its contradictory.

We may illustrate by applying the terms in question to the following propositions,—Few S are P; At any rate, he was not the only one who cheated; Two-thirds of the army are abroad.

(1) "Few S are P"="Most S are not P," and we might hastily be inclined to say that the contradictory is "Most S are P." Both these propositions would however be false in the case in which exactly one half S was P. The true contradictory therefore is "At least one half S is P." The contrary is "All S is P." Similarly the contradictory of "Most S are P" is "At least one half S is not P"; and its contrary "No S is P."

These examples shew that if we once travel outside the limits set by the old logic, and recognise the signs of quantity *most* and *few* as well as *all* and *some*, we soon become involved in numerical statements[1]. Propositions of

[1] In seeking the contradictory of *Few S are P*, we cannot treat the proposition as simply equivalent to *Some S is not P*; for this would give for the required contradictory *All S is P*, and this and the original

the above kind are therefore usually relegated to what has been called numerical logic, a topic discussed at length by De Morgan and to some extent by Jevons.

(2) "At any rate, he was not the only one who cheated." A question of interpretation is naturally raised here; does the statement assert that *he* cheated, or is this left an open question? We may I think choose the latter alternative. What the speaker intends to lay stress upon is that some others cheated at any rate, whatever may have been the case with him. The contradictory then becomes "No others cheated"; and we have no distinct contrary.

(3) "Two-thirds of the army are abroad." This may mean "At least two-thirds of the army are abroad," or "Exactly two-thirds of the army are abroad."

On the first interpretation, the contradictory is "Less than two-thirds of the army are abroad"; and the contrary "None of the army are abroad."

On the second interpretation, the contradictory is "Not exactly two-thirds of the army are abroad," *i. e.*, "Either more or less than two-thirds of the army are abroad." With regard to the contrary we are in a certain difficulty; for we may as it were proceed in two directions, and take our choice between "All the army are abroad" and "None of the army are abroad." I hardly see on what principle we are to choose between these.

Contrary opposition, unlike contradictory opposition, is of very little logical importance; and any discussion of contrariety, extended beyond the particular case contemplated in the ordinary square of opposition, almost necessarily involves us in material considerations.

proposition might both be false. If we transform "Few *S* are *P*" into "The number of *S*'s which are *P*'s is small," then the contradictory may be written, "The number of *S*'s which are *P*'s is not small."

57. The Opposition of Singular Propositions.

Take the proposition, Socrates is wise. The contradictory is—Socrates is not wise[1]; and so long as we keep to the same terms, we cannot go beyond this simple denial. We have therefore no contrary distinct from the contradictory. This opposition of singulars has been called *secondary contradiction* (Mansel's *Aldrich*, p. 56).

There are indeed two methods of treatment according to which we might find a distinct contrary and contradictory in the case of singular propositions, but I think that the above treatment according to which they are not distinguished is preferable to either.

(1) We might introduce the material contrary of the predicate instead of its mere contradictory, (compare section 28). Thus we should have—

Original proposition, Socrates is wise;
Contradictory, Socrates is not wise;
Contrary, Socrates has not a grain of sense.

This might be called the *material* contrary of the given proposition[2]. A fresh term is introduced that could not be formally obtained out of the given proposition. It still remains true that the singular proposition has no *formal* contrary distinct from its contradictory.

(2) Some principle of separation into parts might be introduced according to which the subject would be no

[1] This must be regarded as the correct contradictory from the point of view reached in the present chapter. But it will be shewn in section 107, that if the original proposition is understood, (as it probably would be understood), to imply the existence of Socrates, then a strict application of the criterion of contradiction requires that our contradictory be written,—If there is such a man as Socrates, he is unwise.

[2] The same distinction might be applied in the case of general propositions.

longer a whole indivisible; for example, Socrates might be regarded as having different characteristics at different times or under different conditions or from different points of view. The original proposition would then be read Socrates is always (or in all respects) wise, and the contradictory would be Socrates is sometimes (or in some respects) not wise, while the contrary would be Socrates is never (or in no respects) wise. Treated in this manner, however, the proposition does not remain really singular. Its true subject becomes "the judgments or the acts of Socrates."

58. The Opposition of Conditionals[1] and Disjunctives.

It has been already shewn that the ordinary distinctions of quantity and quality may be applied to Conditional Propositions, and it follows that the ordinary doctrine of opposition will also apply to them. We have

Always if *A* is *B*, *C* is *D*. **A.**
Sometimes if *A* is *B*, *C* is *D*. **I.**
Never if *A* is *B*, is it the case that *C* is *D*. **E.**
Sometimes if *A* is *B*, it is not the case that *C* is *D*. **O.**

Then, as in the case of Categoricals,—

A and **I**, **E** and **O** are subalterns.
A and **E** are contraries.
A and **O**, **E** and **I** are contradictories.
I and **O** are subcontraries.

There is more danger of contradictories being confused with contraries in the case of Conditionals than there is in the case of Categoricals. *If A is B, C is not D* is very liable to be given as the contradictory of *If A is B, C is*

[1] For a further discussion of the opposition of Conditionals, and for a consideration of the opposition of Hypotheticals, see section 109.

D. But it clearly is not its contradictory, *so far as they are general propositions*, since both may be false. For example, the two statements,—If the Times says one thing, the Pall Mall says another; If the Times says one thing, the Pall Mall says the same, *i.e.*, does not say another,—are both false; the two papers are sometimes in agreement and sometimes not.

We cannot apply distinctions of quality to Disjunctives, and therefore the ordinary doctrine of opposition cannot be applied to them. We may, however, find the contradictory and the contrary of a disjunctive proposition, such as Every *A* is either *B* or *C*, in accordance with the definitions given in section 56. Its Contradictory is, In some cases *A* is neither *B* nor *C*; its Contrary, No *A* is either *B* or *C*. It will be observed that the contradictory and contrary of a disjunctive are not themselves disjunctive.

A point to which our attention is called by the above is that the relation of reciprocity that holds between contradictories does not, except in the traditional square of opposition, always hold between contraries[1]. If the proposition β is the contradictory of the proposition α, then α is also the contradictory of β; but if δ is the contrary of α, it does not necessarily follow that α is the contrary of δ. Thus, we have seen that the contrary of "Every *A* is either *B* or *C*" is "No *A* is either *B* or *C*." But the contrary of the latter, according to the definition in section 56, is "Every *A* is both *B* and *C*," which is not the original proposition over again[2].

[1] For example, *Few S are P* and *All S is P* are not reciprocal contraries, although the latter may be regarded as the contrary of the former. Consider, again, the relation between *All S is all P* and *No S is P*.

[2] It should however be added that if we simply treat *No A is either B or C* as a categorical proposition with a complex predicate "either *B*

59. What is the precise meaning of the assertion that a proposition—say "All grasses are edible"—is false? [Jevons, *Studies in Deductive Logic*, p. 116.]

Professor Jevons discusses at some length the point here raised, but I find myself unable to agree with the conclusions at which he arrives. He commences by giving an answer, which may be called the orthodox one, and which I should certainly hold to be the correct one. When I assert that a proposition is false, I mean to imply that its contradictory is true. The given proposition is of the form **A**, and its contradictory is the corresponding **O** proposition,—Some grasses are not edible. When, therefore, I say that it is false that all grasses are edible, I mean that some grasses are not edible. Professor Jevons however continues, "But it does not seem to have occurred to logicians in general to inquire how far similar relations could be detected in the case of disjunctive and other more complicated kinds of propositions. Take, for instance, the assertion that 'all endogens are *all* parallel-leaved plants.' If this be false, what is true? Apparently that one or more endogens are not parallel-leaved plants, or else that one or more parallel-leaved plants are not endogens. But it may also happen that no endogen is a parallel-leaved plant at all. There are three alternatives, and the simple falsity of the original does not shew which of the possible contradictories is true."

In this statement, there appear to be two errors. In the first place, in saying that one or more endogens are not parallel-leaved plants, we do not mean to exclude the possibility that no endogen is a parallel-leaved plant at all. Symbolically, *Some S is not P* does not exclude *No S is P*.

or *C*," then its contrary according to the traditional scheme of opposition is *Every A is either B or C*, and here the relation is reciprocal.

The three alternatives are therefore at any rate reduced to the two first given. But in the second place, I think Professor Jevons is in error in regarding each of these alternatives by itself as a contradictory of the original proposition. The true logical contradictory is the affirmation of the truth of *one or other* of these alternatives. If the original complex proposition is false we certainly know that the new complex proposition limiting us to such alternatives is true.

The point at issue may be further illustrated by taking the proposition in question in a symbolic form. *All S is all P* is a complex proposition, resolvable into the form, *All S is P and all P is S.* It has but one contradictory, namely, *Either some S is not P or some P is not S*[1]. If either of these alternatives holds good, the original statement must in its entirety be false; and on the other hand, if the latter is false, one at least of these alternatives must be true. Professor Jevons speaks as if *Some S is not P* were by itself a contradictory of *All S is all P.* But it is merely inconsistent with it. They may both be false. No doubt in ordinary speech *contradictory* frequently implies no more than "inconsistent with," and if Professor Jevons means that we should also use the term contradictory in this sense in Logic, the question becomes a verbal one. But he means more than this; he seems to mean that in some cases we can find no proposition that must be true when a given proposition is false. This view I cannot but regard as erroneous. It is true that if the original proposition is complex, its contradictory will in general be complex too, and possibly still more complex; but that might naturally be expected.

[1] The contradictory of "All *S* is all *P*" may also be expressed "*S* and *P* are not coextensive."

The above will I think indicate how misleading is Professor Jevons's further statement,—"It will be shewn in a subsequent chapter that a proposition of moderate complexity has an almost unlimited number of contradictory propositions, which are more or less in conflict with the original. The truth of any one or more of these contradictories establishes the falsity of the original, but the falsity of the original does not establish the truth of any one or more of its contradictories." No doubt a complex proposition may yield an indefinite number of other propositions the truth of any one of which is *inconsistent with* its own. But it has only one logical *contradictory*, which contradictory as suggested above is likely to be a still more complex proposition affirming a number of alternatives one or other of which must hold good if the original proposition is false.

With the point here raised Professor Jevons mixes up another, in regard to which his view is almost more misleading. He says, "But the question arises whether there is not confusion of ideas in the usual treatment of this ancient doctrine of opposition, and whether a contradictory of a proposition is not any proposition which involves the falsity of the original, but is not the sole condition of it. I apprehend that any assertion is false which is made without sufficient grounds. It is false to assert that the hidden side of the moon is covered with mountains, not because we can prove the contradictory, but because we know that the assertor must have made the assertion without evidence. If a person ignorant of mathematics were to assert that 'all involutes are transcendental curves,' he would be making a false assertion, because, whether they are so or not, he cannot know it." Surely in Logic we cannot regard the truth or falsity of a proposition as depending upon the knowledge of the person who affirms it, so that the same

proposition would now be true, now false. The question "What is truth?" may be an enormously difficult one to answer absolutely, and I need not say that I shall not attempt to deal with it here; but unless we are allowed to proceed from the falsity of "All *S* is *P*" to the truth of "Some *S* is not *P*," I do not think we can go far in Logic.

EXERCISES.

60. On the common view of the opposition of propositions what are the inferences to be drawn (1) from the truth, (2) from the falsity, of each of the four categorical propositions? [L.]

61. Explain the nature of the opposition between each pair of the following propositions:—

None but Liberals voted against the motion.

Amongst those who voted against the motion were some Liberals.

It is untrue that those who voted against the motion were all Liberals.

62. Give the contradictory of the proposition:— Wherever the property *A* is found, either the property *B* or the property *C* will be found with it, but not both of them together[1].

63. Shew that any term distributed in a general proposition[2] is undistributed in its contradictory, and *vice versa*.

[1] For a general discussion of the opposition of complex terms and complex propositions see Part IV.

[2] Including under generals both universals and particulars, but excluding singulars.

64. Assign precisely the meaning of the assertion that it is false to say that some English soldiers did not behave discreditably in South Africa. [L.]

65. Examine in the case of each of the following propositions the precise meaning of the assertion that the proposition is false:—

(i) Some electricity is generated by friction.

(ii) Oxygen and nitrogen are constituents of the air we breathe.

(iii) If a straight line falling upon two other straight lines make the alternate angles equal to each other, these two straight lines shall be parallel.

(iv) Actions are either good, bad, or indifferent.

CHAPTER III.

THE CONVERSION OF PROPOSITIONS[1].

66. The meaning of logical Conversion. The Ordinary Conversion of propositions.

By *Conversion*, in a broad sense, is meant a change in the position of the terms of a proposition[2].

Logic, however, is concerned with conversion only in so far as the truth of the new proposition obtained by the process is a legitimate inference from the truth of the original proposition. This is what Whately means when he says that "no conversion is employed for any logical purpose unless it be *illative*" (*Elements of Logic*, p. 74). For example,

[1] In this and the four following chapters we proceed on the assumption that each class represented by a simple term exists in the universe of discourse, while at the same time it does not exhaust that universe. This assumption appears always to have been implicitly made in the traditional treatment of Logic.

[2] Ueberweg (*Logic*, § 84) defines Conversion thus. Compare also De Morgan, p. 58. In Geometry, "All equilateral triangles are equiangular" would be regarded as the converse of "All equiangular triangles are equilateral."

the change from *All S is P* to *All P is S* is not a logical conversion, since the truth of the latter proposition does not follow from the truth of the former[1]. In other words, logical conversion is a case of *immediate inference*, which may be defined as the inference of a proposition from a single other proposition[2].

The simplest form of logical conversion may be distinguished from other forms by being called *ordinary* conversion, and it may be defined as follows:—By *ordinary conversion* is meant *a process of immediate inference in which from a given proposition we infer another, having the predicate of the original proposition for subject, and its subject for predicate.*

Thus, given a proposition having *S* for its subject and *P* for its predicate, we seek to obtain by immediate inference a new proposition having *P* for its subject and *S* for its predicate; and applying this rule to the four fundamental forms of proposition, we get the following table:—

[1] *All S is P* and *All P is S* may of course happen to be true together, as in the case of the two propositions given in the preceding note. "But it is only knowledge of the matter of fact contained in the judgment in question which can assure us that the relation, upon which this possibility depends, holds good between *S* and *P* in any particular instance" (Lotze, *Logic*, § 80). When it also happens that *all P is S*, the judgment *all S is P* is sometimes said to be *reciprocal*. If this is to be *formally* expressed in a single judgment, we must make use of the form *All S is all P*.

[2] In discussing immediate inferences we "pursue the content of an enunciated judgment into its relations to judgments not yet uttered" (Lotze). Instead of "immediate inferences" Professor Bain prefers to speak of "equivalent propositional forms." We shall find, however, that the new propositions obtained by immediate inference are not always equivalent to the original propositions, *e.g.*, in conversion *per accidens*.

Original Proposition.	*Converse.*
All S is P. **A.**	Some P is S. **I.**
Some S is P. **I.**	Some P is S. **I.**
No S is P. **E.**	No P is S. **E.**
Some S is not P. **O.**	(None.)

We may again call attention to the fact that, generally speaking, in any judgment we have naturally before the mind the objects denoted by the subject, but the qualities connoted by the predicate[1]. In converting a proposition, however, the attributive force of the predicate is dropped, and an import is given to the predicate similar to that of the subject. In other words, the proposition is taken in extension. It is in passing from the predicative to the "class" reading, (*e.g.*, from "all men are mortal" to "all men are mortals"), that the difficulty sometimes found in correctly converting propositions probably consists. We ought at any rate to recognise that conversion and other immediate inferences in general involve a distinct mental act of the above nature.

67. Simple Conversion, and Conversion *per accidens.*

It will be observed that in the case of **I** and **E**, the converse is of exactly the same form as the original proposition (or *convertend*); we do not lose any part of the information given us by the convertend, and we can pass

[1] Compare section 46.

back to it by re-conversion of the converse. The convertend and its converse are *equivalent* propositions. The conversion in both these cases is said to be *simple.*

In the case of **A**, it is different; although we start with a universal proposition, we obtain by conversion a particular one only, and by no means of operating upon the converse can we regain the original proposition. The convertend and its converse are not equivalent propositions. This is called conversion *per accidens*[1], or conversion *by limitation*[2].

68. Particular negative propositions do not admit of *ordinary* conversion.

It is clear that if the converse is to be a legitimate formal inference from the original proposition (or convertend), it must distribute no term that was not distributed in the convertend. From this it follows immediately that *Some S is not P* does not admit of ordinary conversion; for *S* which is undistributed in the convertend would become the predicate of a negative proposition in the converse, and would therefore be distributed. (I may remind the reader that in what I have called ordinary conversion, and

[1] The conversion of **A** is said by Mansel to be called conversion *per accidens* "because it is not a conversion of the universal *per se*, but by reason of its containing the particular. For the proposition 'Some *B* is *A*' is *primarily* the converse of 'Some *A* is *B*,' *secondarily* of 'All *A* is *B*'" (Mansel's *Aldrich*, p. 61). Professor Baynes seems to deny that this is the correct explanation of the use of the term (*New Analytic of Logical Forms*, p. 29); but however this may be, I do not think that we can really regard the converse of **A** as obtained through its subaltern. We proceed directly from "All *A* is *B*" to "Some *B* is *A*" without the intervention of "Some *A* is *B*."

[2] Simple conversion and conversion *per accidens* are also called respectively *conversio pura* and *conversio impura*. Compare Lotze, *Logic*, § 79.

with which alone we are now dealing, we do not admit the contradictory either of the original subject or of the original predicate as one of the terms of our converse.)

I cannot understand why Professor Jevons should say that the fact that the particular negative proposition is incapable of ordinary conversion "constitutes a blot in the ancient logic" (*Studies in Deductive Logic*, p. 37). We shall find subsequently that just as much can be inferred from the particular negative as from the particular affirmative (since the latter unlike the former does not admit of contraposition). No logic, symbolic or other, can actually obtain more from the given information than the ancient logic does.

69. Give the converse of the following propositions:—

(1) A stitch in time saves nine.

(2) None but the brave deserve the fair.

(3) He can't be wrong whose life is in the right.

(4) The virtuous alone are happy.

No difficulty can be found in converting or performing other immediate inferences upon any given proposition when once it has been brought into ordinary logical form, its quantity and quality being determined, its subject, copula and predicate being definitely distinguished from one another, and its predicate as well as its subject being read in extension.

If this rule is neglected, the most absurd results may be elicited. For example, amongst several curious converses of the first of the above propositions I have had seriously given,—Nine stitches save a stitch in time. Here it is of

course overlooked that "save" is not the logical copula. The proposition may be written, All stitches in time are things that save nine stitches. This being an **A** proposition is only convertible *per accidens*, and we have for our converse,—Some things that save nine stitches are stitches in time.

"None but the brave deserve the fair." For the converse of this I have had,—The fair deserve none but the brave; and, again, No one ugly deserves the brave. The proposition may be written, No one who is not brave is deserving of the fair. This, being an **E** proposition, may be converted simply, giving, No one deserving of the fair is not brave.

"He can't be wrong whose life is in the right." Written in ordinary logical form, this proposition becomes,—No one whose life is in the right is able to be in the wrong; and therefore its converse is,—No one who is able to be in the wrong is one whose life is in the right. This proposition may also be written in the more natural form, His life cannot be in the right who can himself be wrong.

"The virtuous alone are happy." This is an *exclusive* proposition, and is not in a form recognised by the traditional logic. It is equivalent to,—Some who are virtuous are all who are happy. As pointed out in section 38, we really convert this proposition in order to bring it into one of the ordinarily recognised forms, namely, All who are happy are virtuous.

Exercises.

70. State in logical form and convert the following propositions:—

(1) There's not a joy the world can give like that it takes away.

(2) He jests at scars who never felt a wound.

(3) Axioms are self-evident.

(4) Natives alone can stand the climate of Africa.

(5) Not one of the Greeks at Thermopylæ escaped.

(6) All that glitters is not gold. [O.]

71. Give all the logical opposites of the proposition :— Some rich men are virtuous; and also the converse of the contrary of its contradictory. How is the latter directly related to the given proposition?

Does it follow that a proposition admits of simple conversion because its predicate is distributed?

72. Point out any possible ambiguities in the following propositions, and give the contradictory and (where possible) the converse of each of them:—

(i) Some of the candidates have been successful.

(ii) All are not happy that seem so.

Read (iii) All the fish weighed five pounds.

CHAPTER IV.

THE OBVERSION AND CONTRAPOSITION OF PROPOSITIONS.

73. The Obversion of Propositions.

Obversion is *a process of immediate inference in which from a given proposition we infer another, having for its predicate the contradictory of the predicate of the original proposition.* The substitution here indicated is always legitimate *if at the same time we change the quality of the proposition.*

Applying this rule, we have the following table:—

Original Proposition.	*Obverse.*
All S is P. **A.**	No S is not-P. **E.**
Some S is P. **I.**	Some S is not not-P. **O.**
No S is P. **E.**	All S is not-P. **A.**
Some S is not P. **O.**	Some S is not-P. **I.**

The term *Obversion* is used by Professor Bain, and is a

convenient one. The process is also called *Permutation* (Fowler), *Aequipollence* (Ueberweg), *Infinitation* (Bowen), *Immediate Inference by Privative Conception* (Jevons), *Contraversion* (De Morgan), *Contraposition* (Spalding).

It will be observed that the obversion of **A** and **I** is based on the Law of Contradiction,—Nothing can at the same time be both *P* and not-*P*; while the obversion of **E** and **O** is based on the Law of Excluded Middle,—Everything is either *P* or not-*P*.

74. Formal Obversion and Material Obversion.

By *Formal Obversion* is meant the kind of obversion discussed in the previous example, and it is the only kind of obversion that Formal Logic should recognise.

Professor Bain uses the expression *Material Obversion*, and by it he means the process of making "Obverse Inferences which are justified only on an examination of the matter of the proposition" (*Logic*, I. p. 111). He gives as examples,—"Warmth is agreeable; therefore, cold is disagreeable. War is productive of evil; therefore, peace is productive of good. Knowledge is good; therefore, ignorance is bad." I am inclined to doubt whether these are legitimate inferences, formal or otherwise. The conclusions appear to require quite independent investigations to establish them. Granted, for example, that warmth is agreeable, it might be that every other state of temperature is agreeable also.

75. Conversion by Contraposition.

Contraposition (also called *Conversion by Negation*) is *a process of immediate inference in which from a given proposition we infer another proposition having the contradictory of*

the original predicate for its subject, and the original subject for its predicate[1].

[1] There is some difference between logicians as to whether the contrapositive of *All S is P* is *No not-P is S* or *All not-P is not-S*. It is merely a verbal question, depending on our original definition of contraposition. It will be observed that *All not-P is not-S* is the obverse of *No not-P is S*, and if we regard *All not-P is not-S* as the contrapositive of *All S is P*, our definition of contraposition must be altered to,—"a process of immediate inference in which from a given proposition we infer another proposition having the contradictory of the original predicate for its subject, and the *contradictory* of the original subject for its predicate." In this case, what I have originally defined as contraposition may be called conversion by negation. Note should be taken of this difference of usage, and then no difficulty is likely to result. Taking the following definition, we might call either form a contrapositive of the original proposition,—"contraposition is a process of immediate inference in which from a given proposition we infer another proposition having the contradictory of the original predicate for its subject." It is here left an open question whether the predicate of the contrapositive is to be the original subject or the contradictory of the original subject.

The following is from Mansel's *Aldrich*, p. 61,—"Conversion by contraposition, which is not employed by Aristotle, is given by Boethius in his first book, *De Syllogismo Categorico*. He is followed by Petrus Hispanus. It should be observed, that the old Logicians, following Boethius, maintain, that in conversion by contraposition, as well as in the others, the *quality* should remain unchanged. Consequently the converse of 'All *A* is *B*' is 'All not-*B* is not-*A*,' and of 'Some *A* is not *B*,' 'Some not-*B* is not not-*A*.' It is simpler, however, to convert **A** into **E**, and **O** into **I**, ('No not-*B* is *A*,' 'Some not-*B* is *A*'), as is done by Wallis and Abp. Whately; and before Boethius by Apuleius and Capella, who notice the conversion, but do not give it a name. The principle of this conversion may be found in Aristotle, *Top.* II. 8. 1, though he does not employ it for logical purposes."

In most text books, no *definition* of contraposition is given at all, and it may be pointed out that in the attempt to generalise from special examples, Jevons in his *Elementary Lessons in Logic* gets into difficulties. For the contrapositive of **A** he gives *All not-P is not-S*; **O** he says has no contrapositive, (but only a converse by negation, *Some not-P is S*);

Thus, given a proposition having S for its subject and P for its predicate, we seek to obtain by immediate inference a new proposition having not-P for its subject and S for its predicate.

From the definition we can immediately deduce the following rule for obtaining the contrapositive of a given proposition:—*Obvert the original proposition, and then convert the proposition thus obtained.* For given a proposition with S for subject and P for predicate, obversion will yield an equivalent proposition with S for subject and not-P for predicate, and the conversion of this will make not-P the subject and S the predicate; *i.e.*, we shall have found the contrapositive of the given proposition.

Applying this rule, we have the following table:—

Original Proposition.	*Obverse.*	*Contrapositive.*
All S is P. **A.**	No S is not-P. **E.**	No not-P is S. **E.**
Some S is P. **I.**	Some S is not not-P. **O.**	(None.)
No S is P. **E.**	All S is not-P. **A.**	Some not-P is S. **I.**
Some S is not P. **O.**	Some S is not-P. **I.**	Some not-P is S. **I.**

It is easy to shew that *Some S is P* has no contrapositive; for when it is obverted, it becomes a particular negative; but particular negatives do not admit of *ordinary* conversion, which is the process that must succeed obversion in order that a contrapositive may be obtained.

and for the contrapositive of **E** he gives *No P is S.* I have failed to discover any definition of contraposition that can yield these results. If in contraposition the quality of the proposition is to remain unchanged as in Jevons's contrapositive of **A**, then the contrapositive of both **E** and **O** will be Some not-P is not not-S.

It may be helpful if we here sum up the immediate inferences that have been obtained up to this point, making use of the symbols explained in section 36, and denoting not-S by S', not-P by P' :—

Original Proposition.	*Converse.*	*Obverse.*	*Contrapositive*[1].	*Obverted Contrapositive*[1].
SaP	PiS	SeP'	$P'eS$	$P'aS'$
SiP	PiS	SoP'		
SeP	PeS	SaP'	$P'iS$	PoS'
SoP		SiP'	$P'iS$	$P'oS'$

It will be shewn presently how this table of Immediate Inferences may be expanded.

With regard to the utility of the investigation as to what contrapositives are logically inferable from given propositions, the following may be quoted from De Morgan :—

"The uneducated acquire easy and accurate use of the very simplest cases of transformation of propositions and of syllogisms. The educated, by a higher kind of practice, arrive at equally easy and accurate use of some more complicated cases: but not of all those which are treated in ordinary logic. Euclid may have been ignorant of the identity of 'Every X is Y' and 'Every not-Y is not-X,' for anything that appears in his writings: he makes the one follow from the other by a new proof each time" (*Syllabus*, p. 32).

[1] It must be remembered, as explained in the preceding note, that what is called the contrapositive above is sometimes called the converse by negation, and what is called the obverted contrapositive above is sometimes called simply the contrapositive.

76. Transform the following propositions in such a way that, without losing any of their force, they may all have the same subject and the same predicate :—No not-P is S; All P is not-S; Some P is S; Some not-P is not not-S.

This problem may be briefly solved as follows :—

No not-P is S = No S is not-P = All S is P.

All P is not-S = No P is S = No S is P.

Some P is S = Some S is P.

Some not-P is not not-S = Some not-P is S
= Some S is not-P = Some S is not P.

77. The Conversion and Contraposition of Conditional Propositions.

In a conditional proposition the antecedent and the consequent correspond respectively to the subject and the predicate of a categorical proposition. In Conversion therefore the old consequent must be the new antecedent, and in Contraposition the denial of the old consequent must be the new antecedent. Proceeding as before, this gives us immediately the following table :—

Original Proposition.	*Converse.*	*Contrapositive.*
Always if A is B, C is D. **A.**	Sometimes if C is D, A is B. **I.**	Never if C is not D, is it the case that A is B. **E.**
Sometimes if A is B, C is D. **I.**	Sometimes if C is D, A is B. **I.**	None.
Never if A is B, is it the case that C is D. **E.**	Never if C is D, is it the case that A is B. **E.**	Sometimes if C is not D, A is B. **I.**
Sometimes if A is B, it is not the case that C is D. **O.**	None.	Sometimes if C is not D, A is B. **I.**

The obverse of a conditional proposition is sometimes awkward to express. We may however always find it if required; *e.g.*, the obverse of "If *A* is *B*, *C* is *D*" is "If *A* is *B*, it is not the case that *C* is not *D*."

Considering the contrapositive of **A** and the converse of **E**, we observe that given a universal conditional proposition we may always obtain by immediate inference a new conditional, the antecedent of which denies the original consequent, while its consequent denies the original antecedent[1].

78. Give the converse and the contrapositive of "If a straight line falling upon two other straight lines make the alternate angles equal to one another, these two straight lines shall be parallel." [L.]

The application of the doctrines of Conversion and Contraposition to Conditionals may be illustrated by means of the above proposition. Since it is a universal affirmative, it is only convertible *per accidens*. This is a point particularly liable to be overlooked where a universal converse can be legitimately inferred (as in the case of the above proposition), though not as an *immediate inference*. We are in no danger of saying, All men are animals, therefore, all animals are men; but we may be in danger of saying, All equilateral triangles are equiangular, therefore, all equiangular triangles are equilateral. From the point of view of Formal Logic the latter inference is as erroneous as the former.

So far as the given proposition is concerned, we have—

Converse, In some cases in which two straight lines are parallel, a straight line falling upon them shall make the alternate angles equal to one another.

[1] This statement holds good also of true hypotheticals. Given *If P is true, Q is true* we may infer by contraposition *If Q is false, P is false.*

Contrapositive, If two straight lines are not parallel, then a straight line falling upon them shall not make the alternate angles equal to one another.

Exercises.

79. Give the obverse of each of the following propositions:—Whatever is, is right; No news is good news; Good orators are not always good statesmen; A stitch in time saves nine.

80. Give the obverse and the contrapositive of each of the following propositions:—(*a*) All animals feed; (*b*) No plants feed; (*c*) Only animals feed. [L.]

81. Give the contrapositive of each of the following propositions:—

(1) A stitch in time saves nine.

(2) None but the brave deserve the fair.

(3) He can't be wrong whose life is in the right.

(4) The virtuous alone are happy.

82. Give the contradictory and the contrapositive of each of the following:—Blessed are the peacemakers; Not every tale we hear is to be believed; No men are perfect.

83. "The angles at the base of an isosceles triangle are equal."

What can be inferred from this proposition by Obversion, Conversion, and Contraposition respectively? [L.]

84. Describe the logical relations, if any, between each of the following propositions, and each of the others :—

(i) There are no inorganic substances which do not contain carbon;

(ii) All organic substances contain carbon;

(iii) Some substances not containing carbon are organic;

(iv) Some inorganic substances do not contain carbon. [c.]

[This question can be most satisfactorily answered by reducing the propositions to such forms that they all have the same subject and the same predicate. Compare section 76.]

85. Give the contradictory, the contrary, the converse, and the contrapositive of the following propositions :—

(1) Things equal to the same thing are equal to one another.

(2) No one is a hero to his valet.

(3) If there is no rain the harvest is never good.

(4) None think the great unhappy but the great.

(5) Fain would I climb but that I fear to fall.

86. Name the form of each of the following propositions; and, where possible, give the converse and the contrapositive of each :—

(i) Some death is better than some life.

(ii) The candidates in each class are not arranged in order of merit.

(iii) Honesty is the best policy.

(iv) Not all that tempts your wand'ring eyes
And heedless hearts is lawful prize.

(v) If an import duty is a source of revenue, it does not afford protection.

(vi) Great is Diana of the Ephesians.

(vii) All these claims upon my time overpower me.

CHAPTER V.

THE INVERSION OF PROPOSITIONS.

87. In what cases can we obtain by immediate inference from a given proposition a new proposition having the contradictory of the original subject for its subject, and the original predicate for its predicate?

A new form of immediate inference is here indicated, by which, given a proposition having *S* for its subject and *P* for its predicate, we seek to obtain a new proposition having not-*S* for its subject and *P* for its predicate.

If such a proposition can be obtained at all, it will be by a certain combination of the elementary processes of ordinary conversion and obversion. We will take each of the fundamental forms of proposition and see what can be obtained (1) by first converting it, and then performing alternately the operations of obversion and conversion; (2) by first obverting it, and then performing alternately the operations of conversion and obversion. We shall find that in each case we can go on till we reach a particular negative proposition whose turn it is to be converted.

(1) The results of performing the processes of conversion and obversion alternately, commencing with the *former*, are as follows:—

(i) All S is P,
therefore (by conversion), Some P is S,
therefore (by obversion), Some P is not not-S.

Here comes the turn for conversion; but we have an **O** proposition, and can therefore proceed no further.

(ii) Some S is P,
therefore (by conversion), Some P is S,
therefore (by obversion), Some P is not not-S;

and we can get no further.

(iii) No S is P,
therefore (by conversion), No P is S,
therefore (by obversion), All P is not-S,
therefore (by conversion), *Some not-S is P*,
therefore (by obversion), Some not-S is not not-P.

In this case the proposition in italics is the immediate inference that was sought.

(iv) Some S is not P.

In this case we are not able even to commence our series of operations.

(2) The results of performing the processes of conversion and obversion alternately, commencing with the *latter*, are as follows:—

(i) All S is P,
therefore (by obversion), No S is not-P,
therefore (by conversion), No not-P is S,
therefore (by obversion), All not-P is not-S,
therefore (by conversion), Some not-S is not-P,
therefore (by obversion), *Some not-S is not P*.

Here again we have obtained the desired form.

(ii) Some S is P,
therefore (by obversion), Some S is not not-P.

(iii) No *S* is *P*,
therefore (by obversion), All *S* is not-*P*,
therefore (by conversion), Some not-*P* is *S*,
therefore (by obversion), Some not-*P* is not not-*S*.

(iv) Some *S* is not *P*,
therefore (by obversion), Some *S* is not-*P*,
therefore (by conversion), Some not-*P* is *S*,
therefore (by obversion), Some not-*P* is not not-*S*.

We can now answer the question with which we commenced this enquiry. The required proposition can be obtained only if the given proposition is universal; we then have, according as it is affirmative or negative,—

All *S* is *P*, therefore, Some not-*S* is not *P*;
No *S* is *P*, therefore, Some not-*S* is *P*[1].

It should be observed that in the case of the former of these we commenced with obversion in order to get the new form, in the latter we commenced with conversion.

This form of immediate inference has been more or less casually recognised by various logicians; but I do not remember that it has ever received any distinctive name. Sometimes it has been vaguely classed under contraposition, (compare Jevons, *Elementary Lessons in Logic*, pp. 185, 6), but it is really as far removed from the process to which that designation has been given as the latter is from ordinary conversion. I venture to suggest the terms *Inversion* and

[1] The reader will remember that we are at present working on the assumption that each class represented by a single letter exists in the universe of discourse, while at the same time it does not exhaust that universe; in other words, we assume that *S*, not-*S*, *P*, not-*P*, all represent existent classes.

Inverse[1]. Thus, *Inversion* is *a process of immediate inference in which from a given proposition we infer another proposition*

[1] Professor Jevons (carrying out a suggestion of Professor Robertson's) has introduced the term *Inverse* in a different sense. I do not however think that for logical purposes we want any new term in the sense in which he uses it; and I have been unable to think of any other equally suitable term for my own purpose, for which a new term really is needed, if the scheme of immediate inferences by means of conversion and obversion is to be made scientifically complete. The term *contraverse* has occurred to me, but I do not like it so well; and this again has been appropriated by De Morgan in another sense.

Professor Jevons's nomenclature is explained in the following passage from his *Studies in Deductive Logic*, p. 32:—"It appears to be indispensable to endeavour to introduce some fixed nomenclature for the relations of propositions involving two terms. Professor Alexander Bain has already made an innovation by using the term *obverse*, and Professor Hirst, Professor Henrici and other reformers of the teaching of geometry have begun to use the terms converse and obverse in meanings inconsistent with those attached to them in logical science (*Mind*, 1876, p. 147). It seems needful, therefore, to state in the most explicit way the nomenclature here proposed to be adopted with the concurrence of Professor Robertson.

Taking as the original proposition 'All *A* are *B*,' the following are what we may call the *related propositions*:—

Inferable.

Converse. Some *B* are *A*.

Obverse. No *A* are not-*B*.

Contrapositive. No not-*B* are *A*, or All not-*B* are not-*A*.

Non-Inferable.

Inverse. All *B* are *A*.

Reciprocal. All not-*A* are not-*B*.

It must be observed that the converse, obverse, and contrapositive are all true if the original proposition is true. The same is not necessarily the case with the *inverse* and *reciprocal*. These latter two names are adopted from the excellent work of Delbœuf, *Prolégomènes Philosophiques de la Géométrie*, pp. 88—91, at the suggestion of Professor Croom Robertson (*Mind*, 1876, p. 425)."

In this scheme what I propose to call the *Inverse* is not recognised

having the contradictory of the original subject for its subject, and the original predicate for its predicate. In other words, given a proposition having S for subject and P for predicate, we obtain by inversion a new proposition having not-S for subject and P for predicate.

We may now sum up the results that have been obtained with regard to immediate inferences. Given two terms S and P, and admitting their contradictories not-S and not-P, we have eight possible forms of proposition as shewn in the following scheme:—

	Subject.	*Predicate.*
(i)	S	P
(ii)	S	not-P
(iii)	P	S
(iv)	P	not-S
(v)	not-P	S
(vi)	not-P	not-S
(vii)	not-S	P
(viii)	not-S	not-P

at all. On the other hand, I hardly see why the *non-inferable* forms need such a distinct logical recognition as is implied by giving them distinct names; while except in books on Logic I anticipate that the term *converse* is likely still to be used in its non-logical sense, (*i.e.*, "All B are A" is likely still to be spoken of as the converse of "All A are

These propositions may be designated respectively:—

(i) The original proposition,
(ii) The obverse,
(iii) The converse,
(iv) The obverted converse,
(v) The contrapositive,
(vi) The obverted contrapositive,
(vii) The inverse,
(viii) The obverted inverse.

It has been shewn in sections 66, 73, 75, and in the present section, that if the original proposition is universal, we can infer from it propositions of all the remaining seven forms; but if it is particular, we can infer only three others.

Working out the different cases in detail we have:—

A. (i) Original proposition, *All S is P.*
(ii) Obverse, *No S is not-P.*
(iii) Converse, *Some P is S.*
(iv) Obverted Converse, *Some P is not not-S.*
(v) Contrapositive, *No not-P is S.*
(vi) Obverted Contrapositive, *All not-P is not-S.*
(vii) Inverse, *Some not-S is not P.*
(viii) Obverted Inverse, *Some not-S is not-P.*

I. (i) Original proposition, *Some S is P.*
(ii) Obverse, *Some S is not not-P.*
(iii) Converse, *Some P is S.*
(iv) Obverted Converse, *Some P is not not-S.*

B"). It may be noted that in Jevons's use of terms, the inverse would be the same as the converse in the case of **E** and **I** propositions. I imagine also that in consistency there should be yet another term to express the relation of "No not-*B* is not-*A*" or "All not-*B* is *A*" to "No *A* is *B*"; it is, in the sense in which Jevons uses these terms, neither the Converse, Obverse, Contrapositive, Inverse, nor Reciprocal.

(v) Contrapositive, none can be inferred.

(vi) Obverted Contrapositive, none.

(vii) Inverse, none.

(viii) Obverted Inverse, none.

E. (i) Original proposition, *No S is P.*

(ii) Obverse, *All S is not-P.*

(iii) Converse, *No P is S.*

(iv) Obverted Converse, *All P is not-S.*

(v) Contrapositive, *Some not-P is S.*

(vi) Obverted Contrapositive, *Some not-P is not not-S.*

(vii) Inverse, *Some not-S is P.*

(viii) Obverted Inverse, *Some not-S is not not-P.*

O. (i) Original proposition, *Some S is not P.*

(ii) Obverse, *Some S is not-P.*

(iii) Converse, none can be inferred.

(iv) Obverted Converse, none.

(v) Contrapositive, *Some not-P is S.*

(vi) Obverted Contrapositive, *Some not-P is not not-S.*

(vii) Inverse, none.

(viii) Obverted Inverse, none.

All the above is summed up in the following Table (using the symbols described in section 36, and denoting not-*S* by *S'*, not-*P* by *P'*):—

		A.	I.	E.	O.
i	Original proposition	SaP	SiP	SeP	SoP
ii	Obverse......................	SeP'	SoP'	SaP'	SiP'
iii	Converse	PiS	PiS	PeS	
iv	Obverted Converse	PoS'	PoS'	PaS'	
v	Contrapositive	$P'eS$		$P'iS$	$P'iS$
vi	Obverted Contrapositive...	$P'aS'$		$P'oS'$	$P'oS'$
vii	Inverse	$S'oP$		$S'iP$	
viii	Obverted Inverse............	$S'iP'$		$S'oP'$	

It is worth noticing that we can infer the same number of propositions from **E** as from **A** (7), and from **O** as from **I** (3), and the same number of universal propositions from **E** as from **A** (3); also that in two cases we can get no more from **A** than from **I**, and no more from **E** than from **O**.

88. Mutual Relations of Propositions.

We may give the following classification of the relations in which any two propositions may stand to one another[1]:—

(1) They may be *equivalent* or *equipollent*, each proposition being formally inferable from the other;

(2) One of them may be formally *inferable* from the other, but not *vice versa*;

[1] Compare Jevons, *Principles of Science*, chapter 6, § 15.

(3) They may be formally *consistent*, but neither inferable from the other;

(4) They may be *contradictory*, so that from the truth of either we may infer the falsity of the other, and from the falsity of either the truth of the other;

(5) They may be formally *inconsistent* although not contradictory, so that from the truth of either we may infer the falsity of the other, but not from the falsity of either the truth of the other.

For example, in their relation to *All S is P*, the following propositions fall into the above classes respectively:—(1) All not-P is not-S; (2) Some P is S; (3) All P is S; (4) Some S is not P; (5) All not-P is S.

EXERCISES.

89. Give the inverse of each of the following propositions:—A stitch in time saves nine; None but the brave deserve the fair; He can't be wrong whose life is in the right.

90. Give the obverse, the contrapositive, and the inverse of each of the following propositions:—The virtuous alone are truly noble; No Athenians are Helots.

Is it possible to find a simple converse or an inverse of the proposition, Some men are not wise; and, if not, why not? [University of Melbourne.]

91. "Logic must admit either negative terms or negative propositions, but has no need for both." Discuss this. [L.]

92. Assign the logical relation, if any, between each pair of the following propositions:—

(1) All crystals are solids.
(2) Some solids are not crystals.
(3) Some not crystals are not solids.
(4) No crystals are not solids.
(5) Some solids are crystals.
(6) Some not solids are not crystals.
(7) All solids are crystals. [L.]

93. "All that love virtue love angling."

Arrange the following propositions in the four following groups:—(α) Those which can be inferred from the above proposition; (β) Those from which it can be inferred; (γ) Those which are consistent with it, but which cannot be inferred from it; (δ) Those which are inconsistent with it.

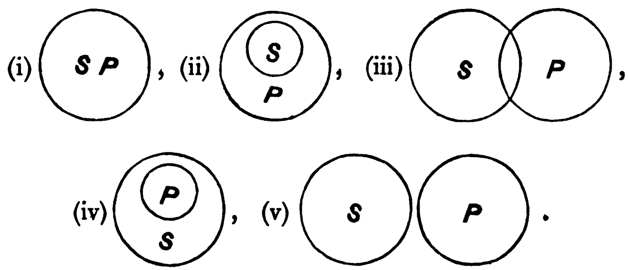
(i) None that love not virtue love angling.
(ii) All that love angling love virtue.
(iii) All that love not angling love virtue.
(iv) None that love not angling love virtue.
(v) Some that love not virtue love angling.
(vi) Some that love not virtue love not angling.
(vii) Some that love not angling love virtue.
(viii) Some that love not angling love not virtue.

CHAPTER VI.

THE DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS.

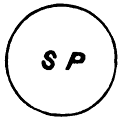
94. Methods of illustrating the ordinary processes of Formal Logic by means of Diagrams.

Representing the individuals included in any class, or denoted by any name, by a circle, it will be obvious that the five following diagrams represent all possible relations between any two classes:—

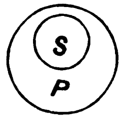


The force of the different propositional forms is to exclude one or more of these possibilities.

All S is P limits us to

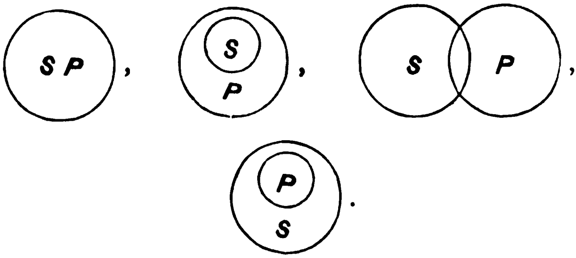


or

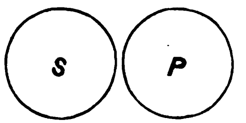


.

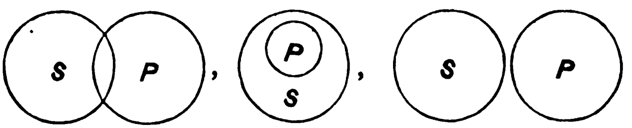
Some S is P to one of the four



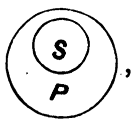
No S is P to



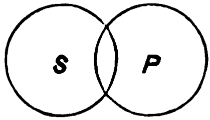
Some S is not P to one of the three



To represent *All S is P* by a single diagram, thus



or *Some S is P* by a single diagram, thus

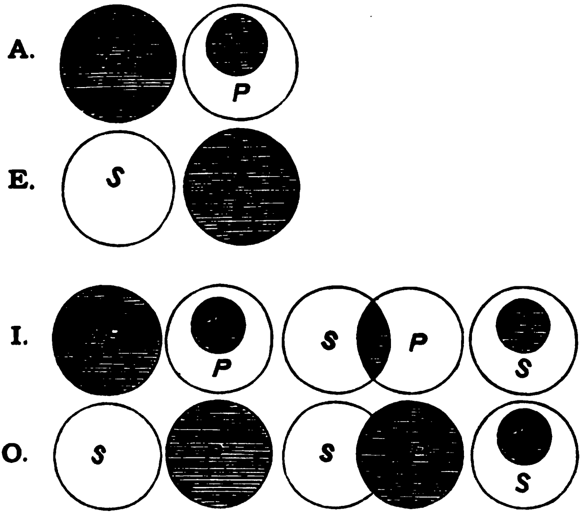


is most misleading; since in each case the proposition really leaves us with other alternatives. This method of employing the diagrams is however adopted by most logicians who have used them, including Sir William Hamilton (*Logic*, I. p. 255), and Professor Jevons (*Elementary Lessons in Logic*, pp. 72—75); and the attempt at such simplification has brought their use into undeserved disrepute. Thus, Dr Venn remarks,—"The common practice, adopted in so many manuals, of appealing to these diagrams,—Eulerian diagrams as they are often called,—seems to me very questionable. The old four propositions **A, E, I, O**, do not exactly correspond to the five diagrams, and consequently none of the moods in the syllogism can in strict propriety be represented by these diagrams" (*Symbolic Logic*, p. 15; compare also pp. 424, 425). This is undoubtedly sound as against the use of Euler's circles by Hamilton and Jevons; but I do not admit its force as against their use in the manner described above[1]. Many of the operations of Formal Logic can be satisfactorily illustrated by their aid; though it is true that they become somewhat cumbrous in relation to the Syllogism. Thus they may be employed:—

(1) To illustrate the distribution of the predicate in a proposition. In the case of each of the four fundamental propositions we may shade the part of the predicate concerning which knowledge is given us.

We then have,—

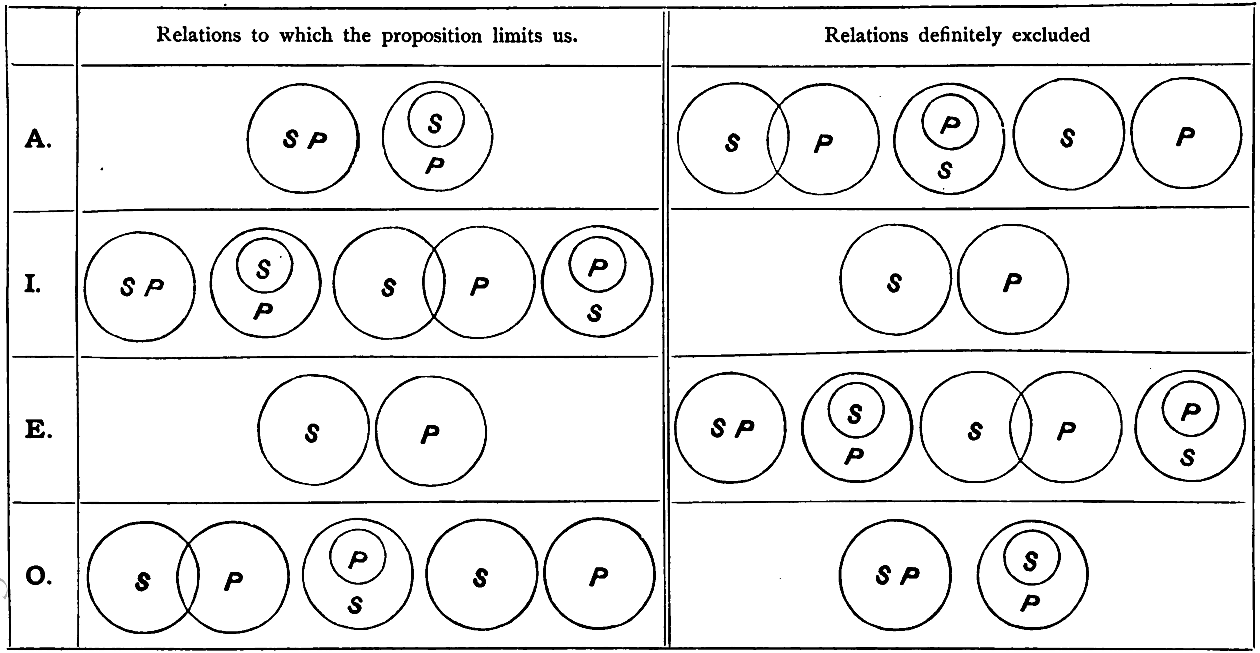
[1] The diagrams are used correctly by Ueberweg; also by Professor Ray in his *Text Book of Deductive Logic*.



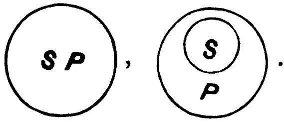
The result is that with **A** and **I** there are cases in which only part of *P* is shaded; whereas with **E** and **O**, the whole of *P* is in every case shaded; and it is made clear that negative propositions distribute, while affirmative propositions do not distribute, their predicates.

(2) To illustrate the Opposition of Propositions. Comparing two contradictory propositions, *e.g.*, **A** and **O**, we see that they have no case in common, but that between them they exhaust all possible cases. Hence the truth, that two contradictory propositions cannot be true together but that one of them must be true, is brought home to us under a new aspect. Again, comparing two subaltern propositions, *e.g.*, **A** and **I**, we notice that the former gives us all the information given by the latter and something more, since it still further limits the possibilities.

To make this point the more clear the following table is appended :—

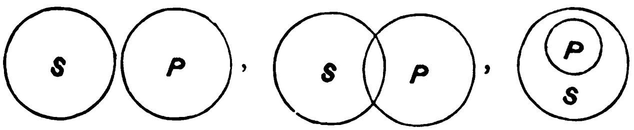

Relations to which the proposition limits us.
Relations definitely excluded
A.
I.
E.
O.
S P
S
P

(3) To illustrate the Conversion of Propositions. Thus it is made quite clear how it is that **A** admits only of Conversion *per accidens*. *All S is P* limits us to one or other of the following



The problem of Conversion is,—What do we know of *P* in either case? In the first we have All *P* is *S*, but in the second Some *P* is *S*; *i.e.*, taking the cases indifferently, we have *Some P is S* and nothing more.

Again, it is made clear how it is that **O** is inconvertible. *Some S is not P* limits us to one or other of the following,—



What then do we know concerning *P*? The three cases give us respectively

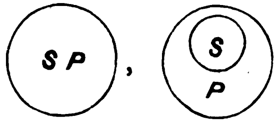
(i) No *P* is *S*,

(ii) Some *P* is *S*, and Some *P* is not *S*,

(iii) All *P* is *S*.

(i) and (iii) are contraries, and (ii) is inconsistent with both of them. Hence *nothing* can be affirmed of *P* that is true in all three cases indifferently.

(4) To illustrate the more complicated forms of immediate inference. Taking, for example, the proposition *All S is P*, we may ask, What does this enable us to assert about not-*P* and not-*S* respectively? We have one or other of these cases



With regard to not-*P*, these yield respectively,

(i) No not-*P* is *S*,

(ii) No not-*P* is *S*. And thus we obtain the contrapositive of the given proposition.

With regard to not-*S* we have

(i) All not-*S* is not-*P*,

(ii) Some not-*S* is not-*P*, (unless *P* constitutes the entire universe of discourse, a point that is further discussed subsequently); *i.e.*, in either case we may infer *Some not-S is not-P*. **E, I, O** may be dealt with similarly.

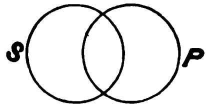
The application of the diagrams to syllogisms and to special problems will be shewn in subsequent sections.

With regard to all the above, it may be said that the use of the circles gives us nothing that could not easily have been obtained independently. This is of course true; but no one, who has had experience of the difficulty that is sometimes found by students in really mastering the elementary principles of Formal Logic, and especially in dealing with immediate inferences, will despise any means of illustrating afresh the old truths, and presenting them under a new aspect.

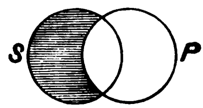
The fact that we have not a single diagram corresponding to each fundamental form of proposition is fatal if we wish to illustrate any complicated train of reasoning in this way; but in indicating the real nature of the knowledge given by the propositions themselves, it is rather an advan-

tage as shewing how limited in some cases this knowledge actually is.

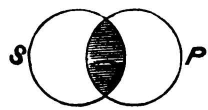
The diagrams invented and used by Dr Venn (*Symbolic Logic*, chapter 5) are extremely interesting and valuable. In this scheme the diagram



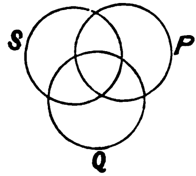
does not itself represent any proposition, but the framework into which propositions may be fitted. Denoting not-*S* by *S'* and what is both *S* and *P* by *SP*, &c., it is clear that everything must be contained in one or more of the four classes *SP*, *SP'*, *S'P*, *S'P'*; and the above diagram shews four compartments, (one being that which lies outside both the circles), corresponding to these four classes. Every universal proposition denies the existence of one or more of such classes, and it may therefore be diagrammatically represented by shading out the corresponding compartment or compartments. Thus, *All S is P*, which denies the existence of *SP'*, is represented by



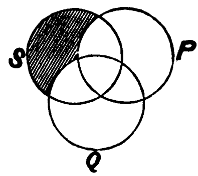
No S is P by



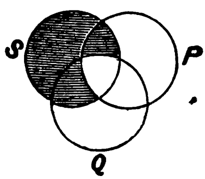
If we have three terms we have three circles and eight compartments, thus:—



All S is P or Q is represented by



All S is P and Q by



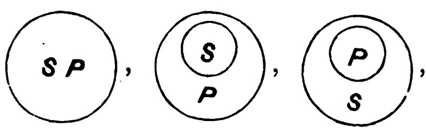
It is in cases involving three or more terms that the advantage of this scheme over the Eulerian scheme is most manifest. It is not however so easy to apply these diagrams to the case of particular propositions[1].

[1] "If we introduce particular propositions we must of course employ some additional form of diagrammatical notation......We might, for example, just draw a bar across the compartments declared to be saved; remembering of course that, whereas destruction is distributive, *i.e.*, *every* included sub-section is destroyed, the salvation is only alternative or partial, *i.e.*, we can only be sure that some of the included sub-sections are saved. Thus, 'No x is y,' leading to the destruction of xy, will destroy both xyz and $xy\bar{z}$, ($\bar{z}$ denoting not-z), if z has to be taken account of. But 'Some x is y', saving a part of xy, does not in the

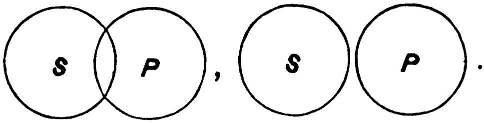
Lambert's scheme of representing propositions by combinations of straight lines will be touched upon in connexion with the syllogism; see section 178.

[A passing reference may be made to the fundamental objection raised by Mansel against the introduction of any such aids at all. "If Logic is exclusively concerned with Thought, and Thought is exclusively concerned with Concepts, it is impossible to approve of a practice, sanctioned by some eminent Logicians, of representing the relation of terms in a syllogism by that of figures in a diagram. To illustrate, for example, the position of the terms in *Barbara* by a diagram of three circles, one within another, is to lose sight of the distinctive mark of a concept, that it cannot be presented to the sense, and tends to confuse the mental inclusion of one notion in the sphere of another, with the local inclusion of a smaller portion of space in a larger" (*Prolegomena Logica*, p. 55). In answering this objection, it seems sufficient to point out that even conceptualist logicians must recognise and deal with the *extension* of concepts, and that the Eulerian diagrams make no pretence of representing the concepts themselves, but only their extension.]

95. Any information given with respect to two classes limits the possible relations between them to one or more of the five following,—



least indicate whether such part is xyz or $xy\bar{z}$......If it were worth while thus to illustrate complicated groups of propositions of the kind in question, it could, I fancy, be done with very tolerable success." Venn in *Mind*, 1883, pp. 599, 600.



Such information may in all cases be expressed by means of the propositional forms **A**, **I**, **E**, **O**.

Let the five relations be designated respectively α, β, γ, δ, ϵ[1]. Information is given when the possibility of one or more of these is denied; in other words, when we are limited to one, two, three, or four of them. Let limitation to α or β, (*i.e.*, the exclusion of γ, δ and ϵ), be denoted by α, β; limitation to α, β, or γ, (*i.e.*, the exclusion of δ and ϵ), by α, β, γ; and so on.

Now if we wish to express such information by means of the four ordinary propositional forms, we find that sometimes a single proposition will suffice for our purpose; thus α, β is expressed by "All S is P." Sometimes we require a combination of propositions; thus α is expressed by saying that "All S is P, and also All P is S," (since All S is P excludes γ, δ, ϵ, and All P is S further excludes β). Some other cases are still more complicated; thus the fact that we are limited to α or δ cannot be expressed more simply than by saying, "Either All S is P and All P is S, or else Some S is P, Some S is not P, and some P is not S."

Let $\mathbf{A}$ = All S is P, $\mathbf{A}_1$ = All P is S, and similarly for the other propositions. Also let $\mathbf{AA}_1$ = All S is P and All P is S, &c. Then the following is a scheme for all possible cases:—

[1] Thus, the classes being S and P, α denotes that S and P are wholly coincident; β that P contains S and more besides; γ that S contains P and more besides; δ that S and P overlap each other, but that each includes something not included by the other; ϵ that S and P have nothing whatever in common.

limitation to	*denoted by*	*limitation to*	*denoted by*
α	**AA_1**	α, β, γ	**A** or **A_1**
β	**AO_1**	α, β, δ	**A** or **IO_1**
γ	**A_1O**	α, β, ϵ	**A** or **E**
δ	**IOO_1**	α, γ, δ	**A_1** or **IO**
ϵ	**E**	α, γ, ϵ	**A_1** or **E**
α, β	**A**	α, δ, ϵ	**AA_1** or **OO_1**
α, γ	**A_1**	β, γ, δ	**IO** or **IO_1**
α, δ	**AA_1** or **IOO_1**	β, γ, ϵ	**AO_1** or **A_1O** or **E**
α, ϵ	**AA_1** or **E**	β, δ, ϵ	**O_1**
β, γ	**AO_1** or **A_1O**	γ, δ, ϵ	**O**
β, δ	**IO_1**	$\alpha, \beta, \gamma, \delta$	**I**
β, ϵ	**AO_1** or **E**	$\alpha, \beta, \gamma, \epsilon$	**A** or **A_1** or **E**
γ, δ	**IO**	$\alpha, \beta, \delta, \epsilon$	**A** or **O_1**
γ, ϵ	**A_1O** or **E**	$\alpha, \gamma, \delta, \epsilon$	**A_1** or **O**
δ, ϵ	**OO_1**	$\beta, \gamma, \delta, \epsilon$	**O** or **O_1**

It will be found that any other combinations of propositions than those here given involve either contradictions or redundancies, or else no information is given because all the five relations that are *a priori* possible still remain possible.

For example, **AI** is clearly redundant; **AO** is self-con-

tradictory; **A** or **A_1O** is redundant (since the same information is given by **A** or **A_1**); **A** or **O** gives no information (since it excludes no possible case). The student is recommended to test other combinations similarly. It must be remembered that **I_1 = I**, and **E_1 = E**.

It should be noticed that if we read the first column downwards and the second column upwards we get pairs of contradictories.

EXERCISE.

96. Illustrate by means of the Eulerian diagrams:—(1) the relation between **A** and **E**, (2) the relation between **I** and **O**, (3) the conversion of **I**, (4) the contraposition of **O**, (5) the inversion of **E**.

CHAPTER VII.

THE LOGICAL FOUNDATION OF IMMEDIATE INFERENCES.

97. Legitimacy of Inferences based on the Square of Opposition; and Legitimacy of Obversion.

The inferences based on the square of opposition may be considered to depend exclusively on the three fundamental laws of thought: namely, the Law of Identity,—*A* is *A*; the Law of Contradiction,—*A* is not not-*A*; the Law of Excluded Middle,—*A* is either *B* or not-*B*. For example, from the truth of All *S* is *P* we may infer the truth of Some *S* is *P* by the Law of Identity, and the falsity of Some *S* is not *P* by the Law of Contradiction; from the falsity of All *S* is *P* we may infer the truth of Some *S* is not *P* by the Law of Excluded Middle.

Obversion also may be based entirely on the Laws of Contradiction and Excluded Middle. From All *S* is *P* we get No *S* is not-*P* by the Law of Contradiction; and from No *S* is *P* we get All *S* is not-*P* by the Law of Excluded Middle.

98. Legitimacy of the various processes of Conversion.

Aristotle proves the conversion of **E** *indirectly*, as

follows[1]:—No S is P, therefore, No P is S; for if not, Some individual P, say Q, is S; Q then is both S and P; but this is inconsistent with the original proposition.

Having shewn that the simple conversion of **E** is legitimate, we can prove that the conversion *per accidens* of **A** is also legitimate.

All S is P,
therefore, Some P is S;
for, if not, No P is S,
and therefore (by conversion) No S is P;

but this is inconsistent with the original supposition.

The legitimacy of the simple conversion of **I** follows similarly.

Having established the validity of the processes of obversion and conversion, nothing further is needed in order to shew the legitimacy of contraposition and inversion[2].

In the above nothing seems to be required beyond the principles of contradiction and excluded middle; the proof is not however satisfactory, for it may be plausibly maintained that in passing from "Some individual P, say Q, is S," to "Some Q is both S and P," we have already practically assumed the process of conversion[3].

[1] "By the method called ἔκθεσις, *i.e.*, by the *exhibition* of an individual instance." See Mansel's *Aldrich*, pp. 61, 2.

[2] We shall in the following chapter enquire whether or not special implications with regard to existence are involved in any of the processes considered in this and the preceding section.

[3] This I imagine would be the line taken by De Morgan who denies that conversion can be based exclusively on the three fundamental laws of thought. He remarks,—"When any writer attempts to shew *how* the perception of convertibility '*A* is *B* gives *B* is *A*' follows from the principles of identity, difference, and excluded middle, I shall be able to judge of the process; as it is, I find that others do not go beyond the

But, however this may be, it is clear that conversion is capable of being justified without any explicit reference to the above-mentioned principles. For it seems sufficient to say that in the case of each of the four fundamental forms of proposition, its conversion (or in the case of an **O** proposition, the impossibility of converting it) is self-evident. Thus, taking an **E** proposition, it is self-evident that if one class is entirely excluded from another class, this second class is entirely excluded from the first[1].

simple assertion, and that I myself can detect the *petitio principii* in every one of my own attempts" (*Syllabus*, p. 47).

At any rate in basing conversion on the laws of contradiction and excluded middle, recourse must be had to an indirect method of proof; and I do not see how the application of the three laws of thought indicates our inability to convert an **O** proposition. (It may be observed that we can shew the inconvertibility of **O** by proving that *Some S is not P* is compatible with every one of the following propositions,—*All P is S*, *Some P is S*, *No P is S*, *Some P is not S*. My point is that I do not see how any direct application of the three laws of thought aids us in recognising the compatibility of *Some S is not P* and *All P is S*.)

[1] "Aldrich assumes the distribution of the predicate in a negative to prove the simple conversion of **E**. Those who adopt Aristotle's proof of the latter might deduce the former from it. Both however may fairly be allowed to stand on their own evidence." Mansel's *Aldrich*, p. 52.

I cannot at all agree with Professor Bain who would establish the rules of conversion by a kind of inductive proof. He writes as follows:—"When we examine carefully the various processes in Logic, we find them to be material to the very core. Take *Conversion*. How do we know that, if No X is Y, No Y is X? By examining cases in detail, and finding the equivalence to be true. Obvious as the inference seems on the mere formal ground, we do not content ourselves with the formal aspect. If we did, we should be as likely to say, All X is Y gives All Y is X; we are prevented from this leap merely by the examination of cases" (*Logic, Deduction*, p. 251). No one on reflection would maintain it to be self-evident that the simple conversion of **A** is legitimate;

In the case of an **A** proposition it is clear on reflection that the statement *All S is P* may include one or other of the two relations of classes,—either S and P coincident, or P containing S and more besides,—but that these are the only two possible relations to which it can be applied. It is self-evident that in each of these cases *Some P is S*; and hence the inference by conversion from an **A** proposition is shewn to be justified[1]. In the case of an **O** proposition, if we consider all the relationships of classes in which it holds good, we find that nothing is true of P in terms of S in *all* of them. Hence **O** is inconvertible[2]. Similarly, without assuming conversion, we might logically justify the process of contraposition.

99. Reduction of immediate inferences to the mediate form[3].

We have defined immediate inference as the inference of a proposition from a single other proposition; mediate inference is the inference of a proposition from at least two other propositions.

(1) One of the old Greek logicians, Alexander of Aphrodisias, establishes the conversion of **E** by means of a syllogism in *Ferio*.

No S is P,
therefore, No P is S;

for when the case is put to us we recognise immediately that the *contradictory* of *All P is S* is compatible with *All S is P*. But we can maintain it to be self-evident that the simple conversion of **E** is legitimate.

[1] Compare section 94, where these inferences are illustrated by the aid of the Eulerian diagrams.

[2] Again, compare Euler's diagrams. See also note 3 on page 131.

[3] Students who have not already a technical knowledge of the syllogism may omit this section until they have read the earlier chapters of Part III.

for, if not, then by the law of contradiction, Some P is S; and we have this syllogism,—

No S is P,
Some P is S,

therefore, Some P is not P,

a *reductio ad absurdum*[1].

(2) The conversion of **A** may be established similarly by means of a syllogism in *Celarent*.

All S is P,
therefore, Some P is S;

for, if not, then No P is S, and we have this syllogism,—

No P is S,
All S is P,

therefore, No S is S,

which is absurd.

(3) It may be plausibly maintained that in Aristotle's proof of the conversion of **E** (given in the preceding section), there is an implicit syllogism: namely,—Q is P, Q is S, therefore, Some S is P.

(4) The contraposition of **A** may be established by means of a syllogism in *Camestres* as follows,—

Given All S is P,
we have also No not-P is P, by the law of contradiction,

therefore, No not-P is S.

(5) We might also obtain the contrapositive of All S is P as follows:—

[1] Compare Mansel's *Aldrich*, p. 62.

By the law of excluded middle, All not-*P* is *S* or not-*S*, and, by hypothesis, All *S* is *P*,

therefore, All not-*P* is *P* or not-*S*;

but, by the law of contradiction, No not-*P* is *P*,

therefore, All not-*P* is not-*S*[1].

(6) The contraposition of **A** may also be established indirectly by means of a syllogism in *Darii*:—

All *S* is *P*,

therefore, No not-*P* is *S*;

for, if not, Some not-*P* is *S*; and we have the following syllogism,—

All *S* is *P*,
Some not-*P* is *S*,

therefore, Some not-*P* is *P*,

which is absurd[2].

All the above are interesting, as illustrating the processes of immediate inference; but regarded as proofs they labour under the disadvantage of deducing the less complex by means of the more complex.

The following are only interesting as curiosities:—

(7) Wolf obtains the subaltern of a universal proposition by a syllogism in *Darii*.

Given All *S* is *P*,
we have also Some *S* is *S*, by the law of identity

therefore, Some *S* is *P*.

(Compare Mansel, *Prolegomena Logica*, p. 217.)

(8) "Still more absurd is the elaborate system which

[1] Compare Jevons, *Principles of Science*, chapter 6, § 2; and *Studies in Deductive Logic*, p. 44.

[2] Compare De Morgan, *Formal Logic*, p. 25.

Krug, after a hint from Wolf, has constructed in which all immediate inferences appear as hypothetical syllogisms; a major premiss being supplied in the form, 'If all *A* is *B*, some *A* is *B*.' The author appears to have forgotten, that either this premiss is an additional empirical truth, in which case the immediate reasoning is not a logical process at all; or it is a formal inference, presupposing the very reasoning to which it is prefixed, and thus begging the whole question" (Mansel, *Prolegomena Logica*, p. 217).

CHAPTER VIII.

THE EXISTENTIAL IMPORT OF CATEGORICAL PROPOSITIONS[1].

100. Existence and the Universe of Discourse.

The discussion of "existence" upon which we are about to enter is no kind of metaphysical enquiry[2]. By existence

[1] It may be advisable for students, on a first reading, to omit this chapter.

[2] "As to the nature of this existence, or what may really be meant by it, we have hardly any need to trouble ourselves, for almost any possible sense in which the logician can understand it will involve precisely the same difficulties and call for the same solution of them. We may leave it to any one to define the existence as he pleases, but when he has done this it will always be reasonable to enquire whether there is anything existing corresponding to the *X* or *Y* which constitute our subject and predicate. There can in fact be no fixed tests for this existence, for it will vary widely according to the nature of the subject-matter with which we are concerned in our reasonings. For instance, we may happen to be speaking of ordinary phenomenal existence, and at the time present; by the distinction in question is then meant nothing more and nothing deeper than what is meant by saying that there are such things as antelopes and elephants in existence, but not such things as unicorns or mastodons. If again we are referring to the sum-total of all that is conceivable, whether real or imaginary, then we should mean what is meant by saying that everything must be regarded as existent which does not involve a contradiction in terms, and nothing which does. Or if we were concerned with Wonderland and its occupants we need not go deeper down than they do who tell us that March hares

we mean merely membership of the universe of discourse, whatever that may happen to be. It may be the whole universe of things, using the word "thing" in its very widest signification; more usually the reference is to some limited universe[1]. The question what is the universe of discourse in any particular case belongs of course to the matter and not to the form of thought. What we have now to discuss is the extent, if any, to which membership of the universe of discourse is implied in categorical propositions; and the ways in which different answers that may be given to this question affect ordinary logical doctrines.

101. Formal Logic and the Existential Import of Propositions.

We might naturally be inclined to think that Formal Logic cannot possibly have any concern with questions relating to Existence. On reflection however it becomes clear that some questions of this kind necessarily present themselves for consideration. Even granting that the

exist there. In other words, the interpretation of the distinction will vary very widely in different cases, and consequently the tests by which it would have in the last resort to be verified; but it must always exist as a real distinction, and there is a sufficient identity of sense and application pervading its various significations to enable us to talk of it in common terms." Venn, *Symbolic Logic*, pp. 127, 128.

[1] "The universe of discourse is sometimes limited to a small portion of the actual universe of things, and is sometimes co-extensive with that universe." Boole, *Laws of Thought*, p. 166. Compare also De Morgan, *Syllabus of a Proposed System of Logic*, §§ 122, 3; and Jevons, *Principles of Science*, chapter 3, § 4.

It must be clearly understood that the universe of discourse is by no means necessarily identical with the region of what we ordinarily call "fact"; it may be the universe of dreams, or of imagination, or of some particular realm of imagination, *e.g.*, modern fiction, or fairy land, or the world of the Homeric poems.

formal logician may hold that given the proposition *All S is P*, it is no concern of his whether or not there are any individuals actually belonging to the classes *S* and *P*, nevertheless he must admit that the proposition at least involves that if there are any *S* there must be some *P*, while it does not involve that if there are any *P* there must be some *S*. But now convert the proposition. We obtain *Some P is S*, and this does involve that if there are any *P* there must be some *S*. I do not therefore see how in converting the given proposition this assumption can be avoided. Thus, from "All dragons are serpents," we may infer by conversion "Some serpents are dragons," and this proposition implies that if there are serpents there are also dragons. Similarly, in passing from *All S is P* to *Some not-S is not-P*, it must at least be assumed that if *S* does not constitute the entire universe of discourse, neither does *P* do so.

Jevons indeed remarks that he does not see how there can be in deductive logic any question about existence[1]; and with reference to the opposite view taken by De Morgan, he says, "This is one of the few points in which it is possible to suspect him of unsoundness." I can however attach no meaning to Jevons's own "Criterion of Consistency" (*Studies in Deductive Logic*, p. 181) unless it has some reference to "existence." "It is assumed as a necessary law that every term must have its negative. This was called the *Law of Infinity* in my first logical essay (*Pure Logic*, p. 65; see also p. 45); but as pointed out by Mr A. J. Ellis, it is assumed by De Morgan, in his *Syllabus*, Article 16. Thence arises what I propose to call the *Criterion of Consistency*, stated as follows:—*Any two or more propositions are contradictory when, and only when, after all possible substitutions are made,*

[1] *Studies in Deductive Logic*, p. 141.

they occasion the total disappearance of any term, positive or negative, from the Logical Alphabet." What can this mean but that although we may deny the existence of the combination *AB*, we cannot without contradiction deny the existence of *A* itself, or not-*A*, or *B*, or not-*B*? This assumption regarding the existential implication of propositions runs through the whole of Jevons's equational logic. For example, I take the following quite at random,—"There remain four combinations, *ABC*, *aBC*, *abC*, and *abc*. But these do not stand on the same logical footing, because if we were to remove *ABC*, there would be no such thing as *A* left; and if we were to remove *abc* there would be no such thing as *c* left. Now it is the Criterion or condition of logical consistency that every separate term and its negative shall remain. Hence there must exist some things which are described by *ABC*, and other things described by *abc*" (*Studies in Deductive Logic*, p. 216). We will return in section 106 to a consideration of Jevons's Criterion of Consistency itself.

102. Various Suppositions concerning the Existential Import of Categorical Propositions.

Several different views may be taken as to what implication with regard to existence, if any, is involved in categorical propositions. We may formulate the following for special discussion[1]:—

[1] The suppositions that follow are not intended to be exhaustive. We might, for instance, regard propositions as implying the existence both of their subjects and predicates, but not of the contradictories of these; or we might regard affirmatives as always implying the existence of their subjects but negatives as not necessarily implying the existence of theirs. This last supposition represents the view of Ueberweg. I have not space however to give a separate discussion to suppositions other than those mentioned in the text.

(1) It may be held that every categorical proposition implies the existence both of objects denoted by the terms directly involved and also of objects denoted by their contradictories; that, for example, *All S is P* implies the existence of *S*, not-*S*, *P*, not-*P*. This view is implied in Jevons's Criterion of Consistency mentioned in the preceding section. It is also practically adopted by De Morgan[1].

(2) It may be held that every proposition implies simply the existence of its subject. This is Mill's view (with regard to real propositions); for he holds that we cannot give information about a non-existent subject[2].

(3) Dr Venn is of opinion that for purposes of Symbolic Logic, *universal* propositions *should not* be regarded as implying the existence of their subjects, but that *particular* propositions *should* be regarded as doing so. This doctrine might be extended to ordinary Formal Logic. *All S is P* now merely denies the existence of anything that is both *S* and not-*P*; *No S is P* denies the existence of anything that is both *S* and *P*; *Some S is P* affirms the existence of something that is both *S* and *P*; *Some S is not P* affirms the existence of something that is both *S* and not-*P*[3].

[1] "By the *universe* (of a proposition) is meant the collection of all objects which are contemplated as objects about which assertion or denial may take place. *Let every name which belongs to the whole universe be excluded as needless*: this must be particularly remembered. Let every object which has not the name *X* (*of which there are always some*) be conceived as therefore marked with the name *x* meaning not-*X*" (*Syllabus*, pp. 12, 13).

[2] "An accidental or non-essential affirmation does imply the real existence of the subject, because in the case of a non-existent subject there is nothing for the proposition to assert" (*Logic*, I., chapter 6, § 2).

[3] This doctrine is practically identical with one put forward in a more paradoxical form by Professor Brentano. Compare *Mind*, 1876, pp. 289—292. "Where we say *Some man is sick*, Brentano gives as a

(4) It may be held that in Formal Logic we should not regard propositions as necessarily implying the existence either of their subjects or of their predicates. On this view, as on the last, the full implication of *All S is P* with regard to existence may be expressed by saying that it denies the existence of anything that is at the same time *S* and not-*P*. Similarly *No S is P* implies the existence neither of *S* nor of *P*, but merely denies the existence of anything that is both *S* and *P*. *Some S is P* (or *is not P*) may be read Some *S*, if there is any *S*, is *P* (or is not *P*). Here we do not even negative or deny the existence of any class absolutely[1]; the sum total of what we affirm with regard to existence is that *if* any *S* exists, then something which is both *S* and *P* (or *S* and not-*P*) exists[2].

In the three following sections we shall investigate the consequences of the above suppositions respectively; and in section 106 we shall enquire which of them it is most convenient for the logician normally to adopt.

103. Immediate Inferences and the Existential Import of Propositions.

We shall in this section enquire how far different

substitute, *There is a sick man.* Instead of *No stone is alive*, he puts *There is not a live stone. Some man is not learned* becomes *There is an unlearned man.* Finally, *All men are mortal* is to be expressed in his system *There is not an immortal man.*"

[1] I may here call attention to what appears to be an erroneous dictum of Jevons's. "We cannot," he says, "make any statement except a truism without implying that certain combinations of terms are contradictory and excluded from thought" (*Principles of Science*, 2nd edition, p. 32). This is true of universals (though somewhat loosely expressed), but it does not seem to be true of particular propositions, whatever view we may take of them.

[2] It may be observed that on this interpretation, particular propositions have a hypothetical and not a purely categorical character.

suppositions regarding the existential import of propositions affect the validity of Obversion and Conversion and the other immediate inferences based upon these. In the next section we shall consider inferences connected with the Square of Opposition.

We may take in order the suppositions formulated in the preceding section.

(1) *Let every proposition imply the existence of both subject and predicate and their contradictories.*

In this case it is clear that Conversion, Obversion, Contraposition, and Inversion are all valid processes. The existence of all the terms involved in them is guaranteed by the original proposition, and they can carry no existential implication that the original proposition does not itself carry[1].

(2) *Let every proposition imply simply the existence of its subject.*

Then (*a*) Obversion is obviously valid.

(*b*) The Conversion of **A** is valid, and also that of **I**. If *All S is P* and *Some S is P* imply directly the existence of *S*, then they clearly imply indirectly the existence of *P*; and this is all that is required in order that their conversion may be legitimate. The Conversion of **E** is not valid; for *No S is P* implies neither directly nor indirectly the existence of *P*, whilst its converse does imply this.

(*c*) The Contraposition of **E** is valid, and also that of **O**. *No S is P* and *Some S is not P* both imply on our present supposition the existence of *S*, and since by the law of excluded middle every *S* is either *P* or not-*P*, it follows

[1] The reader may be reminded that in our original working out of these immediate inferences we provisionally adopted the supposition in question.

that they imply indirectly the existence of not-*P*. The Contraposition of **A** is not valid; for it involves the conversion of **E**, which we have already seen not to be valid[1].

(*d*) Inversion is not a valid process; for it involves in the case of both **A** and **E** the conversion of an **E** proposition[2].

Of course if along with an **E** proposition we are given specially the information that *P* exists, or if this is implied in some other proposition given us at the same time, then the **E** proposition may be converted. Under corresponding circumstances the contraposition and inversion of **A** and the inversion of **E** may be valid[3]. Or again, given simply "No *S* is *P*," we may infer "Either *P* is non-existent or no *P* is *S*"; and similarly in other cases.

(3) *Let particulars imply, while universals do not imply, the existence of their subjects.*

(*a*) The validity of Obversion is again obviously unaffected.

(*b*) The Conversion of **E** is valid, and also that of **I**, but not that of **A**.

(*c*) The Contraposition of **A** is valid, and also that of **O**, but not that of **E**.

[1] Or we might argue directly that the contraposition of **A** is not valid, since *All S is P* does not imply the existence of not-*P*, whilst its contrapositive does imply this.

[2] Or again we might argue directly from the fact that neither *All S is P* nor *No S is P* implies the existence of not-*S*.

[3] For example, given (α) No *S* is *P*, (β) All *R* is *P*; we may under our present supposition convert (α), since (β) implies indirectly the existence of *P*; and we may contraposit (β), since, as shewn above, (α) implies indirectly the existence of not-*P*. It will also be found that given these two propositions together, they both admit of inversion.

(*d*) Inversion is not a valid process.

These results are obvious; and the final outcome is,—what might have been anticipated,—that we may infer a universal from a universal, or a particular from a particular, but not a particular from a universal[1].

(4) *Let no proposition logically imply the existence either of its subject or of its predicate.*

Having now got rid of the implication of the existence

[1] But, of course, from the two propositions, All S is P, Some P is R, we can infer Some P is S; and similarly in other cases. The impossibility under this supposition of inferring a particular from universals will be touched upon again in connexion with the syllogism.

Dr Venn, indeed, whilst adopting this supposition, considers that three universals can establish a particular. Compare *Symbolic Logic*, pp. 142—149. Any universe of discourse contains *a priori* four classes,—(1) SP, (2) S not-P, (3) P not-S, (4) not-S not-P. *All S is P* negatives (2); *No not-S is P* negatives (3); *All not-S is P* negatives (4). Given these three propositions therefore, we are able to infer that there is some SP, for this is all that we have left in the universe of discourse. Mr Bradley (*Principles of Logic*, p. 153) criticises this conclusion very severely, but in a style that I have no particular desire to imitate. Still I cannot accept Dr Venn's conclusion. The assumption of course is that the universe of discourse can never be entirely emptied of content. No doubt as a matter of fact in most cases it cannot be. But where we admit very limited universes this does not seem a legitimate formal assumption. Four men, for example, may be discussing the nature of the contents of some universe of discourse, (say, those who have passed a certain examination), and each may at the outset possess an item of information not possessed by any of the others. Is there any contradiction in our supposing the result of their discussion to be that the contents in question are found to be *nil*? To follow up our concrete example, I can see no contradiction in supposing their respective contributions to be as follows: No Trinity men are in the first class; No Trinity men are in any other class than the first; No non-Trinity men are in the first class; No non-Trinity men are in any other class than the first.

either of subject or predicate in the case of all propositions, we might naturally suppose that in no case in which we make an immediate inference need we trouble ourselves with any question of existence at all. As already suggested, however, this conclusion would be erroneous.

(*a*) Obversion is still a valid process. Take, for example, the obversion of *No S is P*. All that the obverse *All S is not-P* implies with regard to existence is that if there is any *S* there is also some not-*P*. But this is necessarily implied in the proposition *No S is P* itself. If there is any *S* it is by the law of excluded middle either *P* or not-*P*; therefore, given that *No S is P*, it follows immediately that if there is any *S* there is some not-*P*.

(*b*) The Conversion of **E** is valid. Since *No S is P* denies the existence of anything that is both *S* and *P*, it implies that if there is any *P* there is some not-*S*; and this is the only implication with regard to existence involved in its converse. The Conversion of **A**, however, is not valid; nor is that of **I**. For *Some P is S* implies that if there is any *P* there is also some *S*; but this is not implied either in *All S is P* or in *Some S is P*.

(*c*) That the Contraposition of **A** is valid follows from the fact that the obversion of **A** and the conversion of **E** are both valid[1]. That the Contraposition of **E** and that of **O** are invalid follows from the fact that the conversion of **A** and that of **I** are both invalid.

(*d*) That Inversion is invalid follows similarly.

[1] Or we might argue directly as follows: since the proposition *All S is P* denies the existence of anything that is both *S* and not-*P*, it implies that if there is any not-*P* there is some not-*S*; and this is the only implication with regard to existence involved in its contrapositive.

On our present supposition then the following are valid: the obversion and contraposition of **A**, the obversion of **I**, the obversion and conversion of **E**, the obversion of **O**; the following are invalid: the conversion and inversion of **A**, the conversion of **I**, the contraposition and inversion of **E**, the contraposition of **O**. We find therefore that inferences to universals remain in all cases valid; inferences to particulars are rendered invalid except in the case of obversion[1].

The following is a summary of results, the figures indicating under which suppositions each process is valid: Obversion[2] of **A**, **I**, **E**, **O**,—(1), (2), (3), (4); Conversion of **A**, and Contraposition of **E**,—(1), (2); Conversion of **I**, and Contraposition of **O**,—(1), (2), (3); Conversion of **E**, and Contraposition of **A**,—(1), (3), (4); Inversion of **A**, **E**,—(1).

104. The Doctrine of Opposition and the Existential Import of Propositions.

The ordinary doctrine of Opposition is the following: (*a*) The truth of *Some S is P* follows from that of *All S is P* and the truth of *Some S is not P* from that of *No S is P*, (doctrine of Subalternation); (*b*) *All S is P* and *Some S is not P* cannot both be true and they cannot both be false, similarly for *Some S is P* and *No S is P*, (doctrine of Contradiction); (*c*) *All S is P* and *No S is P* cannot both be true but they may both be false, (doctrine of Contrariety);

[1] We are not in this section considering the process of subalternation.

[2] We find that obversion remains valid on all the suppositions we have been specially discussing. If however we regard affirmatives as implying the existence of their subjects while negatives do not, then of course we cannot pass by obversion from **E** to **A**, or from **O** to **I**.

(*d*) *Some S is P* and *Some S is not P* may both be true but they cannot both be false, (doctrine of Sub-contrariety). We will now examine how far these several doctrines hold good under various suppositions with regard to the existential import of propositions.

(1) *Let every proposition imply the existence of both subject and predicate and their contradictories.*

On this supposition, any proposition containing a term, which is either unrepresented in the universe of discourse or which exhausts that universe, is false[1]; for it implies what is inconsistent with fact. It follows that a pair of contradictories as usually stated, and also a pair of sub-contraries, may both be false. For example, *All S is P* and *Some S is not P* both imply the existence of *S*. In the case then in which *S* does not exist, these propositions are not true contradictories. We must not of course say that under our present supposition true contradictories cannot be found; for this is always possible. The true contradictory of *All S is P* is *Either some S is not P, or else either S, or not-S, or P, or not-P is non-existent.* Similarly in other cases. The ordinary doctrines of Subalternation and Contrariety remain unaffected.

(2) *Let every proposition imply simply the existence of its subject.*

For reasons similar to those stated above, the ordinary doctrines of Contradiction and Sub-contrariety again fail to hold good. The true contradictory of *All S is P* now becomes *Either some S is not P, or S is non-existent.* The ordinary doctrines of Subalternation and Contrariety again remain unaffected.

[1] The hypothesis that no proposition *can* contain such a term is obviously arbitrary.

(3) *Let particulars imply, while universals do not imply, the existence of their subjects.*

(*a*) The ordinary doctrine of Subalternation does not hold good. *Some S is P*, for example, implies the existence of *S*, while this is not implied by *All S is P*.

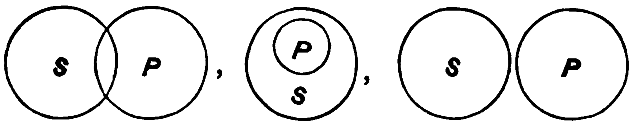
(*b*) The ordinary doctrine of Contradiction does hold good. For example, *All S is P* denies that there is any *S* that is not-*P*; *Some S is not P* affirms that there is some *S* that is not-*P*. It is clear that these propositions cannot both be true; it is also clear that they cannot both be false.

(*c*) The ordinary doctrine of Contrariety does not hold good. For if there is no implication of the existence of the subject in universal propositions we are not actually precluded from asserting together two propositions that are ordinarily given[1] as contraries. We may say *All S is P*

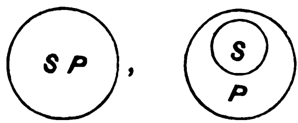
[1] Of course on the view under consideration it is somewhat misleading to continue to speak of these two propositions as contraries. Thus, Mr Bradley, in reference to Dr Venn's treatment of this question, remarks on "the extraordinary assertion that contrary judgments, such as 'All x is y' and 'No x is y,' can be compatible." He continues,—"It is not worth while to enter into a discussion of this matter. They are of course compatible if you allow yourself to play on their ambiguity; but how in that case they can be said to be contrary I have no conception. 'The interesting and unexpected application' is to me, I confess, not anything beyond a confused example of a well-known doctrine concerning the relations of possibility and existence. But I confess besides that I have never been much used 'to discuss the question in a perfectly matter of fact way'" (*Principles of Logic*, p. 154). This is scornful, after Mr Bradley's wont, but it is hardly fair. Granted that universals do not imply the existence of their subjects, we get the following result: if by contraries are meant propositions that are incompatible, then all x is y and no x is y are not contraries; if by contraries are meant (as in the ordinary square of opposition) the two propositions all x is y and no x is y, then so-called contraries are not necessarily incompatible. Here there is nothing extraordinary,

and *No S is P*; but this is virtually to deny the existence of *S*. This point may be illustrated by aid of the Eulerian diagrams[1].

All S is P excludes



No S is P does not definitely affirm the third of these cases, but besides excluding the first two of them also excludes



Thus the two propositions between them exclude all possible cases, supposing *S* to exist.

(*d*) The ordinary doctrine of Sub-contrariety does not hold good. *Some S is P* and *Some S is not P* are both false in the case in which *S* does not exist.

(4) *Let no proposition imply the existence either of its subject or of its predicate.*

(*a*) The ordinary doctrine of Subalternation holds good.

though there may be something unexpected; nor is there anything confused. Is it not really Mr Bradley who is "playing upon an ambiguity"?

[1] It will be observed that, numbering the different possible cases, i, ii, iii, iv, v, as in section 94, **A** on the supposition under discussion *excludes* iii, iv, v; **E** *excludes* i, ii, iii, iv; **I** *affirms* i, ii, iii, or iv; **O** *affirms* iii, iv, or v.

(*b*) The ordinary doctrine of Contradiction does not hold good. *All S is P*, for example, merely denies the existence of any *S*'s that are not *P*'s; *Some S is not P* merely asserts that *if* there are any *S*'s, some of them are not *P*'s. In the case in which *S* does not exist in the universe of discourse we cannot affirm the falsity of either of these propositions.

(*c*) The ordinary doctrine of Contrariety does not hold good. This follows just as in the case of our third supposition.

(*d*) The ordinary doctrine of Sub-contrariety remains unaffected.

The following is a summary of results, the figures indicating under which suppositions each of the ordinary doctrines holds good: the ordinary doctrine of Subalternation,—(1), (2), (4); the ordinary doctrine of Contradiction,—(3); the ordinary doctrine of Contrariety,—(1), (2); the ordinary doctrine of Sub-contrariety,—(4).

Some of the results obtained in this and in the preceding section are both curious and interesting. In Part III, Chapter XI, we shall make a similar enquiry in reference to the Syllogism. The whole question has hitherto failed to receive the amount of attention it deserves in consequence of the prevalence of the notion that Formal Logic cannot possibly have any concern with considerations of existence[1].

105. The relation between the propositions All *S* is *P* and All not-*S* is *P*.

[1] Dr Venn discusses the question fully in its relation to Symbolic Logic. He does not profess to deal with it fully in its relation to ordinary Formal Logic; but the germs of a complete treatment from this point of view are contained in what he says.

This is an interesting case to notice in connexion with the discussion raised in the preceding sections.

All S is P = No S is not-P = No not-P is S.
All not-S is P = No not-S is not-P = No not-P is not-S
= All not-P is S.

The given propositions come out therefore as contraries.

On the view that we ought not to enter into any discussion concerning existence in connexion with immediate inference, we must I suppose rest content with this statement of the case. It seems however sufficiently curious to demand further investigation and explanation. We may as before take different suppositions with regard to the existential import of propositions.

(1) If every proposition implies the existence of both subject and predicate and their contradictories, then it is at once clear that the two propositions cannot both be true together; for between them they deny the existence of not-P.

(2) On the view that propositions imply simply the existence of their subjects, we have found in section 103, that we are not justified in passing from All not-S is P to All not-P is S unless we are given independently the existence of not-P. But it will be observed that in the case before us, the given propositions make this impossible. Since all S is P and all not-S is P, and everything is either S or not-S by the law of excluded middle, it follows that nothing is not-P. In order therefore to reduce the given propositions to such a form that they appear as contraries, (and consequently[1] as inconsistent with each other), we

[1] It will be remembered that under suppositions (1) and (2) the ordinary doctrine of contrariety holds good.

have to assume the very thing that taken together they really deny.

(3) and (4) On the view that at any rate universal propositions do not imply the existence of their subjects, we have found in the preceding section that the propositions No not-P is S, All not-P is S, are not necessarily inconsistent, for they may express the fact that P constitutes the entire universe of discourse. But this fact is just what is given us by the propositions in their original form.

In each case, then, the result obtained is satisfactorily accounted for and explained.

106. The Existential Import of Categorical Propositions[1].

In discussing this question we may consider first the Criterion of Consistency which Jevons (following De Morgan) lays down. It amounts to this, that every proposition implies the existence of things denoted by every term contained in it, and also of things denoted by the contradictories of such terms. If, for example, we have the proposition *All S is P*, this implies that among the members of the universe of discourse are to be found S's and P's, not-S's and not-P's. In defence of this doctrine it will be said that we ought never to make use of a term which either represents a non-existent class, or which represents a class that exhausts the entire universe of discourse. But to this argument two replies may be made. *First*, the universe of discourse may be a very limited one, so that S or P may easily either exhaust it or be absent from it[2]. *Secondly*,

[1] In this section we shall consider only general propositions. Singular propositions will be discussed separately.

[2] In one passage in his *Principles of Science* (chapter 5, § 5) Jevons remarks somewhat tentatively: "If A were identical with 'B or not-B', its negative not-A would be non-existent. This result would generally

with very complex subjects and predicates the contradictory of one or both of our terms may easily exhaust even an extended universe[1]. Further, if I am not allowed to negative *X*, why should I be allowed to negative *AB*? There is nothing to prevent *X* from representing a class formed by taking the part common to two other classes. In certain combinations indeed it may be convenient to substitute *X* for *AB*, or *vice versa*. It would appear then that what is contradictory when we use a certain set of symbols may not be contradictory when we use another set of symbols[2].

I consider that Jevons's criterion is sometimes a convenient assumption to make; provisionally, for example, in working out the doctrine of immediate inferences on the traditional lines. But it is an assumption that should always be explicitly referred to when made; and I certainly

be an absurd one, and I see much reason to think that in a strictly logical point of view it would always be absurd. In all probability we ought to assume as a fundamental logical axiom that every term has its negative in thought. We cannot think at all without separating what we think about from other things, and these things necessarily form the negative notion. If so, it follows that any term of the form '*B* or not-*B*' is just as self-contradictory as one of the form '*B* and not-*B*'." It is clear however that this psychological argument falls away as soon as it is allowed that we may be confining ourselves to a limited universe of discourse, or indeed if we confine ourselves to any universe less extensive than that which covers the whole realm of the conceivable.

[1] Take, for example, this proposition,—No satisfactory solution of the problem of squaring the circle has ever been published by Mr A. Here the subject is non-existent; and it may happen also that Mr A has never published anything at all.

[2] This argument has a special bearing on the complex propositions which are usually relegated to Symbolic Logic, but to which Jevons's criterion is intended specially to apply.

do not think that the whole of Logic should be based upon it.

We may now turn to the more limited question whether general categorical propositions should be regarded as logically implying the existence of their subjects. Our answer will depend to some extent on popular usage, and to some extent on logical convenience. So far as *universal* propositions are concerned, I am inclined on both grounds to answer the question in the negative.

In the first place, I do not think that in ordinary speech we always intend to imply the existence of the subjects of our propositions. No doubt we usually regard them as existing; but it is not difficult to cite exceptions: *e.g.*, No unicorns have ever been seen; All candidates arriving five minutes late are fined one shilling[1]. We may make the first of these assertions without intending to imply that unicorns exist unseen[2]; and the second does not commit us to the prophecy that any candidates will arrive five minutes late[3]. Again, a mathematician may assert that a rectilinear figure having a million equal sides and inscribable in a circle has a million equal angles, without intending to imply the actual

[1] As another example take the proposition,—Every body, not compelled by impressed forces to change its state, continues in a state of rest or of uniform motion in a straight line.

[2] I regard the universe of discourse here to be the actual material universe. If we suppose reference to be made to the universe of imagination, then this particular example is not to the point. To avoid misapprehension it may be worth while to mention that such examples as the following would certainly not be to the point: The wrath of the Homeric gods is very terrible; Fairies are able to assume different forms. The subjects of these propositions exist in the particular universes to which reference is obviously made.

[3] For other examples, see Venn, *Symbolic Logic*, pp. 130, 131.

existence of such a figure[1]; or if I know that A is X, B is Y, C is Z, I may affirm that ABC is XYZ without wishing to commit myself to the view that the combination ABC does ever really occur[2]. Taking complex subjects, and limiting our conception of existence as we not unfrequently do to some particular universe, cases of this kind might be multiplied indefinitely.

But if it is granted that in ordinary thought the existence of the subject of a proposition sometimes is and sometimes is not implied, it follows that since the logician cannot discriminate between these cases, he had better, unless there are very strong reasons to the contrary, content himself with leaving the question open, that is, he should regard such existence as not necessarily or logically implied[3].

Further, to adopt this alternative is logically more convenient; since if we do so, two important logical

[1] Mill argues that a synthetical proposition implies "the real existence of the subject, because in the case of a non-existent subject there is nothing for the proposition to assert" (*Logic*, Book i, Chapter 6, § 2). In answer to this it is sufficient to point out that a non-existent thing will be described as possessing attributes which are separately attributes of existing things, although that particular combination of them may not be anywhere to be found, and if we know (as we may do) that certain of these attributes are always accompanied by other attributes we may predicate the latter of the non-existent thing, thereby obtaining a real proposition which does not involve the actual existence of its subject. As an argument *ad hominem* it may be pointed out that Mill inclines to deny the existence of perfect straight lines or perfect circles. Would he therefore affirm that we can make no real assertions about such things?

[2] Is it not sometimes the case that in order to disprove the existence of some combination, say AB, we establish a self-contradictory proposition of the form AB is both C and not-C?

[3] Of course if ever it is important specially to affirm the existence in the universe of discourse of the subject of a proposition, this can be done by means of a separate statement.

operations, namely the conversion of **E** and the contraposition of **A**, are legitimate without requiring any special assumptions to be made; whereas if we regard universal propositions as implying the existence of their subjects we have seen in section 103 that these operations are not valid, irrespective of some further assumption[1].

The case of *particular* propositions still remains; and here again I am inclined to agree with the view taken by Dr Venn in his *Symbolic Logic*, namely that such propositions should be regarded as logically implying the existence of their subjects. One ground for adopting this view is that "an assertion confined to 'some' of a class generally rests upon observation or testimony rather than on reasoning or imagination, and therefore almost necessarily postulates existent data, though the nature of this observation and consequent existence is, as already remarked, a perfectly open question" (*Symbolic Logic*, p. 131). I think the cases are exceedingly rare in which in ordinary speech we predicate anything of a non-existent subject without doing so universally[2].

[1] Dr Venn shews that the importance of the question here raised is more particularly manifest when we are dealing with very complex propositions.

[2] Exceptional cases in which particular propositions appearing in categorical form cannot be regarded as implying the existence of their subjects must on the above view be transformed into the hypothetical or conditional form. Thus if we do not intend to imply the existence of *S*, instead of writing *Some S's are P's*, we must write,—*If there are any S's, then in some such cases they are also P's.*

I cannot agree with those who regard *Some S is P* and *S may be P* as equivalent forms. ("The particular judgment Some *S* is *P* is the same as the judgment *S* may be *P*." Bradley, *Principles of Logic*, p. 197.) I should say that the former at least implies that if there are any *S*'s there are also *P*'s, while the latter does not imply this. Or we may perhaps put it in this way, that the reference is not in each case to the

An objection to the above view is no doubt the paradox which follows from it, namely that we are not without qualification justified in inferring from *All S is P* that *Some S is P*, (since the latter proposition implies the existence of *S*, while the former does not). But this objection is far more than counterbalanced by the fact that **A** and **O**, **E** and **I**, are now true contradictories. This is not the case under any of the other suppositions that we discussed in section 104. And we have here in my opinion a very strong argument in favour of Dr Venn's doctrine[1].

The conclusion then at which we arrive is that logically *All S is P* should be regarded as implying only the non-existence of anything that is both *S* and not-*P*; *No S is P* as implying only the non-existence of anything that is both *S* and *P*. *Some S is P*, on the other hand, should be regarded as logically implying the existence of something that is both *S* and *P*; *Some S is not P* as implying the existence of something that is both *S* and not-*P*[2].

Adopting this view we have the results (with regard to

same universe of discourse; in the former case the reference is to an actual universe of some kind, in the latter to a conceivable universe of some kind.

[1] Another argument in favour of the above interpretation, especially as compared with interpretation (4) in section 102, is that it allows each propositional form ordinarily regarded as categorical to be really resolved into a single categorical statement, without admixture of hypothesis.

[2] I ought perhaps to say that I regard this solution as partly of the nature of a convention. But in dealing with complex propositions especially, it is very important that there should be some such convention. What I regard as of still more importance for ordinary Formal Logic is a clear recognition of the way in which the existential import of propositions affects the validity of logical operations.

immediate inferences and the square of opposition) that were worked out under supposition (3) in sections 103, 104. We may briefly recapitulate them for **A** as follows:—From the truth of *All S is P* we may infer—If *S* exists, some *S* is *P*; It is false that some *S* is not *P*; If any *S* exists, it is false that no *S* is *P*; If any *S* exists, some *P* is *S*; No *S* is not-*P*; All not-*P* is not-*S*; If any not-*P* exists, some not-*S* is not *P*. From the falsity of *All S is P* we may infer that Some *S* is not *P*.

It will be a useful exercise for the student to draw up a similar table for **E**, **I**, and **O**.

107. The Existential Import of Singular Propositions.

The result arrived at in the preceding section with regard to Universals cannot so satisfactorily be applied to Singulars.

In their case the implication that their subjects exist (in the universe appropriate to the discourse) is generally in common thought very distinct. This is certainly the case when the subject is a proper name, or when it is a general name individualised by an individualising prefix. Take, for example, such propositions as the following: King John signed the Great Charter; This hat is an old one; The present Emperor of Germany is nearly ninety years old. To avoid too great a divergence from common thought therefore I would differentiate singulars from general universals in this respect, and would regard them as implying the existence of their subjects[1].

But here it is necessary to add a word with regard to the Opposition of Singulars. "Socrates lived in Greece"

[1] Except of course when the express object of the proposition is to deny the existence of its subject.

and "Socrates did not live in Greece" cannot on the above view be regarded as true contradictories, since they would both be false in case Socrates turned out to be a myth. The true contradictory of a singular proposition will now take the form of a hypothetical; *e.g.*, the contradictory of the first of the above propositions will be, "If there ever was such a man as Socrates, he did not live in Greece."

CHAPTER IX.

HYPOTHETICAL AND DISJUNCTIVE PROPOSITIONS[1].

108. The Import of Conditional and Hypothetical Propositions.

(A) *Conditionals.*

A conditional proposition may be defined as one which, appearing in the form *If A is B, C is D*, or *Whenever A is B, C is D*, affirms a connexion between phenomena; it may be a co-inherence of attributes in a common subject or a relation in time or space between certain occurrences. The following are examples: If a child is spoilt, he is sure to be troublesome; If a barometer is carried up a mountain, the mercury in it will fall; If there are jackals about, a lion will not be far off.

It is sometimes held that the real *differentia* of all propositions of the form *If A is B, C is D* is "to express human doubt." But so far as *conditionals* are concerned, the doubt which they may imply is incidental rather than the fundamental or differentiating characteristic belonging to them. Materially indeed I think that they do sometimes imply the actual occurrence of their antecedents.

[1] This chapter may be omitted on a first reading.

Whenever the connexion between the antecedent and the consequent in a conditional proposition can be inferred from the nature of the antecedent independently of specific experience, (and this may be the more usual case), then the actual happening of the antecedent is not in any sense involved; but if our knowledge of the connexion does depend on specific experience, (as it sometimes may), and could not have been otherwise obtained, then such actual happening is materially involved. For example, the statement, "If we descend into the earth, the temperature increases at a nearly uniform rate of 1° Fahr. for every 50 feet of descent down to almost a mile," requires that actual descents into the earth should have been made, for otherwise the truth of the statement could not have been known.

In all cases the relation between the antecedent and the consequent of a conditional proposition is analogous to that between the subject and the predicate of a categorical proposition. But whether the conditional and the categorical forms can be regarded as mutually interchangeable will depend on whether we interpret the conditional with regard to the occurrence of its antecedent in the same way as we interpret the categorical with regard to the existence of its subject.

So far as *universals* are concerned, it seems clear that a conditional proposition does not *necessarily* imply the actual occurrence of its antecedent; and therefore, if the view is taken that a universal categorical proposition does necessarily imply the actual existence of its subject, we have a marked distinction between the two kinds of propositions[1]. "If anything is *B*, it is *C*" cannot be resolved into "All *B*

[1] This is Ueberweg's view. "The categorical judgment, in distinction from the hypothetical, always includes the pre-supposition of the existence of the subject" (*Logic*, § 122).

is *C*", since the latter implies the existence of *B* while the former does not.

If, on the other hand, we do not regard universal categorical propositions as logically implying the existence of their subjects, then conditionals and categoricals may be resolved into one another. We may say indifferently "All *B* is *C*" or "If anything is *B*, it is *C*"; "If ever *A* is *B*, then on all such occasions *C* is *D*" or "All occasions of *A* being *B* are occasions of *C* being *D*." Some conditional propositions are indeed so obviously equivalent to categoricals that they seem hardly to require a separate consideration[1].

As to *particular* conditionals, these seem usually to imply, in the same way as particular categoricals, the occurrence of their antecedents; *e.g.*, Sometimes when Parliament meets, it is opened by the Queen in person. On this view the two forms are again mutually interchangeable. But if we regard particular, as well as universal, conditionals as not implying the occurrence of their antecedents, then unless we also adopt the corresponding interpretation of particular categoricals, the two forms will not be mutually resolvable into one another. For particular conditionals will contain a hypothetical element which particular categoricals are supposed not to contain; and this it is which prevents the transformation from being valid.

The conclusion then at which we arrive is that there is no vital distinction between conditionals and categoricals

[1] The examples given at the commencement of this section are reducible to the following categoricals: All spoilt children are troublesome; All occasions on which a barometer is carried up a mountain are occasions on which there is a fall in the mercury which it contains; All places frequented by jackals are places where there are lions.

except in so far as the former are regarded as containing a really hypothetical element which is wanting in the latter. This leads up to a consideration of true hypotheticals.

(B) *Hypotheticals.*

A true hypothetical proposition expresses not a connexion between phenomena but a relation of dependence between two truths. And all such propositions are singular; for since their antecedents must be simply true or false, there can be no element of plurality in them; for example,—If God is just, the wicked will be punished; If the angles of every triangle are together equal to two right angles, then the two acute angles of every right angled triangle are together equal to the third angle; If hydrogen is a metal, then at least one metal exists normally in the gaseous form.

The true hypothetical may be written symbolically in the form *If P is true, Q is true*; and its import is that the truth of Q follows from that of P. The actual truth of P is clearly not implied. We may, as Kant puts it, "combine two false judgments." But can we not in a categorical judgment combine two non-existent entities, and will this not be the precisely corresponding case? In answering this question it is first to be observed that since all true hypotheticals are singular, the corresponding categoricals will be singular; and if singular categoricals are regarded as implying the existence of their subjects, the transformation is clearly illegitimate. But leaving this point on one side, a more fundamental consideration is that we cannot really obtain a categorical the relation between whose subject and predicate will be equivalent to the relation between the antecedent and the consequent of a true hypothetical. It is here that the real force of the distinction between true

hypotheticals on the one hand and either categoricals or conditionals on the other hand becomes apparent. It is true that the proposition *If P is true, Q is true* may be expressed in the categorical form *The truth of P is a truth from which the truth of Q follows*; but the predicate of this proposition is not equivalent to the consequent of the hypothetical proposition[1]. In short, the relation of dependence of one truth upon another is not analogous to the relation between the predicate and the subject of a categorical proposition. Hence a true hypothetical cannot be reduced to a categorical in which we have a subject and predicate corresponding precisely to the old antecedent and consequent[2].

109. The Opposition of Hypotheticals and Conditionals, and Immediate Inferences from them.

(A) *Conditionals.*

Taking the view that no conditionals imply the actual occurrence of their antecedents[3], we must apply to them the results obtained under supposition (4) in sections 103, 104. There are only two points to which attention need here be specially directed. The first is that the contraposition of

[1] It is clear that "*a truth from which the truth of Q follows*" is not the same as "*the truth of Q.*" Amongst other differences we may observe that the contrapositives of the two given forms will not be the same.

[2] No doubt hypotheticals can in all cases be shewn ultimately to imply categorical propositions. This however is a question distinct from that raised in the text.

[3] If we here make an exception for particulars, and so assimilate conditionals altogether with categoricals (according to the view of the latter taken in the preceding chapter), then of course conditionals require no separate discussion.

A and the conversion of **E** are valid processes, and there is therefore no need to modify the rule laid down in the concluding paragraph of section 77. The second point is that **A** and **O**, **E** and **I**, are not in conditionals true contradictories. Take for example the following: *If ever P occurs, then on all such occasions Q occurs; If ever P occurs, then on some such occasions Q does not occur.* We cannot say that either of these propositions is false supposing that *P* never occurs. The true contradictories of the above respectively are: *P sometimes occurs, and on some such occasions Q does not occur; P sometimes occurs, and on all such occasions Q occurs also.* This complexity is due to the hypothetical element involved in the proposed interpretation of particular conditionals.

(B) *Hypotheticals.*

Here again there is no need to modify the rule that from any hypothetical we may obtain by immediate inference another hypothetical, the antecedent of which denies the old consequent, and its consequent the old antecedent. *If P is true, Q is true*; therefore, *If Q is not true, P is not true.* But again attention must be drawn to the fact that the two following propositions are not strict contradictories: *If P is true, Q is true; If P is true, Q is not true.* For supposing *P* not to be true, we cannot say that either of these propositions is false[1]. Their true contradictories respectively are: *P is true but Q is not true; P and Q are both true.* We may look at it in this way. Denote the truth of *P* by *P* and its falsity by P', and use similar symbols in the case of *Q*. Then there are four possibilities, namely, PQ, PQ', $P'Q$, $P'Q'$, one of which must hold good, but any pair of which are mutually incon-

[1] For example: If two and two make five, all men are liars; If two and two make five, some men are not liars.

sistent. The proposition *If P is true, Q is true* merely excludes PQ'; and the proposition *If P is true, Q is not true* merely excludes PQ. But in denying that *If P is true, Q is true*, we must affirm PQ'; and this requires the exclusion of the three other possibilities PQ, $P'Q$, $P'Q'$. This yields the true contradictory as stated above. Similarly in the other case[1].

110. The Interpretation of Disjunctive Propositions.

It is a disputed question whether in a disjunctive proposition the alternatives should be regarded as in all cases mutually exclusive; whether, for example, in the proposition *A is either B or C* it is necessarily implied that *A* cannot be both *B* and *C*[2].

There are really involved here two questions which should be distinguished.

(1) In ordinary speech do we intend that the alternatives in a disjunctive proposition should be necessarily[3] understood as excluding one another? A very few instances will I think enable us to decide in the negative. Take, for example, the proposition,—"He has either used bad text-books, or he has been badly taught." Would anyone understand this to exclude the possibility of his having been

[1] The validity of these results is still more clearly seen by substituting for the hypotheticals their disjunctive equivalents, namely, *Either P is not true or Q is true, Either P is not true or Q is not true.*

[2] Whately, Mansel, Mill, and Jevons would answer this question in the negative; Kant, Hamilton, Thomson, Boole, Bain, and Fowler in the affirmative.

[3] There are of course many cases in which we understand them to be mutually exclusive; the only point in dispute is whether we can lay this down as a universal rule.

badly taught and used bad text-books as well? Or suppose it laid down as a condition of eligibility for some appointment that every candidate must be a member either of the University of Oxford, or of the University of Cambridge, or of the University of London. Would anyone regard this as implying the ineligibility of persons who happened to be members of more than one of these Universities? Jevons (*Pure Logic*, pp. 76, 77) instances the following proposition,—"A peer is either a duke, or a marquis, or an earl, or a viscount, or a baron." We do not consider this statement incorrect because many peers as a matter of fact possess two or more titles.

(2) Still this does not definitely settle the question. Granted that in common speech the alternatives of a disjunction may or may not be mutually exclusive, it may nevertheless be maintained that this is only because common speech is elliptical, that in Logic we should be more precise, and that the statement "*A* is either *B* or *C*" (where it may be both) should therefore be written "*A* is either *B* and not *C*, or *C* and not *B*, or both *B* and *C*."

This is a question of interpretation or method, and I do not apprehend that any burning principle is involved in the answer that we may give. For my own part I do not find any sufficient reason for diverging from the usage of every day language[1]. On the other hand, I think that if Logic is to be of practical utility, the less logical forms diverge from those of ordinary speech the better. And further, condensed forms of expression do not conduce to clearness[2], or even ultimately to conciseness. For

[1] At any rate apart from the exigencies of particular systems of Symbolic Logic.

[2] Professor Fowler indicates this view in his statement that "it is the object of Logic not to state our thoughts in a condensed form but to

where our information is meagre, a condensed form is likely to express more than we intend, and in order to keep within the mark we must indicate additional alternatives. Certainly the "necessarily exclusive" interpretation of disjunctives very much complicates the manipulation of complex propositions[1].

analyse them into their simplest elements" (*Deductive Logic*, p. 32); though he does not apply it to the case before us. Cf. Mansel, *Prolegomena Logica*, p. 238. Obviously a disjunctive proposition is a more condensed form of expression on the exclusive than it is on the non-exclusive interpretation.

[1] A view strongly opposed to that adopted in the text is taken by Mr Bradley. His argument is as follows:—"The commonest way of regarding disjunction is to take it as a combination of hypotheses. This view in itself is somewhat superficial, and it is possible even to state it incorrectly. 'Either A is B or C is D' means, we are told, that if A is not B then C is D, and if C is not D then A is B. But a moment's reflection shews us that here two cases are omitted. Supposing, in the one case, that *A is B*, and supposing, in the other, that *C is D*, are we able in these cases to say nothing at all? Our 'either—or' can certainly assure us that, if A is B, C—D must be false, and that, if C is D, then A—B is false. We have not exhausted the disjunctive statement, until we have provided for four possibilities, B and not-B, C and not-C" (*Principles of Logic*, p. 121). My difference with Mr Bradley is that I regard the question as really one of interpretation. In my view, it is open to a logician to choose either of the two ways of interpreting a disjunctive proposition, provided that he makes it quite clear which he has selected; but I can see no justification whatever for dogmatising as in the following passage,—"Our slovenly habits of expression and thought are no real evidence against the exclusive character of disjunction. 'A is b or c' does strictly exclude 'A is both b and c.' When a speaker asserts that a given person is a fool or a rogue, he may not *mean* to deny that he is both. But, having no interest in shewing that he is both, being perfectly satisfied provided he is one, either b or c, the speaker has not the possibility bc in his mind. Ignoring it as irrelevant, he argues as if it did not exist. And thus he may practically be right in what he says, though formally his statement is downright false: for he has excluded the alternative bc" (p. 124).

It is of course possible to make the alternatives *logically* incompatible or exclusive. Thus, not wishing to exclude the case of *A* being both *B* and *C*, we may write *A is B or bC*[1]; or, wishing to exclude that case, *A is Bc or bC*[1]. But in neither of these instances can we say that the incompatibility of the alternatives is really given by the disjunctive proposition. It is a merely formal proposition that *No A is both B and bC*, or that *No A is both Bc and bC*. The proposition *Every A is Bc or bC* does however tell us that no *A* is both *B* and *C*. And when from our knowledge of the subject-matter it is obvious that a disjunctive is intended to be understood in the exclusive sense, (and no doubt this is a very frequent case), we have in the above form a means of correctly and unambiguously expressing the fact. Where it is inconvenient to use this form it is open to us to make a separate statement to the effect that *No A is both B and C*. All that is here contended for is that the bare symbolic form *A is either B or C* is not to be interpreted as equivalent to *A is either Bc or bC*.

111. The Resolution of Disjunctive Propositions.

Adopting the view that in a disjunctive proposition the alternatives are not necessarily exclusive, *Either A is B or C is D* is primarily reducible to two hypotheticals, namely, *If A is not B, C is D*, and *If C is not D, A is B*. But each of these is the contrapositive of the other, and may therefore be inferred from it. Hence the full meaning of the disjunctive is expressed by means of *either* of these hypotheticals[2].

[1] Where b = not-B, and c = not-C.

[2] Compare Professor Croom Robertson in *Mind*, 1877, p. 266. If the alternatives of a disjunction are regarded as necessarily exclusive,

The resolution of a disjunctive in the manner here indicated may yield either a conditional or what we have called a true hypothetical. For example, "Every blood-vessel is either a vein or an artery" will yield a conditional; "Either free-will is a fact or the sense of obligation an illusion" will yield a true hypothetical.

then we get primarily four hypotheticals, namely, *If A is B, C is not D*, and *If C is D, A is not B*, in addition to the two given above. But these again are logical contrapositives each of the other.

Mr Bradley (*Principles of Logic*, p. 121) lays it down that "disjunctive judgments cannot really be reduced to hypotheticals" at all; but I hardly care to disagree with him since he admits all that I should contend for. He distinctly resolves "*A* is *b* or *c*" into hypotheticals (p. 130); but, he adds, although the meaning of disjunctives can thus "be given hypothetically; we must not go on to argue from this that they *are* hypothetical" (p. 121). They "declare a fact without any supposition" (p. 122). But so does the hypothetical itself, namely, the connexion between the antecedent and the consequent. Further, "a *combination* of hypotheticals surely does not lie in the hypotheticals themselves" (p. 122). Undoubtedly, by means of a combination of hypotheticals, we may make a most categorical statement: *e.g.*, If *A* is *B*, *C* is *D*; and if *A* is not *B*, *C* is *D*.

PART III.

SYLLOGISMS.

CHAPTER I.

THE RULES OF THE SYLLOGISM.

112. The Terms of the Syllogism.

A reasoning consisting of three categorical propositions (of which one is the conclusion), and containing three and only three terms, is called a Categorical Syllogism[1].

Every categorical syllogism therefore contains three and only three terms, of which two appear in the conclusion and also in one or other of the premisses, and one in the premisses only. That which appears as the predicate of the conclusion, and in one of the premisses, is called the *major term*; that which appears as the subject of the conclusion, and in one of the premisses, is called the *minor term*; and that which appears in both the premisses, but not in the conclusion, (being that term by their relations to which the mutual

[1] It is incorrect to define a syllogism as any combination of two propositions yielding as a conclusion a third proposition. This would include the argument *a fortiori* and other deductive inferences, which as such are never regarded as syllogistic. Cf. sections 114, 120, and 191.

relation of the two other terms is determined), is called the *middle term.*

Thus, in the syllogism,—

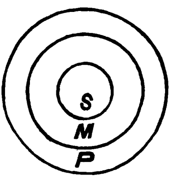
All M is P,
All S is M,
therefore, *All S is P;*

P is the major term, *S* is the minor term, and *M* is the middle term.

[These respective designations of the terms of a syllogism resulted from such a syllogism as,—

All M is P,
All S is M,
therefore, *All S is P,*

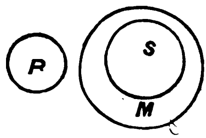
being taken as the type of syllogism. With the exception of the somewhat rare case in which the terms of a proposition are coextensive, such a syllogism as the above may be represented by the following diagram. Here clearly the



major term is the largest in extent, and the minor the smallest, while the middle occupies an intermediate position. But we have no guarantee that the same relation between the terms of a syllogism will hold, when one of the premisses is a negative or a particular proposition; *e.g.*, the following syllogism,—

No M is P,
All S is M,
therefore, *No S is P,*

gives as one case

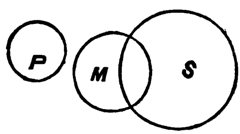


where the major term may be the smallest in extent, and the middle the largest.

Again, the following syllogism,—

No M is P,
Some S is M,
therefore, *Some S is not P,*

gives as one case



where the major term may be the smallest in extent and the minor the largest.

With regard to the middle term, however, we may note that although it is not always a middle term in extent, it is always a middle term in the sense that by its means the two other terms are connected, and their mutual relation determined.]

113. The Propositions of the Syllogism.

Every categorical syllogism consists of three propositions.

Of these one is the *conclusion.* The premisses are called the *major premiss* and the *minor premiss* according as they contain the major term or the minor term respectively.

Thus, *All M is P*, (major premiss),
All S is M, (minor premiss),
therefore, *All S is P*, (conclusion).

It is usual (as in the above syllogism) to state the major premiss first and the conclusion last. This is, however, nothing more than a convention. The order of the premisses in no way affects the validity of a syllogism, and has indeed no logical significance, though in certain cases it may be of some rhetorical importance. Professor Jevons argues that the cogency of a syllogism is more clearly recognisable when the minor premiss is stated first[1]. But it is doubtful whether any general rule of this kind can be laid down. In favour of the traditional order, it is to be said that in what is usually regarded as the typical syllogism[2], (*All M is P*, *All S is M*, therefore, *All S is P*), there is a philosophical ground for stating the major premiss first, since that gives the general rule, of which the minor premiss enables us to make a particular application.

114. The Rules of the Syllogism; and the Deduction of the Corollaries.

The rules of the Categorical Syllogism as usually stated are as follows:—

(1) *Every syllogism contains three and only three terms.*

(2) *Every syllogism consists of three and only three propositions.*

[1] *Principles of Science*, Chapter 6, § 14.
[2] Technically, the syllogism in *Barbara*.

It may be observed that these are not so much rules, as a general description of the nature of the syllogism. A reasoning which does not fulfil these conditions may be formally valid, but we do not call it a syllogism[1]. The four following rules are really rules in the sense that if, when we have got the reasoning into the form of a syllogism, they are not fulfilled, then the reasoning is invalid.

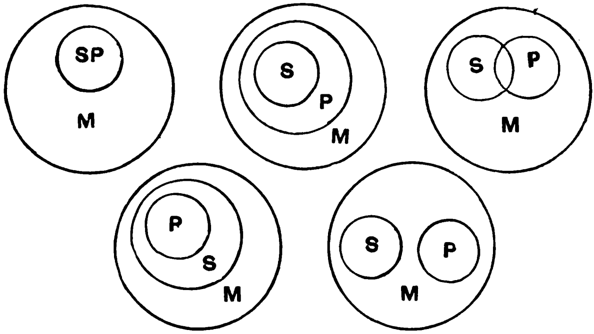
(3) *No one of the three terms of the syllogism must be used ambiguously; and the middle term must be distributed once at least in the premisses.*

This rule is frequently given in the form: "The middle term must be distributed once at least, and must not be ambiguous." But it is obvious that we have to guard against ambiguous major and ambiguous minor as well as against ambiguous middle. The fallacy resulting from the ambiguity of one of the terms of a syllogism is a case of *quaternio terminorum*, *i.e.*, a fallacy of four terms.

The necessity of distributing the middle term may be illustrated by the aid of the Eulerian diagrams. Given, for instance, *All P is M* and *All S is M*, we may have any one of the five following cases:—

[1] For example, *B is greater than C*,
A is greater than B,
therefore, *A is greater than C*.

Here is a valid reasoning consisting of three propositions. But it contains more than three terms; for the predicate of the second premiss is "greater than *B*," while the subject of the first premiss is "*B*." It is therefore as it stands not a syllogism. Whether reasonings of this kind admit of being reduced to syllogistic form is a problem which we discuss subsequently.



Here all the five relations that are *a priori* possible between *S* and *P* are still possible. We have therefore no conclusion.

If in a syllogism the middle term is distributed in neither premiss, we are said to have the fallacy of *undistributed middle.*

(4) *No term may be distributed in the conclusion which was not distributed in one of the premisses.*

The breach of this rule is called *illicit process of the major*, or *illicit process of the minor*, as the case may be; or, more briefly, *illicit major* or *illicit minor.*

(5) *From two negative premisses nothing can be inferred*[1].

(6) *If one premiss is negative, the conclusion must be negative; and to prove a negative conclusion, one of the premisses must be negative.*

From these rules, three corollaries may be deduced :—

(i) *From two particular premisses nothing can be inferred.*

[1] Rule 5 might like rule 3 be illustrated by means of the Eulerian diagrams. The student is recommended to work this out for himself.

Two particular premisses must be either

(α) both negative,

or (β) both affirmative,

or (γ) one negative and one affirmative.

But in case (α) no conclusion follows by rule 5.

In case (β) since no term can be distributed in two particular affirmative propositions, the middle term cannot be distributed, and therefore by rule 3 no conclusion follows.

In case (γ) if we can have a conclusion it must be negative (rule 6). The major term therefore will be distributed in the conclusion; and hence we must have two terms distributed in the premisses, namely, the middle and the major (rules 3, 4). But a particular negative proposition and a particular affirmative proposition between them distribute only one term. Therefore, no conclusion can be obtained.

[De Morgan (*Formal Logic*, p. 14) proves this corollary as follows:—"Since both premisses are particular in form, the middle term can only enter one of them universally by being the predicate of a negative proposition; consequently the other premiss must be affirmative, and, being particular, neither of its terms is universal. Consequently both the terms as to which the conclusion is to be drawn enter partially, and the conclusion can only be a particular *affirmative* proposition. But if one of the premisses be negative, the conclusion must be *negative*. This contradiction shews that the supposition of particular premisses producing a legitimate result is inadmissible."]

(ii) *If one premiss is particular, so must be the conclusion*[1].

[1] This and the sixth rule are sometimes combined into the one rule, *Conclusio sequitur partem deteriorem*,—*i.e.*, the conclusion follows the worse or weaker premiss both in quality and in quantity; a negative being considered weaker than an affirmative, and a particular than a universal.

We must have either

(α) two negative premisses, but this case is rejected by rule 5;

or (β) two affirmative premisses;

or (γ) one affirmative and one negative.

In case (β) the premisses, being both affirmative and one of them particular, can distribute but one term between them. This must be the middle term by rule 3. The minor term is therefore undistributed in the premisses, and the conclusion must be particular by rule 4.

In case (γ) the premisses will between them distribute two and only two terms. These must be the middle by rule 3, and the major by rule 4, (since we have a negative premiss, necessitating a negative conclusion by rule 6, and therefore the distribution of the major term in the conclusion). Again, therefore, the minor cannot be distributed in the premisses, and the conclusion must be particular by rule 4[1].

[De Morgan (*Formal Logic*, p. 14) gives the following very ingenious proof of this corollary:—"If two propositions *P* and *Q* together prove a third *R*, it is plain that *P* and the denial of *R* prove the denial of *Q*. For *P* and *Q* cannot be true together without *R*. Now, if possible, let *P* (a particular) and *Q* (a universal) prove *R* (a universal). Then *P* (particular) and the denial of *R* (particular) prove the denial of *Q*. But two particulars can prove nothing."]

[1] Spalding (*Logic*, p. 209) remarks,—"When one of the premisses is particular, the conclusion must be particular. The transgression of this rule is a symptom of illicit process of the minor." It is not however the case that if we infer a universal conclusion from a particular premiss we necessarily commit the fallacy of illicit minor. For example, we may have,—*Some M is P*, *All S is M*, therefore, *All S is P*; where the fallacy committed is that of undistributed middle.

(iii) *From a particular major and a negative minor nothing can be inferred.*

Since the minor premiss is given negative, the major premiss must by rule 5 be affirmative. But it is also particular, and it therefore follows that the major term cannot be distributed in it. Hence, by rule 4, it must be undistributed in the conclusion, *i.e.*, the conclusion must be *affirmative.* But also, by rule 6, since we have a negative premiss, it must be *negative.* This contradiction establishes the corollary that under the supposed circumstances no conclusion is possible.

115. Simplification of the Rules of the Syllogism.

The rules given above may be reduced to four. It has already been pointed out that rules 1 and 2 should be regarded as a description of the syllogism rather than as rules for its validity. Again, the part of rule 3 relating to ambiguity may be regarded as contained in the proviso that there shall be only three terms; for, if one of the terms is ambiguous, we have really four terms, and therefore have not a syllogism according to our definition of syllogism. We are then left with the four following rules:—

A. Two rules relating to the distribution of terms.

(1) The middle term must be distributed once at least in the premisses.

(2) No term may be distributed in the conclusion which was not distributed in one of the premisses.

B. Two rules relating to quality.

(3) From two negative premisses no conclusion follows.

(4) If one premiss is negative, the conclusion must be negative; and to prove a negative conclusion, one of the premisses must be negative.

We may now shew that even these four rules are not independent of one another. A breach of the second, or of the third, or of the first part of the fourth involves *indirectly* a breach of the first.

116. The rule that "from two negative premisses nothing can be inferred" may be established as a corollary from the rule that "the middle term must be distributed once at least in the premisses."

This is shewn by De Morgan[1]. He takes two universal negative premisses *E*, *E*. In whatever figure they are, they can be reduced by conversion to

No P is M,
No S is M.

Then by obversion they become (without losing any of their force),—

All P is not-M,
All S is not-M;

and we have undistributed middle. Hence rule 3 is exhibited as a corollary from rule 1.

An objection may perhaps be taken to the above on the ground that the premisses might also be reduced to,—

All M is not-P,
All M is not-S;

where the middle term is distributed in both premisses. Here however it is to be observed that we have no longer a middle term *connecting S and P* at all. We shall return

[1] *Formal Logic*, p. 13.

subsequently to this method of dealing with two negative premisses.

The case in which one of the premisses is particular is dealt with by De Morgan as follows:—"Again, *No Y is X*, *Some Ys are not Zs*, may be converted into

Every X is (*a thing which is not Y*),
Some (*things which are not Zs*) *are Ys*,

in which there is no middle term."

This is not satisfactory, since we may often exhibit a valid syllogism in such a form that there appear to be four terms; *e.g.*, *All M is P*, *All S is M*, may be converted into

All M is P,
No S is not-M,

in which there is no middle term.

The case in question may however be disposed of by saying that if we can infer nothing from two universal negative premisses, *a fortiori* we cannot from two negative premisses, one of which is particular.

117. The rule that "if one premiss is negative the conclusion must be negative" may be established as a corollary from the rule that "from two negative premisses nothing can be inferred."

The following proof is suggested by De Morgan's deduction of corollary ii. (cf. section 114):—If two propositions *P* and *Q* together prove a third *R*, it is plain that *P* and the denial of *R* prove the denial of *Q*. For *P* and *Q* cannot be true together without *R*. Now if possible let *P* (a negative) and *Q* (an affirmative) prove *R* (an affirmative). Then *P* (a negative) and the denial of *R* (a negative) prove the denial of *Q*. But two negatives prove nothing.

118. Any syllogism involving directly an illicit process of major or minor involves indirectly a fallacy of undistributed middle.

Let *P* and *Q* be the premisses and *R* the conclusion of a syllogism involving illicit major or minor, a term *X* which is undistributed in *P* being distributed in *R*. Then the contradictory of *R* combined with *P* must prove the contradictory of *Q*. But any term distributed in a proposition is undistributed in its contradictory. *X* is therefore undistributed in the contradictory of *R*, and by hypothesis it is undistributed in *P*. But *X* is the middle term of the new syllogism, which is therefore guilty of the fallacy of undistributed middle. It is thus shewn that any syllogism involving directly a fallacy of illicit major or minor involves indirectly a fallacy of undistributed middle[1].

119. Further Reduction of the independent Rules of the Syllogism.

Rules 2 and 3 (as given in section 115) are thus shewn to be corollaries from rule 1; and the first part of rule 4 to be a corollary from rule 3. The independent rules of the syllogism are thus reduced to the two following:

[1] For this section I am indebted to Mr W. E. Johnson, who points out that we might also proceed in the opposite direction, and exhibit the rule relating to the distribution of the middle term as a corollary from the rule relating to the distribution of the major and minor terms. This order of procedure is in one respect to be preferred, since the two independent rules for the syllogism will then be identical with the two rules for subalternation and ordinary conversion (which are the only immediate inferences in which from a given proposition we obtain another *containing the same terms*), namely, that the quality of the given proposition must be preserved and that no term may be distributed in the inferred that was not distributed in the original proposition.

(α) The middle term must be distributed once at least in the premisses;

(β) To prove a negative conclusion one of the premisses must be negative[1].

It should be clearly understood that it is not meant that every invalid syllogism will offend *directly* against one of these two rules. As a direct test for the detection of invalid syllogisms we must still fall back upon the *four* rules given in section 115[2]. All that we have succeeded in shewing is that ultimately these rules are not independent of one another.

120. Two negative premisses may yield a valid conclusion; but not syllogistically.

[1] On examination it will be found that the only syllogism rejected by this rule and not also rejected directly or indirectly by the preceding rule is the following:—*All P is M*, *All M is S*, therefore, *Some S is not P*. In the technical language explained in the following chapter, this is *AAO* in Figure 4. So far therefore as the first three figures are concerned, we are left with the single rule of *Distributed Middle*. Students are recommended to work this point out more in detail for themselves.

[2] For example, the invalid syllogism,—

All M is P,
No S is M,
therefore, *No S is P*,

does not directly involve a breach of either of the two rules given above. But if this syllogism is valid, then must also the following syllogism be valid:

All M is P, (original major),
Some S is P, (contradictory of original conclusion),
therefore, *Some S is M*, (contradictory of original minor);

and here we have undistributed middle. This rule then establishes *indirectly* the invalidity of the syllogism in question. The principle involved is the same as that on which we shall find the process of indirect reduction to be based.

Professor Jevons remarks: "The old rules of logic informed us that from two negative premisses no conclusion could be drawn, but it is a fact that the rule in this bare form does not hold universally true; and I am not aware that any precise explanation has been given of the conditions under which it is or is not imperative. Consider the following example,—Whatever is not metallic is not capable of powerful magnetic influence, Carbon is not metallic, therefore, Carbon is not capable of powerful magnetic influence. Here we have two distinctly negative premisses, and yet they yield a perfectly valid negative conclusion. The syllogistic rule is actually falsified in its bare and general statement" (*Principles of Science*, Chapter 4, § 10).

This apparent exception is however no real exception. The reasoning, (which may be expressed symbolically,—*No not-M is P*, *No S is M*, therefore, *No S is P*), is certainly valid; but if we regard the premisses as negative it has four terms *S*, *P*, *M*, and not-*M*, and is therefore no syllogism. Reducing it to syllogistic form, the minor becomes by obversion *All S is not-M*, an affirmative proposition[1]. It is not the case therefore that we have succeeded in finding a valid *syllogism* with two negative premisses. In other words, while we must not say that from two negative premisses nothing follows, it remains true that if a syllogism regularly expressed has two negative premisses it is invalid[2].

Mr Bradley (*Principles of Logic*, p. 254) returns to the position taken by Professor Jevons. In reference to the example given above, he says, "This argument no doubt

[1] It may be added that it is in this form that the cogency of the argument is most easily to be recognised.

[2] This is clearly explained by Jevons himself in his *Elementary Lessons in Logic*, p. 134.

has *quaternio terminorum* and is vicious technically, but the fact remains that from two denials you somehow *have* proved a further denial. '*A* is not *B*, what is not *B* is not *C*, therefore *A* is not *C*'; the premisses are surely negative to start with, and it appears pedantic either to urge on one side that '*A* is not-*B*' is simply positive, or on the other that *B* and not-*B* afford no junction. If from negative premisses I can get my conclusion, it seems idle to object that I have first transformed one premiss; for that objection does not shew that the premisses are not negative, and it does not shew that I have failed to get my conclusion."

This is somewhat beside the mark; and if the points on both sides are clearly stated there appears no room for further controversy. On the one hand, it is implicitly admitted both by Professor Jevons (*Studies in Deductive Logic*, p. 89), and by Mr Bradley, that two negative premisses invalidate a *syllogism*, *i.e.*, understanding by a syllogism a mediate reasoning containing three and only three terms. On the other hand, everyone would allow that from two propositions which may both be regarded as negative, a conclusion may sometimes be obtained; for example, the propositions which constitute the premisses of a syllogism in *Barbara*[1] may be written in a negative form, thus, *No M is not-P*, *No S is not-M*, and the conclusion *All S is P* still follows. We must not, in fact, attach undue importance to the distinction between positive and negative propositions[2]. By means of the process of Ob-

[1] *All M is P*,
All S is M,
therefore, *All S is P*. Cf. section 158.

[2] Lotze (*Logic*, § 89) seems to imply that the possibility of writing both premisses in a negative form is confined to reasonings which syllogistically fall into Figure 3. The above example shews that this is not the case.

version, the logician may at will regard any given proposition as either the one or the other.

[A case similar to that adduced by Professor Jevons is dealt with in the *Port Royal Logic* (Professor Baynes's translation, p. 211) as follows:—

"There are many reasonings, of which all the propositions appear negative, and which are, nevertheless, very good, because there is in them one which is negative only in appearance, and in reality affirmative, as we have already shewn, and as we may still further see by this example:

That which has no parts cannot perish by the dissolution of its parts;

The soul has no parts;

Therefore, the soul cannot perish by the dissolution of its parts.

There are several who advance such syllogisms to shew that we have no right to maintain unconditionally this axiom of logic, *Nothing can be inferred from pure negatives;* but they have not observed that, in sense, the minor of this and such other syllogisms is affirmative, since the middle, which is the subject of the major, is in it the attribute. Now the subject of the major is not that which has parts, but that which has not parts, and thus the sense of the minor is, *The soul is a thing without parts*, which is a proposition affirmative of a negative attribute."

Ueberweg also, who himself gives a clear explanation of the case, shews that it was not overlooked by the older logicians; and he thinks it not improbable that the doctrine of qualitative Æquipollence between two judgments (*i.e.* obversion) resulted from the consideration of this very question[1].]

[1] *System of Logic*, § 106.

121. What conclusion can be inferred from the premisses,—(i) *All P is M*, (ii) *All S is M*, (iii) *M does not constitute the entire universe of discourse?* Is the third premiss necessary in order that the conclusion may be legitimate?

From (i) we obtain by immediate inference *All not-M is not-P*, and from (ii) *All not-M is not-S*; and these premisses yield the conclusion,—

Some not-S is not-P.

Or, we might reason as follows:—Since *S* and *P* are both entirely included in *M*, there must be outside *M* some not-*S* and some not-*P* that are coincident; and this is the same conclusion as before.

Now in the latter form of the reasoning it would seem that we have assumed that there is *some* not-*M*, *i.e.*, that *M* does not constitute the entire universe of discourse. How is it that the necessity of this assumption is not also apparent in our first method of treatment?

The truth appears to be that we have here an illustration of De Morgan's view (*Formal Logic*, p. 112) that in all syllogisms the existence of the middle term is a *datum*. From the premisses *All B is C*, *All B is A*, we cannot obtain the conclusion *Some A is C* without implicitly assuming the existence of *B*. Take as an example,—All witches ride through the air on broomsticks; All witches are old women; therefore, Some old women ride through the air on broomsticks. This point is further discussed in chapter xi.

We may note that the reasoning,—

All P is M,
All S is M,
therefore, *Some not-S is not-P*,

does not invalidate the *syllogistic* rule that the middle term must be distributed once at least in the premisses, since as it stands it contains more than three terms and is therefore not a syllogism at all.

122. Can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference?

Professor Bain answers this question in the negative (*Logic*, *Deduction*, p. 159), and he illustrates his view by reference to the following syllogism:

Socrates fought at Delium,
Socrates was the master of Plato,
therefore, The master of Plato fought at Delium.

"But," he adds, "the proposition 'Socrates was the master of Plato and fought at Delium,' compounded out of the two premisses is nothing more than a grammatical abbreviation"; and the step hence to the conclusion is a mere omission of something that had previously been said. "Now, we never consider that we have made a real inference, a step in advance, when we repeat *less* than we are entitled to say, or drop from a complex statement some portion not desired at the moment. Such an operation keeps strictly within the domain of Equivalence or Immediate Inference. In no way, therefore, can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference."

This argument leads up to some very interesting considerations, but it proves too much. In the following syllogisms the premisses may be similarly compounded together:

$$\left.\begin{array}{l}\text{all men are mortal,}\\ \text{all men are rational,}\end{array}\right\}\text{ all men are mortal and rational;}$$

therefore, some rational beings are mortal.

all men are mortal, } all men including kings are mortal;
all kings are men, }
therefore, all kings are mortal[1].

Do not Dr Bain's criticisms apply to these syllogisms as much as to the syllogism with two singular premisses? The method of treatment adopted is indeed particularly applicable to syllogisms in which the middle term is subject in both premisses[2]; but in any case it is true that the conclusion of a syllogism contains a part of, and only a part of, the information contained in the two premisses taken together. Also, we may always combine the two premisses in a single statement; and thus we may always get Dr Bain's result[3]. In other words, in the conclusion of every syllogism "we repeat less than we are entitled to say," or, if we care to put it so, "drop from a complex statement some portion not desired at the moment."

123. Is the ordinary syllogistic conclusion open to the charge of incompleteness?

This charge (a consideration of which will appropriately

[1] Compare with the above the following syllogism which has two singular premisses:—

The Lord Chancellor receives a higher salary than the Prime Minister,

Lord Selborne is the Lord Chancellor,

therefore, Lord Selborne receives a higher salary than the Prime Minister.

The premisses here would similarly, I suppose, be compounded by Professor Bain into "The Lord Chancellor, Lord Selborne, receives a higher salary than the Prime Minister."

[2] Such syllogisms are said to be in Figure 3. Cf. section 143.

[3] It may be pointed out that the general method adopted by Boole in his *Laws of Thought* is to sum up all his given propositions in a single proposition, and then eliminate the terms that are not required. Compare also the methods employed in Part IV.

supplement the discussion contained in the preceding section) is brought by Professor Jevons (*Principles of Science*, I. p. 71) against the ordinary syllogistic conclusion. The premisses "Potassium floats on water, Potassium is a metal" yield, according to him, the conclusion "Potassium metal is potassium floating on water." But "Aristotle would have inferred that some metals float on water. Hence Aristotle's conclusion simply *leaves out some of the information afforded in the premisses*; it even leaves us open to interpret the *some metals* in a wider sense than we are warranted in doing."

In reply to this it may be remarked: first, that the Aristotelian conclusion does not profess to sum up the whole of the information contained in the premisses of the syllogism; secondly, that *some* in Logic means merely "not none," "one at least." The conclusion of the above syllogism might perhaps better be written "some metal floats on water," or "some metal or metals, &c." Lotze remarks in criticism of Jevons: "His whole procedure is simply a repetition or at the outside an addition of his two premisses; thus it merely adheres to the given facts, and such a process has never been taken for a *Syllogism*, which always means a movement of thought that uses what is given for the purpose of advancing beyond it...... The meaning of the Syllogism, as Aristotle framed it, would in this case be that the occurrence of a floating metal Potassium proves that the property of being so light is not incompatible with the character of metal in general"[1]. This criticism is perhaps pushed a little

[1] *Logic*, book ii, chapter 3, note. Compare also, Dr Venn in the *Academy*, Oct. 3, 1874, and Professor Croom Robertson in *Mind*, 1876, p. 219. Dr Venn remarks, "Surely, as the old expression 'discursive thought' implies, we designedly pass on from premisses to conclusion, and then drop the premisses from sight. If we want to keep them in

too far. Jevons's conclusion is hardly fairly described as the mere sum of the premisses; for it brings out a relation between two terms which was not immediately apparent in the premisses as they originally stood. Still there can be no doubt that the elimination of the middle term is the very gist of syllogistic reasoning as ordinarily understood.

It may be added, as an *argumentum ad hominem* against Jevons, that his own conclusion *leaves out some of the information afforded in the premisses.* For we cannot pass back from the proposition that "Potassium metal is potassium floating on water" to either of the original premisses.

124. The connexion between the *Dictum de omni et nullo* and the ordinary rules of the syllogism.

The *Dictum de omni et nullo* was given by Aristotle as the axiom on which all syllogistic inference is based. It applies directly, however, to those syllogisms only in which the major term is predicate in the major premiss, and the minor term subject in the minor premiss, (*i.e.*, to what are called syllogisms in Figure 1). The rules of the syllogism, on the other hand, apply independently of the position of the terms in the premisses. Nevertheless, it is interesting to trace the connexion between them. We shall find all the rules really involved in the *Dictum*, but some of them in a less general form, in consequence of the distinction pointed out above.

The *Dictum* may be stated as follows:—"Whatever is predicated, whether affirmatively or negatively, of a term

sight we can perfectly well retain them as premisses; if not, if all that we want is the final fact, it is no use to burden our minds or paper with premisses as well as conclusion. All reasoning is derived from data which under conceivable circumstances might be useful again, but which we are satisfied to recover when we want them."

distributed may be predicated in like manner of everything contained under it."

(1) The *Dictum* provides for three and only three terms; namely, (i) a certain term which must be distributed, (ii) something predicated of this term, (iii) something contained under it. These terms are respectively the middle, major, and minor. We may consider the rule relating to the ambiguity of terms also contained here, since if any term is ambiguous we have practically more than three terms.

(2) The *Dictum* provides for three and only three propositions; namely, (i) a proposition predicating something of a term distributed, (ii) a proposition declaring something to be contained under this term, (iii) a proposition making the original predication of the contained term. These propositions constitute respectively the major premiss, the minor premiss, and the conclusion of the syllogism.

(3) The *Dictum* prescribes not merely that the middle term shall be distributed once at least in the premisses, but more definitely that it shall be distributed in the major premiss,—"Whatever is predicated of a term *distributed*." [This is really another form of what we shall find to be a special rule of Figure 1, namely that the major premiss must be universal. Cf. section 144.]

(4) The proposition declaring that something is contained under the term distributed must necessarily be an affirmative proposition. The *Dictum* provides therefore that the premisses shall not be both negative. [It really provides that the *minor* premiss shall be affirmative, which again is one of the special rules of Figure 1.]

(5) The words "in like manner" clearly provide against a breach of rule 6, namely that if one premiss is negative, the conclusion must be negative, and *vice versa*.

(6) Illicit process of the major is provided against indi-

rectly. We can commit this fallacy only if we have a negative conclusion; but the words "in like manner" declare that if we have a negative conclusion, we must have a negative major premiss; and since in any syllogism to which the *Dictum* directly applies, the major term is predicate of this premiss, it will be distributed in its premiss as well as in the conclusion. Illicit process of the minor is provided against inasmuch as the *Dictum* warrants us in making our predication in the conclusion only of what has been shewn in the minor premiss to be contained under the middle term.

CHAPTER II.

SIMPLE EXERCISES ON THE SYLLOGISM.

125. If P is a mark of the presence of Q, and R of that of S, and if P and R are never found together, am I right in inferring that Q and S sometimes exist separately? [V.]

The premisses may be stated,—

All P is Q,
All R is S,
No P is R;

and in order to establish the desired conclusion we must be able to infer at least one of the following,—

Some Q is not S,
Some S is not Q.

But neither of these propositions can be inferred; for they distribute respectively S and Q, and neither of these terms is distributed in the given premisses. The question is therefore to be answered in the negative.

126. If it be known concerning a syllogism in the Aristotelian system that the middle term is dis-

tributed in both premisses, what can we infer as to the conclusion? [C.]

If both premisses are affirmative, they can between them distribute only two terms; but by hypothesis the middle term is distributed twice in the premisses, the minor term cannot therefore be distributed, and it follows that the conclusion must be particular.

If one of the premisses is negative, we may have three terms distributed in the premisses; these must, however, be the middle term twice (by hypothesis), and the major term (since the conclusion must now be negative and the major term will therefore be distributed in it); hence the minor term cannot be distributed in the premisses, and it again follows that the conclusion must be particular.

But either both premisses will be affirmative, or one affirmative and the other negative; in any case, therefore, we can infer that the conclusion will be particular.

127. Shew *directly* in how many ways it is possible to prove the conclusions *SaP*, *SeP*; point out those that conform immediately to the *Dictum de omni et nullo*; and exhibit the equivalence between these and the remainder. [W.]

(1) To prove *All S is P.*

Both premisses must be affirmative, and both must be universal.

S being distributed in the conclusion must be distributed in the minor premiss, which must therefore be *All S is M.*

M not being distributed in the minor must be distributed in the major which must therefore be *All M is P.*

SaP can therefore be proved in only one way, namely,

All M is P,
All S is M,
therefore, *All S is P;*

and this syllogism conforms immediately to the *Dictum.*

(2) To prove *No S is P.*

Both premisses must be universal, and one must be negative while the other is affirmative; *i.e.*, one premiss must be *E* and the other *A*.

First, let the major be *E*, *i.e.*,

either *No M is P* or *No P is M.*

In each case the minor must be affirmative and must distribute *S*; therefore, it will be *All S is M.*

Secondly, let the minor be *E*, *i.e.*,

either *No M is S* or *No S is M.*

In each case the major must be affirmative and must distribute *P*; therefore, it will be *All P is M.*

We can then prove *SeP* in four ways, thus,—

(i)	(ii)	(iii)	(iv)
MeP,	*PeM,*	*PaM,*	*PaM,*
SaM,	*SaM,*	*MeS,*	*SeM,*
SeP.	*SeP.*	*SeP.*	*SeP.*

Of these, (i) only conforms immediately to the *Dictum*, and we have to shew the equivalence between it and the others.

The only difference between (i) and (ii) is that the major premiss of the one is the simple converse of the major premiss of the other; they are therefore equivalent. Similarly the only difference between (iii) and (iv) is that the

minor premiss of the one is the simple converse of the minor premiss of the other; they are therefore equivalent.

Finally, we may shew that (iii) is equivalent to (i) by transposing the premisses and converting the conclusion.

EXERCISES[1].

128. Explain what is meant by a *Syllogism*; and put the following argument into syllogistic form:—"We have no right to treat heat as a substance, for it may be transformed into something which is not heat, and is certainly not a substance at all, namely, mechanical work." [N.]

129. Put the following argument into syllogistic form:—How can any one maintain that pain is always an evil, who admits that remorse involves pain, and yet may sometimes be a real good? [V.]

130. "There is no Englishman among the wounded, so no officer can have received a wound." Supply a premiss that will make this reasoning correct. Can you supply any premiss that will make it (i) guilty of illicit process of the major, (ii) guilty of illicit process of the minor?

131. It has been pointed out by Ohm that reasoning to the following effect occurs in some works on mathematics:—"A magnitude required for the solution of a problem must satisfy a particular equation, and as the

[1] The following exercises may be solved without any knowledge beyond what is contained in the preceding chapter, the assumption however being made that if no rule of the syllogism as given in section 114 or section 115 is broken, then the syllogism is valid.

magnitude x satisfies this equation, it is therefore the magnitude required."

Examine the logical validity of this argument. [C.]

132. Obtain a conclusion from the two negative premisses,—*No P is M, No S is M.*

133. If it is false that the attribute *B* is ever found coexisting with *A*, and not less false that the attribute *C* is sometimes found absent from *A*, can you assert anything about *B* in terms of *C*? [C.]

134. Enumerate the cases in which no valid conclusion can be drawn from two premisses.

135. Give examples in which, attempting to infer a universal conclusion where we have a particular premiss, we commit respectively one but one only of each of the following fallacies,—(*a*) undistributed middle, (*b*) illicit major, (*c*) illicit minor. Give also an example in which, making the same attempt, we commit none of the above fallacies.

136. Shew that

(i) If both premisses of a syllogism are affirmative, and one but only one of them universal, they will between them distribute only one term;

(ii) If both premisses are affirmative and both universal, they will between them distribute two terms;

(iii) If one but only one premiss is negative, and one but only one premiss universal, they will between them distribute two terms;

(iv) If one but only one premiss is negative, and both premisses are universal, they will between them distribute three terms.

137. Ascertain how many distributed terms there may be in the premisses of a syllogism more than in the conclusion. [L.]

138. Prove that, when the minor term is predicate in its premiss, the conclusion cannot be *A*. [L.]

139. If the major term of a syllogism be the predicate of the major premiss, what do we know about the minor premiss? [L.]

140. How much can you tell about a valid syllogism if you know

(1) that only the middle term is distributed;

(2) that only the middle and minor terms are distributed;

(3) that all three terms are distributed? [W.]

141. Shew that if the conclusion of a syllogism be a universal proposition, the middle term can be but once distributed in the premisses. [L.]

142. Shew *directly* in how many ways it is possible to prove the conclusions *SiP*, *SoP*. [W.]

CHAPTER III.

THE FIGURES AND MOODS OF THE SYLLOGISM.

143. Figure and Mood.

By the *Figure* of a syllogism is meant the position of the terms in the premisses.

Denoting the major, middle and minor terms by the letters P, M, S respectively, and stating the major premiss first, we have four figures of the syllogism as shewn in the following table :—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
$M-P$	$P-M$	$M-P$	$P-M$
$S-M$	$S-M$	$M-S$	$M-S$
$S-P$	$S-P$	$S-P$	$S-P$.

By the *Mood* of a Syllogism is meant the quantity and quality of the premisses and conclusion. For example, AAA is a mood in which both the premisses and the conclusion are universal affirmatives; EIO is a mood in which the major is a universal negative, the minor a particular affirmative, and the conclusion a particular negative. It is clear that if figure and mood are both given, the syllogism is given.

144. The Special Rules of the Figures; and the Determination of the Legitimate Moods in each Figure[1].

It may first of all be shewn that certain combinations of premisses are incapable of yielding a valid conclusion in any figure. *A priori*, there are possible the following sixteen different combinations of premisses, the major premiss being always stated first:—*AA*, *AI*, *AE*, *AO*, *IA*, *II*, *IE*, *IO*, *EA*, *EI*, *EE*, *EO*, *OA*, *OI*, *OE*, *OO*. Referring back however to the syllogistic rules (section 114), we find that of these, *EE*, *EO*, *OE*, *OO*, (being combinations of negative premisses), give no conclusion by rule 5; again, *II*, *IO*, *OI*, (being combinations of particular premisses), are excluded by corollary i.; and *IE* is excluded by corollary iii., which tells us that nothing follows from a particular major and a negative minor.

We are left then with the following eight possible combinations:—*AA*, *AI*, *AE*, *AO*, *IA*, *EA*, *EI*, *OA*; and we may now go on to determine in which figures these will yield conclusions.

The special rules[2] *and the legitimate moods of Figure* 1.

The position of the terms in Figure 1 is shewn thus,—

$$\begin{array}{c} M-P \\ S-M \\ \hline S-P \end{array}$$

and we can prove that in this figure:—

[1] The method of determination here adopted is only one amongst several possible methods. Another is suggested, for example, in sections 127, 142.

[2] As indicated in section 124, the special rules of Figure 1 follow immediately from the *Dictum de omni et nullo*.

(1) *The minor premiss must be affirmative.* For if it were negative, the major premiss would have to be affirmative by rule 5, and the conclusion negative by rule 6. The major term would therefore be distributed in the conclusion, and undistributed in its premiss; and the syllogism would be invalid by rule 4.

(2) *The major premiss must be universal.* For the middle term cannot be distributed in the minor premiss since this is affirmative, and must therefore be distributed in the major premiss.

Rule (1) shews that *AE* and *AO*, and rule (2) that *IA* and *OA* yield no conclusions in this figure. We are therefore left with only four combinations, namely, *AA*, *AI*, *EA*, *EI*. Applying the rules that a negative premiss gives a negative conclusion, while conversely a negative conclusion requires a negative premiss, and that a particular premiss gives a particular conclusion only, we find that *AA* will justify either of the conclusions *A* or *I*, *EA* either *E* or *O*, *AI* only *I*, *EI* only *O*. We have then six moods in Figure 1 which do not offend against any of the rules of the syllogism[1], namely, *AAA*, *AAI*, *AII*, *EAE*, *EAO*, *EIO*.

[1] Rule (2) provides against undistributed middle, and rule (1) against illicit major. We cannot have illicit minor, unless we have a universal conclusion with a particular premiss, and this also has been provided against.

Mr Johnson points out that the following symmetrical rules may be laid down for the correct distribution of terms in the different figures; and that these rules (three in each figure) taken together with the *rules of quality* are sufficient to ensure that *no* syllogistic rule is broken.

(i) To avoid undistributed middle: In figure 1, If the minor is affirmative, the major must be universal; In figure 4, If the major is affirmative, the minor must be universal; In figure 2, One premiss must be negative; In figure 3, one premiss must be universal. (The last of these rules is given for the sake of completeness. It is however

We may establish the actual validity of these moods by shewing that the axiom of the syllogism, the *Dictum de omni et nullo*, applies to them; or by taking them severally and shewing that in each case the cogency of the reasoning is self-evident.

The special rules and the legitimate moods of Figure 2.

The position of the terms in Figure 2 is shewn thus,—

$$\begin{array}{c} P-M \\ \underline{S-M} \\ S-P \end{array}$$

and its special rules, (which the student is recommended to deduce from the general rules of syllogism for himself), are,—

(1) *One premiss must be negative;*

(2) *The major premiss must be universal.*

Applying these rules, we shall find that we are again left with six moods, namely, *AEE*, *AEO*, *AOO*, *EAE*, *EAO*, *EIO*.

We cannot now immediately apply the *Dictum de omni et nullo* to shew positively that these moods are legitimate. We may however as before establish the cogency of the reasoning in each case by shewing it to be self-evident.

superfluous if we have already proved the corollaries given in section 114).

(ii) To avoid illicit major: In figures 1 and 3, If the conclusion is negative, the major must be negative and therefore the minor affirmative; In figures 2 and 4, If the conclusion is negative, the major must be universal.

(iii) To avoid illicit minor: In figures 1 and 2, If the minor is particular, the conclusion must be particular; In figures 3 and 4, If the minor is affirmative, the conclusion must be particular. (The first of these two rules is again superfluous as a special rule.)

These rules are substantially identical with those given in the text.

The older logicians do not adopt this course; their method is to prove that by means of immediate inferences each mood can be reduced to such a form that the *Dictum* does apply directly to it. This is the doctrine of Reduction which we shall discuss in the following chapter.

The special rules and the legitimate moods of Figure 3.

The position of the terms in this figure is shewn thus,—

$$\begin{array}{c} M - P \\ \underline{M - S} \\ S - P \end{array}$$

and its special rules are,—

(1) *The minor must be affirmative;*

(2) *The conclusion must be particular.*

Proceeding as before, we shall find ourselves left with six valid moods,—*AAI*, *AII*, *EAO*, *EIO*, *IAI*, *OAO*.

The special rules and the legitimate moods of Figure 4.

The position of the terms in this figure is shewn thus,—

$$\begin{array}{c} P - M \\ \underline{M - S} \\ S - P \end{array}$$

and its special rules are,—

(1) *If the major is affirmative, the minor must be universal;*

(2) *If either premiss is negative, the major must be universal;*

(3) *If the minor is affirmative, the conclusion must be particular.*

The result of the application of these rules is again six valid moods:—*AAI*, *AEE*, *AEO*, *EAO*, *EIO*, *IAI*.

Our final conclusion then is that there are 24 valid moods, namely, six in each figure.

In Figure 1, *AAA*, *AAI*, *EAE*, *EAO*, *AII*, *EIO*.

In Figure 2, *EAE*, *EAO*, *AEE*, *AEO*, *EIO*, *AOO*.

In Figure 3, *AAI*, *IAI*, *AII*, *EAO*, *OAO*, *EIO*.

In Figure 4, *AAI*, *AEE*, *AEO*, *EAO*, *IAI*, *EIO*.

145. Weakened Conclusions and Subaltern Moods.

When from premisses that would have justified a universal conclusion we content ourselves with inferring a particular, (as, for example, in the syllogism All M is P, All S is M, therefore, Some S is P), we are said to have a *weakened conclusion*, and the syllogism is said to be a *weakened syllogism* or to be in a *subaltern mood*, (because the conclusion might be obtained by subaltern inference from the conclusion of the corresponding strong mood).

In the preceding section it has been shewn that in each figure there are six moods which do not offend against any of the syllogistic rules; so that in all we have 24 distinct valid moods. Five of these however have weakened conclusions; and, since we are not likely to be satisfied with a particular conclusion when the corresponding universal can be obtained from the same premisses, these moods are of no practical importance. Accordingly when the moods of the various figures are enumerated (as in the mnemonic verses) they are usually omitted. Still, their recognition gives a completeness to the theory of the syllogism, which it cannot otherwise possess. There is also a symmetry in the result of their recognition as yielding exactly six legitimate moods in each figure[1].

The subaltern moods are,—

[1] It has been remarked that 19 being a prime number at once suggests incompleteness or artificiality in the common enumeration.

In Figure 1, *AAI*, *EAO*;
In Figure 2, *EAO*, *AEO*;
In Figure 4, *AEO*.

146. In what figure can there be no weakened conclusion and why? Do any of the 19 moods commonly recognised give a weaker conclusion than the premisses would warrant? [L.]

It is obvious that there can be no weakened conclusion in Figure 3, since in no case can we infer more than a particular conclusion in this figure.

I should answer the question, "whether any of the 19 moods commonly recognised yield a weaker conclusion than the premisses would warrant," in the negative. Professor Jevons (*Studies in Deductive Logic*, p. 87) apparently answers it in the affirmative, having in view *AAI* in Figure 4.

With the premisses

All P is M,
All M is S,

the conclusion *Some S is P* is certainly in one sense weaker than the premisses would warrant since we might have inferred the universal conclusion *All P is S*. But *All P is S* is not the universal corresponding to *Some S is P*. The subjects of these two propositions are different; and we infer all that we possibly can about *S* when we say *Some S is P*. In other words, regarded as a mood of Figure 4, this mood is not a subaltern. *AAI* in Figure 4 is thus differentiated from *AAI* in Figure 1, and its recognition in the mnemonic verses justified.

147. Strengthened Syllogisms.

If in a syllogism the same conclusion can still be obtained although for one of the premisses we substitute

its subaltern, the syllogism is said to be a *strengthened syllogism*. A strengthened syllogism is thus a syllogism with an unnecessarily strengthened premiss[1].

For example, the conclusion of the syllogism,—

All M is P,
All M is S,
therefore, *Some S is P,*

could equally be obtained from the premisses,—

All M is P,
Some M is S;

or from the premisses,—

Some M is P,
All M is S.

By trial we may find that *every syllogism in which there are two universal premisses with a particular conclusion is a strengthened syllogism, with the single exception of AEO in the Fourth Figure*[2].

In a full enumeration there are two strengthened syllogisms in each figure:—

In Figure 1, *AAI, EAO;*
In Figure 2, *EAO, AEO;*
In Figure 3, *AAI, EAO;*
In Figure 4, *AAI, EAO.*

The distinction between a strengthened syllogism, (that is, a syllogism with a strengthened premiss), and a weakened syllogism, (that is, a syllogism with a weakened conclusion), should be carefully noted.

It will be observed that in Figures 1 and 2, a syllogism having a strengthened premiss may also be regarded as a

[1] Cf. De Morgan, *Formal Logic*, pp. 91, 130.

[2] A general proof of this proposition is given in section 266.

syllogism having a weakened conclusion, and *vice versa;* but in Figures 3 and 4, the contrary holds in both cases. The only syllogism with a weakened conclusion in either of these figures is *AEO* in Figure 4, but this does not contain a strengthened premiss. That is, having

All P is M,
No M is S,
therefore, *Some S is not P*;

the syllogism becomes invalid, if for either of the premisses we substitute its subaltern.

148. The peculiarities and uses of each of the four figures of the syllogism.

Figure 1. In this figure we can prove conclusions of all the forms *A*, *E*, *I*, *O*; and it is the *only* figure in which we can prove a universal affirmative conclusion. This alone makes it by far the most useful and important of the syllogistic figures. All deductive science, the object of which is to establish universal affirmatives, tends to work in *AAA* in this figure.

Another point to notice is that only in this figure have we both the subject of the conclusion as subject in the premisses, and the predicate of the conclusion as predicate in the premisses. (In Figure 2 the predicate of the conclusion is subject in the major premiss; in Figure 3 the subject of the conclusion is predicate in the minor premiss; and in Figure 4 we have a double inversion[1].) This no doubt partly accounts for the fact that reasoning in Figure 1 so often seems more natural than the same reasoning expressed in any of the other figures[2].

[1] The double inversion in Figure 4 is one of the reasons given by Thomson for rejecting that figure altogether. Cf. section 165.

[2] Compare Solly, *Syllabus of Logic*, pp. 130—132.

Figure 2. In this figure we can prove negatives only; and therefore it is chiefly used for purposes of disproof. For example, Every real natural poem is naïve; those poems of Ossian which Macpherson pretended to discover are not naïve (but sentimental); hence they are not real natural poems. (Ueberweg, *System of Logic*, § 113.) It has been called the *exclusive* figure; because by means of it we may go on excluding various suppositions as to the nature of something under investigation, whose real character we wish to ascertain, (a process called *abscissio infiniti*).

For example,

Such and such an order has such and such properties,
This plant has not those properties;
therefore, It does not belong to that order.

A syllogism of this kind may be repeated with a number of different orders till the enquiry is so narrowed down that the place of the plant is easily determined. Whately (*Elements of Logic*, p. 92) gives an example from the diagnosis of a disease.

Figure 3. In this figure we can prove particulars only. It is frequently useful when we wish to take objection to a universal proposition laid down by an opponent, by establishing an instance in which such universal proposition does not hold good.

It is the natural figure when the middle term is a singular term, especially if the other terms are general. We have already shewn that if one and only one term of an affirmative proposition is singular it is almost necessarily the subject. For example, such a reasoning as,—

Socrates was wise,
Socrates was a philosopher,
therefore, Some philosophers are wise,

can only with great awkwardness be expressed in any figure other than Figure 3.

Figure 4. This figure is seldom used, and some logicians have altogether refused to recognise it. We shall return to a discussion of it subsequently. See section 165.

[Lambert (a distinguished mathematician as well as logician, whose *Neues Organon* appeared in 1764) expresses the uses of the different syllogistic figures as follows: "The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinctions between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus."

De Morgan (*Syllabus*, p. 30) thus characterizes the different figures: "The first figure may be called the figure of *direct transition;* the fourth, which is nothing but the first with a *converted conclusion*[1], the figure of *inverted transition;* the second, the figure of *reference to* (the middle term); the third, the figure of *reference from* (the middle term)."]

EXERCISES.

149. Why is *IE* an inadmissible, while *EI* is an admissible, mood in every figure of the syllogism? [L.]

150. Which of the following conjunctions of propositions make valid syllogisms? In the case of those which you regard as invalid, give your reasons for so treating them.

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
AEE	*AAA*	*AOE*	*AII*
AOO	*AOE*	*AEO*	
	IEA	*AOO*	
	AEE	*IEO*	

[1] Compare, however, section 165.

151. What moods are good in the first figure and faulty in the second, and *vice versa?* Why are they excluded in one figure and not in the other? [o.]

152. Shew that *O* cannot stand as premiss in Figure 1, as major in Figure 2, as minor in Figure 3, as premiss in Figure 4. [c.]

153. Shew that it is impossible to have the conclusion in *A* in any figure but the first. What fallacies would be committed if there were such a conclusion to a reasoning in any other figure? [c.]

154. Find in what figures the following moods are valid :—*AAI*, *AII*, *IAI*.

155. Prove that in Figure 4, if the minor premiss is negative, both the premisses must be universal.

CHAPTER IV.

THE REDUCTION OF SYLLOGISMS.

156. The Problem of Reduction.

By *Reduction* is meant the process of expressing the reasoning contained in a given syllogism in some other mood or figure. Unless otherwise stated, Reduction is always supposed to be to Figure 1.

As an example, we may take the following syllogism in Figure 3,—

All M is P,
Some M is S,
therefore, *Some S is P.*

It will be seen that by simply converting the minor premiss, we have precisely the same reasoning in Figure 1.

This is an example of *direct* or *ostensive* reduction.

157. Indirect Reduction.

We prove a proposition *indirectly* when we prove its contradictory to be false; and this may be done by shewing that an ultimate consequence of the truth of its contradictory is the truth of some proposition that is self-evidently false.

The method of indirect proof is in several cases adopted by Euclid; and it is sometimes employed in the reduction

of syllogisms from one mood to another. Thus, *AOO* in Figure 2 is usually reduced in this manner. From the premisses,—

All P is M,
Some S is not M,
it follows that *Some S is not P*;

for if this conclusion is not true, its contradictory (namely, *All S is P*) must be so, and the premisses being given true we shall have true together the three propositions,—

All P is M; (1)
Some S is not M; (2)
All S is P. (3)

But combining (1) and (3) we have a syllogism in Figure 1,—

All P is M,
All S is P,
yielding the conclusion *All S is M*. (4)

Some S is not M (2) and *All S is M* (4) are therefore true together; but this is self-evidently absurd, since they are contradictories.

Hence it has been shewn that the consequence of supposing *Some S is not P* false is a self-contradiction; and we may therefore infer that it is true.

It will be observed that the only explicit syllogism that has been made use of in the above is in Figure 1[1]; and the

[1] Solly (*Syllabus of Logic*, p. 104) maintains that a full analysis of the reasoning will shew that three distinct syllogisms are really involved,—

"Let *A* and *B* represent the premisses, and *C* the conclusion of any syllogism. In order to prove *C* by the indirect method, we commence with assuming that *C* is not true. The three syllogisms may be then stated as follows:

process is therefore regarded as a reduction of the reasoning to Figure 1.

This method of reduction is called *Reductio ad impossibile*, or *Reductio per impossibile*[1], or *Deductio ad impossibile*, or *Deductio ad absurdum*. It is the only way of reducing *AOO* (Figure 2), or *OAO* (Figure 3), to Figure 1, unless we make use of negative terms (as in obversion and contraposition); and it was adopted by the old writers in consequence of their objection to negative terms.

158. The mnemonic lines *Barbara, Celarent*, &c.

The mnemonic hexameter verses, (which are spoken of by De Morgan as "the magic words by which the different moods have been denoted for many centuries, words which I take to be more full of meaning than any that ever were made"), are usually given as follows,—

> *Barbără, Cēlārent, Dărīī, Fĕrīōque* prioris:
> *Cēsărĕ, Cāmēstres, Festīnŏ, Bărōcŏ*, secundae:
> Tertia, *Dāraptī, Dīsămis, Dātīsī, Fĕlapton,*
> *Bōcardō, Fĕrīsŏn*, habet: Quarta insuper addit
> *Brāmantip, Cămĕnes, Dīmăris, Fēsāpŏ, Frĕsīson.*

First syllogism: '*A* is; *C* is not; therefore *B* is not'.

Second syllogism: 'If *A* is, and *C* is not, it follows that *B* is not; but *B* is; therefore it is false that *A* is and *C* is not.'

Third syllogism: 'Either both propositions *A* is and *C* is not are false, or else one of them is false; but that *A* is is not false; therefore that *C* is not is false, (*i.e.*, *C* is).'"

I do not see any flaw in this analysis; at any rate it must be admitted that the reasoning involved in Indirect Reduction is highly complex, and since the two moods to which it is generally applied can also be reduced directly (see section 159), some modern logicians are inclined to banish it entirely from their treatment of the syllogism.

[1] Cf. Mansel's *Aldrich*, pp. 88, 89.

Each valid mood in every figure, unless it be a subaltern mood, is here represented by a separate word; and in the case of a mood in any of the so-called imperfect figures, (*i.e.*, Figures 2, 3, 4), the mnemonic gives full information for its reduction to Figure 1, the so-called perfect figure.

The only meaningless letters are *b* (not initial), *d* (not initial), *l*, *n*, *r*, *t*; the signification of the remainder is as follows:—

The *vowels* give the quality and quantity of the propositions of which the syllogism is composed; and therefore really give the syllogism itself. Thus, *Camenes* being in Figure 4, represents the syllogism,—

	All P is M,
	No M is S,
therefore,	*No S is P.*

The *initial letters* in the case of Figures 2, 3, 4 shew to which of the moods of Figure 1 the given mood is to be reduced, namely to that which has the same initial letter. [The letters *B*, *C*, *D*, *F* were chosen for the moods of Figure 1 as being the first four consonants in the alphabet.]

Thus, *Camestres* is reduced to *Celarent*,—

	All P is M,		*No M is S,*
	No S is M,	×	*All P is M,*
therefore,	*No S is P.*	therefore,	*No P is S,*
		therefore,	*No S is P.*

s (in the middle of a word) indicates that in the process of reduction the preceding proposition is to be simply converted. Thus, in reducing *Camestres* to *Celarent*, as shewn above, the minor premiss is simply converted.

s (at the end of a word[1]) shews that the conclusion of

[1] This peculiarity in the signification of *s* and *p* when they are *final* letters is frequently overlooked.

the *new* syllogism has to be simply converted in order to obtain the given conclusion. This again is illustrated in the reduction of *Camestres*. The final *s* does not affect the conclusion of *Camestres* itself, but the conclusion of *Celarent* to which it is reduced.

p (in the middle of a word) signifies that the preceding proposition is to be converted *per accidens*. Thus, in the reduction of *Darapti* to *Darii*,—

	All M is P,		*All M is P*,
	All M is S,		*Some S is M*,
therefore,	*Some S is P*.	therefore,	*Some S is P*.

p (at the end of a word[1]) implies that the conclusion *obtained by reduction* is to be converted *per accidens*. Thus, in *Bramantip*, the *p* obviously cannot affect the *I* conclusion of the mood itself[2]; it really affects the *A* conclusion of the syllogism in *Barbara* which is given by reduction. Thus,—

	All P is M,		*All M is S*,
	All M is S,		*All P is M*,
therefore,	*Some S is P*.	therefore,	*All P is S*,
		therefore,	*Some S is P*.

m indicates that in reduction the premisses have to be transposed, (*metathesis præmissarum*); as just shewn in the case of *Bramantip*, and also in the case of *Camestres*.

c signifies that the mood is to be reduced *indirectly*, (*i.e.*, by *reductio per impossibile* in the manner indicated in the preceding section); and the position of the letter indicates that in this process of indirect reduction the first step is to omit the premiss preceding it, *i.e.*, the other premiss is to be

[1] See note on the preceding page.

[2] Compare however Hamilton, *Logic*, I. p. 264, and Spalding, *Logic*, pp. 230, 1.

combined with the contradictory of the conclusion, (*conversio syllogismi*, or *ductio per contradictoriam propositionem sive per impossibile*). *c* is by some writers replaced by *k*, thus *Baroko* and *Bokardo* instead of *Baroco* and *Bocardo*.

The following lines are sometimes added to the verses given above, in order to meet the case of the subaltern moods:—

Quinque Subalterni, totidem Generalibus orti,
Nomen habent nullum, nec, si bene colligis, usum[1].

[1] The mnemonics have been written in various forms. Those given above are from Aldrich, and they are the ones that are in general use in England. Wallis in his *Institutio Logicæ* (1687) gives for Figure 4, *Balani*, *Cadere*, *Digami*, *Fegano*, *Fedibo*. P. van Musschenbroek in his *Institutiones Logicæ* (1748) gives *Barbari*, *Calentes*, *Dibatis*, *Fespamo*, *Fresisom*. This variety of forms for the moods of Figure 4 is no doubt due to the fact that the recognition of this figure at all was quite exceptional until comparatively recently. Compare section 166.

According to Ueberweg, the mnemonics run,—

Barbara, *Celarent* primæ, *Darii Ferio*que.
Cesare, *Camestres*, *Festino*, *Baroco* secundæ.
Tertia grande sonans recitat *Darapti*, *Felapton*,
Disamis, *Datisi*, *Bocardo*, *Ferison*. Quartæ
Sunt *Bamalip*, *Calemes*, *Dimatis*, *Fesapo*, *Fresison*.

Mr Carveth Read (*Mind*, 1882, p. 440) suggests an ingenious modification of the verses, so as to make each mnemonic immediately suggest the figure to which the corresponding mood belongs, at the same time abolishing all the unmeaning letters. He takes *l* as the sign of the first figure, *n* of the second, *r* of the third, and *t* of the fourth. The lines then run

Ballala, *Celallel*, *Dalii*, *Felio*que prioris.
Cesane, *Camesnes*, *Fesinon*, *Banoco* secundæ.
Tertia *Darapri*, *Drisamis*, *Darisi*, *Ferapro*,
Bocaro, *Ferisor* habet. Quarta insuper addit
Bamatip, *Cametes*, *Dimatis*, *Fesapto*, *Fesistot*.

On the same principle, Miss Christine Ladd (*Studies in Logic*, Johns

159. The *direct* reduction of *Baroco* and *Bocardo*.

Baroco:—

All P is M,
Some S is not M,
therefore, *Some S is not P,*

may be reduced to *Ferio* by contrapositing the major premiss, and obverting the minor premiss, thus,—

No not-M is P,
Some S is not-M,
therefore, *Some S is not P.*

Faksoko has been suggested as a mnemonic for this method of reduction, *k* denoting obversion, so that *ks* denotes obversion followed by conversion (*i.e.*, contraposition).

Whately's mnemonic *Fakoro* (*Elements of Logic*, p. 97) does not indicate the obversion of the minor premiss (*r* being with him an unmeaning letter).

Bocardo:—

Some M is not P,
All M is S,
therefore, *Some S is not P,*

may be reduced to *Darii* by contrapositing the major premiss and transposing the premisses, thus,—

All M is S,
Some not-P is M,
therefore, *Some not-P is S.*

Hopkins University, p. 40) suggests that the different figures might be indicated by the letters *r*, *t*, *l* and *n* respectively. Thus,—

Barbara, *Cegare*, *Darii*, *Ferio*que prioris.
Cesate, *Camestes*, *Festivo*, *Batoko* secundæ.
Tertia *Dalipi*, *Disalmis*, *Dalisi*, *Felapo*,
Bokalo, *Feliso* habet. Quarta insuper addit
Bamanip, *Camenes*, *Dimanis*, *Fesanpo*, *Fesison*.

We have first to convert and then to obvert this conclusion, however, in order to get the original conclusion. This process may be indicated by *Doksamosk*, (which again is obviously preferable to *Dokamo* suggested by Whately, since this word would make it appear as if we immediately obtained the original conclusion in *Darii*[1]).

160. Indirect reduction of *Bramantip*.

Just as *Bocardo* and *Baroco* which are usually reduced indirectly may be reduced directly, so other moods which are usually reduced directly may be reduced indirectly. We may take *Bramantip* as an example :—

All P is M,
All M is S,
therefore, *Some S is P*;

for, if not, then *No S is P*; and combining this with the given minor premiss we have a syllogism in *Celarent*,—

No S is P,
All M is S,
therefore, *No M is P*,

which yields by conversion *No P is M*. But this is the contrary of the original major premiss *All P is M*, and it is impossible that they should be true together. Hence we infer the truth of the original conclusion.

[1] Mr Carveth Read (*Mind*, 1882, p. 441) uses the letters *k* and *s* as above; but his mnemonics are required also to indicate the figure to which the moods belong (see the preceding note); and he therefore arrives at *Faksnoko* and *Doksamrosk*.

Spalding (*Logic*, p. 235) suggests *Facoco* and *Docamoc*; but the processes here indicated by the letter *c* are not in all cases the same, and these mnemonics are therefore unsatisfactory.

161. Any syllogistic reasoning may be reduced, directly or indirectly, not merely to Figure 1, but to any given mood of that Figure, provided it be not a subaltern mood.

The doctrine of reduction may be extended, and it can be shewn not merely that any syllogism may be reduced to Figure 1, but also that it may be reduced to any given mood of that figure, provided it is not a subaltern mood. This position will obviously be established if we can shew that *Barbara*, *Celarent*, *Darii*, and *Ferio* are mutually reducible to one another. *Barbara* may be reduced to *Celarent* by obverting the major premiss and also the new conclusion which is thereby obtained. Thus,

All M is P,
All S is M,
therefore, *All S is P;*

becomes *No M is not-P*,
All S is M,
therefore, *No S is not-P*,
therefore, *All S is P.*

Conversely, *Celarent* is reducible to *Barbara*; and in a similar manner by obversion of major premiss and conclusion *Darii* and *Ferio* are reducible to each other.

It will now suffice if we can shew that *Barbara* and *Darii* are mutually reducible to each other. Clearly the only method possible here is the *indirect* method.

Take *Barbara*, MaP,
SaM,
$\overline{SaP};$

for, if not, then we have SoP; and MaP, SaM, SoP must be true together. From SoP by first obverting and then

converting, (and denoting not-*P* by *P'*), we get *P'iS*, and combining this with *SaM* we have a syllogism in *Darii*,—

$$\frac{\begin{array}{c} SaM, \\ P'iS, \end{array}}{P'iM.}$$

P'iM by conversion and obversion becomes *MoP*; and therefore *MaP* and *MoP* are true together; but this is impossible, since they are contradictories.

Therefore, *SoP* cannot be true, *i.e.*, the truth of *SaP* is established.

Similarly, *Darii* may be indirectly reduced to *Barbara*[1].

$$\frac{\begin{array}{ll} MaP, & \text{(i)} \\ SiM, & \text{(ii)} \end{array}}{\begin{array}{ll} SiP. & \text{(iii)} \end{array}}$$

The contradictory of (iii) is *SeP*, from which we obtain *PaS'*. Combining with (i), we have—

$$\frac{\begin{array}{c} PaS' \\ MaP, \end{array}}{MaS' \text{ in } \textit{Barbara}.}$$

But from this conclusion we may obtain *SeM*, which is the contradictory of (ii).

162. *Dicta* for Figures 2 and 3 corresponding to the *Dictum* for Figure 1.

For Figure 2 has been given a *dictum de diverso*,—"If one term is contained in, and another excluded from, a third term, they are mutually excluded." This is at least expressed loosely since it would appear to warrant a universal conclu-

[1] It has been maintained, that this reduction is unnecessary, and that, to all intents and purposes, *Darii* is *Barbara*, since the "some *S*" in the minor is, and is known to be, the *same some* as in the conclusion.

sion in *Festino* and *Baroco.* Mansel (*Aldrich*, p. 86) puts this *Dictum* in a more satisfactory form:—"If a certain attribute can be predicated, affirmatively or negatively, of every member of a class, any subject of which it cannot be so predicated, does not belong to the class." This proposition may claim to be axiomatic, and it can be applied directly to any syllogism in Figure 2.

For Figure 3 is given a *dictum de exemplo*[1],—"Two terms which contain a common part, partly agree, or if one contains a part which the other does not, they partly differ." This formula also is open to exception. The proposition "If one term contains a part which another does not, they partly differ" applied to *No M is P*, *All M is S*, would appear to justify *Some P is not S* just as much as *Some S is not P.* Mansel's amendment here is to give two principles for Figure 3, the *Dictum de exemplo*,—"If a certain attribute can be affirmed of any portion of the members of a class, it is not incompatible with the distinctive attributes of that class"; and the *Dictum de excepto*,—"If a certain attribute can be denied of any portion of the members of a class, it is not inseparable from the distinctive attributes of that class." But is it essential that in the minor premiss we should be predicating the *distinctive attributes* of the class as is here implied? This appears to be a fatal objection to Mansel's *dicta* for Figure 3. Moreover, granted that *P* is *not incompatible* with *S*, are we therefore justified in saying *Some S is P*?

[1] The *dictum de diverso* and the *dictum de exemplo* are usually attributed to Lambert. Thomson, however, remarks that Mill and others are in error "in thinking that Lambert invented these *dicta.* More than a century earlier, Keckermann saw that each figure had its own law and its peculiar use, and stated them as accurately, if less concisely, than Lambert" (*Laws of Thought*, p. 173, note).

I would suggest the following axioms,—"If two terms are both affirmatively predicated of a common third, and one at least of them universally so, they may be partially predicated of each other"; "If one term is denied while another is affirmed of a common third term, either the denial or the affirmation being universal, the former may be partially denied of the latter." These will I think be found to apply respectively to the affirmative and negative moods of Figure 3, and they may be regarded as axiomatic; but they are certainly somewhat laboured.

163. Is *Reduction* an essential part of the doctrine of the syllogism?

According to the original theory of Reduction, the object of the process is to be sure that the conclusion is a valid inference from the premisses. Given a syllogism in Figure 1, we are able to test its validity by reference to the *Dictum de omni et nullo*; but we have no such means of dealing directly with syllogisms in any other figure. Thus, Whately says,—"As it is on the *Dictum de omni et nullo* that all Reasoning *ultimately* depends, so, all arguments may be in one way or other brought into some one of the four Moods in the First Figure: and a Syllogism is, in that case, said to be *reduced*" (*Elements of Logic*, p. 93). Professor Fowler puts the same position in a more guarded manner,—"As we have adopted no canon for the 2nd, 3rd, and 4th figures, we have as yet no positive proof that the six moods remaining in each of those figures are valid; we merely know that they do not offend against any of the syllogistic rules. But if we can *reduce* them, *i.e.*, bring them back to the 1st figure, by shewing that they are only different statements of its moods, or in other words, that precisely the same conclusions can be

obtained from equivalent premisses in the 1st figure, their validity will be proved beyond question" (*Deductive Logic*, p. 97).

On the other hand, by some logicians Reduction is regarded as *unnecessary* and *unnatural*. It is maintained to be *unnecessary* on the ground that it is not true that the *Dictum de omni et nullo* is the paramount law for all perfect inference, or that the first figure alone is perfect[1]. In the preceding section we have discussed *dicta* for the other figures, which may be regarded as making them independent of the first, and putting them on a level with it. It may also be maintained that in any mood the validity of a particular syllogism is as self-evident as that of the *Dictum* itself; and that therefore although axioms of syllogism are useful as *generalisations* of the syllogistic process, they are needless in order to establish the validity of any given syllogism. This view is indicated by Ueberweg.

Again, Reduction is said to be *unnatural*, inasmuch as it often involves the substitution of an unnatural and indirect for a natural and direct predication. Figures 2 and 3 at any rate have their special uses, and certain reasonings naturally fall into these figures rather than into Figure 1[2].

[1] See Thomson, *Laws of Thought*, p. 172.

[2] Sir W. Hamilton (*Logic*, vol. 2, p. 438) quotes from Lambert (*Neues Organon*, §§ 230, 231) as follows:—"For, as the syllogisms of every figure admit of being transmuted into those of the first, and partly also into those of any other, if we rightly convert, or interchange, or turn into propositions of equal value, their premisses; consequently, in this point of view, no difference subsists between them. But whether we in every case should perform such commutations in order to bring a syllogism under a favourite figure, or to assure ourselves of its correctness,—this is a wholly different question. The latter is manifestly futile. For, in the commutation, we must always undertake a conversion of the premisses, and a converted proposition is assuredly not always

This argument is very well elaborated by Archbishop Thomson (*Laws of Thought*, pp. 173—175). He gives this example,—"Thus, when it was desirable to shew by an example that zeal and activity did not always proceed from selfish motives, the natural course would be some such syllogism as the following. The Apostles sought no earthly reward, the Apostles were zealous in their work; therefore, some zealous persons seek not earthly reward." In reducing this syllogism to Figure 1, we have to convert our minor into "Some zealous persons were Apostles," which is awkward and unnatural.

Take again this syllogism,

"Every reasonable man wishes the Reform Bill to pass,
I don't,
therefore, I am not a reasonable man."

Reduced in the regular way to *Celarent*, the major premiss becomes "No person wishing the Reform Bill to pass is I," yielding the conclusion "No reasonable man is I."

Further illustrations of this point will be found if we reduce to Figure 1, syllogisms with such premisses as the following:—All orchids have opposite leaves, this plant has not opposite leaves; Socrates is poor, Socrates is wise.

The above arguments appear conclusively to establish the position that Reduction is not an essential part of the doctrine of the Syllogism, at any rate so far as establishing the validity of the different moods is concerned.

of equal evidence with that which we had to convert, while, at the same time, we are not so well accustomed to it......It is thus apparent that we use every syllogistic figure there, where the propositions, as each figure requires them, are more familiar and more current. The difference of the figures rests, therefore, not only on their form, but extends itself, by relation to their employment, also to things themselves, so that we use each figure where its use is more natural."

It may, however, be doubted whether any treatment of the Syllogism can be regarded as scientific or complete until the *equivalence* between the moods in the different figures has been shewn; and for this purpose, as well as for its utility as a logical exercise, a full treatment of the problem of Reduction should be retained.

164. Are Figures 2, 3, and 4 genuine and original forms of reasoning?

Hamilton (*Logic*, I. p. 433) answers this question in the negative. "The last three figures," he says, "are virtually identical with the first." This has been recognised by logicians, and hence "the tedious and disgusting rules of their reduction." He himself goes further, and extinguishes these figures altogether, as being merely "accidental modifications of the first," and "the mutilated expressions of a complex mental process."

If the last three figures are admitted as genuine and original forms of reasoning, the following anomalies in Hamilton's opinion result:—

"In the first place, the principle that all reasoning is the recognition of the relation of a least part to a greatest whole, through a lesser whole or greater part, is invalidated." In reply to this, it may simply be asked whether it really requires the last three figures in order to invalidate this principle.

"In the second place, the second general rule I gave you for categorical syllogisms, is invalidated in both its clauses." It does not occur to Hamilton that his rules may have been needlessly limited in their application. He has indeed with a great flourish of trumpets *simplified* the rules of the syllogism by replacing those usually given by

the special rules of Figure 1[1]; and he is now shocked to find that these do not apply to Figures 2, 3, 4. This whole reasoning of Hamilton's is a flagrant example of *petitio principii*.

The question at issue is really this,—can we formulate a principle which shall be accepted as axiomatic, and which shall apply immediately to syllogisms in other figures than the first?

Now take a syllogism in *Cesare*:

No P is M,
All S is M,
therefore, *No S is P.*

Hamilton maintains (*Logic*, I. pp. 434, 435) that we can only properly see the force of this reasoning by mentally converting the major premiss to *No M is P.* But will not the following which applies immediately to *Cesare* be accepted as axiomatic,—"If one class is excluded from and another is contained in a third class, these two classes are excluded from one another"? This simple case seems to me sufficient to overthrow the whole of Hamilton's elaborate but confused reasoning[2].

Baroco may be taken as another example:

[1] "Had Dr Whately looked a little closer into the matter, he might have seen that the six rules which he and other logicians enumerate, may, without any sacrifice of precision, and with even an increase of perspicuity, be reduced to three.......These three simple laws comprise all the rules which logicians lay down with so confusing a minuteness" (*Logic*, I. pp. 305, 6). The simplification is obtained solely by giving laws which have a more limited application than other logicians had contemplated.

[2] It may be pointed out that Hamilton himself elsewhere (*Logic*, vol. II. p. 358) gives special Canons for Figures 2, 3.

All P is M,
Some S is not M,
therefore, *Some S is not P.*

Axiom: "If one class is totally contained in, and another partially excluded from a third class, the second class is partially excluded from the first." Now compare Hamilton's elaborate explication (*Logic*, I. pp. 438, 9),—"The formula of *Baroco* is:—

All P are M;
But *some S are not M;*
Therefore, *some S are not P.*

But the following is the full mental process:—

Sumption,........................ *All P are M;*
Real Subsumption,............ (*Some not-M are S*);
which gives the
Expressed Subsumption,... {Then, *Some S are not-M;* Or, *Some S are not M;*
Real Conclusion,...............(Therefore, *Some not-P are S*);
which gives the
Expressed Conclusion, ...{Then, *Some S are not-P;* Or, *Some S are not P.*"

It is surely absurd to say that we go through this complex mental process in order to discover the validity of a syllogism in *Baroco.*

But even granting that this is the case, I cannot see how on his own grounds Hamilton succeeds in getting rid of the necessity of "the tedious and disgusting" rules of reduction; nor that he has advanced beyond the logicians who reject the independent validity of Figures 2, 3, 4, and consequently establish the necessity of the process of reduction, and naturally along with it of rules for conducting the process.

It would seem that only those logicians who, like Thomson, maintain the independent validity of other figures than the first have any justification whatever for ignoring the doctrine of reduction.

The position taken by Hamilton is somewhat similar to that taken by Kant in his essay "On the Mistaken Subtilty of the Four Figures." Kant's argument is virtually based on the two following propositions: (1) Reasonings in Figures 2, 3, 4 require to be implicitly, if not explicitly, reduced to Figure 1, in order that their validity may be apparent[1]; (2) No reasonings ever fall naturally into any of the moods of Figures 2, 3, 4, which are, therefore, a mere useless invention of logicians. On grounds already indicated, I cannot but regard both these propositions as erroneous[2].

165. *The Fourth Figure.*

Figure 4 was not as such recognised by Aristotle; and its introduction having been attributed by Averroes to Galen, it is frequently spoken of as the *Galenian* figure. It does not usually appear in works on Logic before the beginning of the last century, and even by modern logicians its use is sometimes condemned. Thus, Bowen (*Logic*, p. 192) holds that "what is called the Fourth Figure is only the First with a converted conclusion; that is, we do

[1] For example, he remarks that in *Cesare* we must have covertly performed the conversion of the major premiss in thought, since otherwise our premisses would not be conclusive.

[2] A further error seems to be involved in the following: "It cannot be denied that we can draw conclusions legitimately in all these figures. But it is incontestable that all except the first determine the conclusion only by a roundabout way, and by interpolated inferences, and that *the very same conclusion would follow from the same middle term in the first figure by pure and unmixed reasoning.*" At least I do not see how the latter part of this statement can be justified in the case of *Baroco*.

not actually reason in the Fourth, but only in the First, and then if occasion requires, convert the conclusion of the First." This account of the Fourth Figure cannot, however, be accepted, for it will not apply to *Fesapo* or *Fresison*. For example, the premisses of *Fesapo* are

No P is M,
All M is S;

and, as they stand, we cannot obtain any conclusion whatever from them in Figure 1.

Thomson's ground of rejection is that "in the fourth figure the order of thought is wholly inverted, the subject of the conclusion had only been a predicate, whilst the predicate had been the leading subject in the premiss. Against this the mind rebels; and we can ascertain that the conclusion is only the converse of the real one, by proposing to ourselves similar sets of premisses, to which we shall always find ourselves supplying a conclusion so arranged that the syllogism is the first figure, with the second premiss first" (*Laws of Thought*, p. 178). With regard to the first part of this argument, Thomson himself points out that the same objection applies partially to Figures 2 and 3. It is no doubt a reason why as a matter of fact Figure 4 is seldom used; but I cannot see that it is a reason for altogether refusing to recognise it. The second part of Thomson's argument is, for a reason already stated, unsound. The conclusion, for example, of *Fresison* cannot be "the converse of the real conclusion," since (being an **O** proposition) it is the converse of nothing at all.

For my own part, I do not see how we can treat the syllogism scientifically and completely without admitting Figure 4. In an *a priori* separation of figures according to the position of the terms in the premisses, it necessarily

appears, and we find that valid reasoning is possible in it. It is not actually in frequent use, but reasonings may sometimes not unnaturally fall into it; for example,—

None of the apostles were Greeks,
Some Greeks are worthy of all honour,
therefore, Some worthy of all honour are not apostles.

166. The moods of Figure 4 regarded as indirect moods of Figure 1.

The earliest form in which the mnemonic verses appeared was as follows:—

Barbara, Celarent, Darii, Ferio, Baralipton,
Celantes, Dabitis, Fapesmo, Frisesomorum,
Cesare, Camestres, Festino, Baroco, Darapti,
Felapton, Disamis, Datisi, Bocardo, Ferison[1].

Aristotle recognised only three figures: the first figure, which he considered the type of all syllogisms and which he called the perfect figure, the *Dictum de omni et nullo* being directly applicable to it alone; and the second and third figures, which he called imperfect figures, it being necessary to reduce them to Figure 1, in order to obtain a test of their validity.

Before the fourth figure, however, was commonly recognised as such, its moods were recognised in another form, namely, as *indirect* moods of the first figure; and the above mnemonics,—*Baralipton, Celantes, Dabitis, Fapesmo, Frisesomorum,*—represent these moods so regarded[2].

[1] First given by Petrus Hispanus, afterwards Pope John XXI., who died in 1277.

[2] From the 14th to the 17th century the mnemonics found in works on Logic usually give the moods of Figure 4 in this form, or else omit them altogether. Wallis (1687) recognises them in both forms, giving two sets of mnemonics.

Mansel (*Aldrich*, p. 78) defines an *indirect* mood as "one in which we do not infer the immediate conclusion, but its converse."

Thus,— *All M is P*,
All S is M,

yields the direct conclusion, *All S is P*, in *Barbara*, or the indirect conclusion, *Some P is S*, in *Baralipton*.

Similarly *Celantes* corresponds to *Celarent*, and *Dabitis* to *Darii*.

Nevertheless, exception must be taken to Mansel's definition, since in *Fapesmo* and *Frisesomorum* we have indirect moods to which there are no corresponding valid direct moods. In these we do not infer "the converse of the immediate conclusion" since there is no immediate conclusion. Mansel deals with these two moods very awkwardly. He says, "*Fapesmo* and *Frisesomorum* have negative minor premisses, and thus offend against a special rule of the first figure; but this is checked by a counter-balancing transgression. For by simply converting **O**, we alter the distribution of the terms, so as to avoid an illicit process." But surely we cannot counterbalance one violation of law by committing a second. The truth of course is that, in the first place, the special rules of Figure 1 as ordinarily given do not apply to the indirect moods; and in the second place, the conclusion **O** is not obtained by conversion at all.

The real distinction between direct and indirect moods is that the terms which are major and minor in the one become respectively minor and major in the other.

Taking *Ferio*, and making this inversion, we have no valid conclusion, therefore, *Ferio* has no corresponding indirect mood. Similarly, *Fapesmo* and *Frisesomorum*, (the *Fesapo* and *Fresison* of Figure 4), have no corresponding direct moods.

It is a merely formal difference whether we recognise the five moods in question in this way, or as constituting a distinct figure; but the latter alternative seems far less likely to give rise to confusion.

We have also indirect moods in Figures 2 and 3, but they are merely reproductions of other of the direct moods, as noticed by Professor Fowler (*Deductive Logic*, p. 104). Thus, the premisses,—

No P is M,
All S is M,

may yield both the conclusions *No S is P*, and *No P is S*; and if we call the former the *direct* conclusion the latter will be the *indirect*, (the former being *Cesare*, and the latter *Camestres* with the premisses transposed).

Some P is M,
No S is M,

yields no direct conclusion, but has an indirect conclusion *Some P is not S.* This however merely gives *Festino* over again.

We get a similar result, in all cases in which this conception is applied within the limits of Figures 2 and 3.

Exercises.

167. "*Barbara*, *Baroco* and *Bocardo* cannot be ostensively reduced to any other figure except by the use of conversion by contraposition." Shew by general reasoning that this follows from the rules relating to the conversion of propositions. Also reduce each of the above moods directly to Figure 4 by the aid of conversion by contraposition.

168. Reduce *Ferio* to Figure 2, *Festino* to Figure 3, *Felapton* to Figure 4.

169. State the following argument in a syllogism of the third figure, and reduce it, both directly and indirectly, to the first:—Some things worthy of being known are not directly useful, for every truth is worthy of being known, while not every truth is directly useful.

[University of Melbourne.]

170. Express the following argument in as many moods of the third figure as you can, using any process of immediate inference which may be necessary:—Some things which have a practical worth are also of theoretical value; for every science has a theoretical as well as a practical value. [University of Melbourne.]

171. "Rejecting the fourth figure and the subaltern moods, we may say with Aristotle; **A** is proved only in one figure and one mood, **E** in two figures and three moods, **I** in two figures and four moods, **O** in three figures and six moods. For this reason, **A** is declared by Aristotle to be the most difficult proposition to establish, and the easiest to overthrow; **O**, the reverse." Discuss the fitness of these data to establish the conclusion.

172. Shew that any mood may be directly reduced to any other mood, provided (1) that the latter contains neither a strengthened premiss nor a weakened conclusion, and (2) that if the conclusion of the former is universal, the conclusion of the latter is also universal.

173. Shew that any mood may be directly or indirectly reduced to any other mood provided that the latter

contains neither a strengthened premiss nor a weakened conclusion.

174. Examine the following statement of De Morgan's:—"There are but six distinct syllogisms. All others are made from them by strengthening one of the premisses, or converting one or both of the premisses, where such conversion is allowable; or else by first making the conversion, and then strengthening one of the premisses."

CHAPTER V.

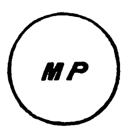
THE DIAGRAMMATIC REPRESENTATION OF SYLLOGISMS.

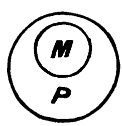
175. The application of the Eulerian diagrams to syllogistic reasonings.

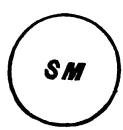
To illustrate the application of the Eulerian diagrams to syllogistic reasonings we may take a syllogism in *Barbara*:

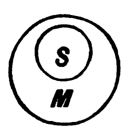
All M is P,
All S is M,
therefore, *All S is P*.

We must first represent the premisses separately by means of the diagrams. They each yield two cases; thus,—

All M is P, (i) 

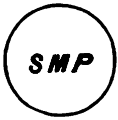
(ii) 

All S is M, (*a*) 

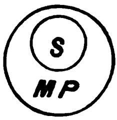
(*b*) 

To obtain the conclusion, each of the cases yielded by the major premiss must now be combined with each of those yielded by the minor. This gives four combinations, and whatever is true of *S* in terms of *P* in *all* of them, is the conclusion required.

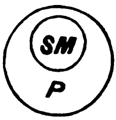
(i) and (*a*) yield



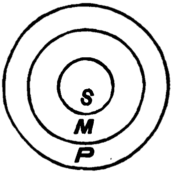
(i) and (*b*)



(ii) and (*a*)



(ii) and (*b*)



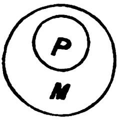
We find that in all these cases *all S is P*, which conclusion may therefore be inferred from the given premisses.

Next, take a syllogism in *Bocardo*. The application of the diagrams is now more complicated. The premisses are,—

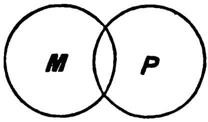
Some M is not P,
All M is S.

The major premiss gives three possible cases,—

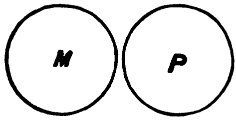
(i)



(ii)

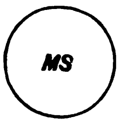


(iii)

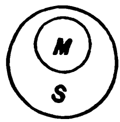


and the minor premiss gives two possible cases,—

(*a*)

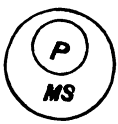


(*b*)

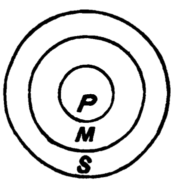


Taking them together we have six combinations, some of which themselves yield more than one case :—

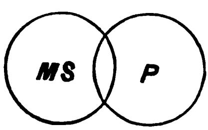
(i) and (*a*)



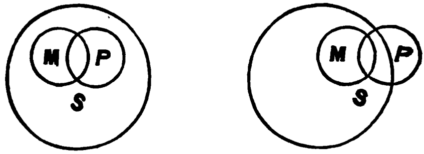
(i) and (*b*)



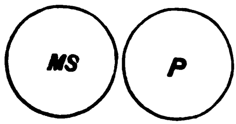
(ii) and (*a*)



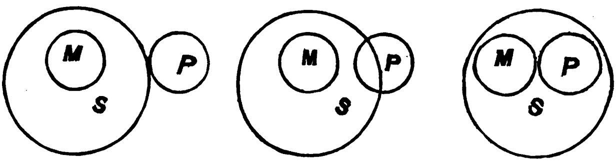
(ii) and (*b*)



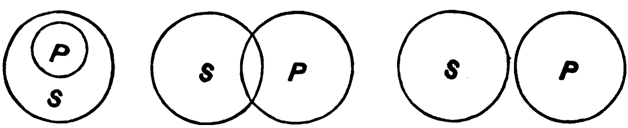
(iii) and (*a*)



(iii) and (*b*)



So far as *S* and *P* are concerned, (*i.e.*, leaving *M* out of account), it will be found that these nine cases are reducible to the following three:

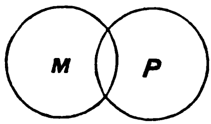


The conclusion therefore is *Some S is not P.*

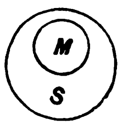
It must be admitted that this is very complex, and it would be a serious matter if in the first instance we had to work through all the different moods in this manner. Still, for purposes of illustration, this very complexity has a certain advantage. It shews how many relations between three terms in respect of extension are left to us, even with two premisses given.

176. What is all that we can infer about *S* and *P* respectively from the following premisses,—*Some M is not P*, *Some M is P*, *Some P is not M*, *All M is S*, *Some S is not M*?

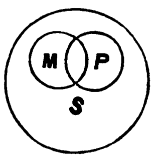
The first three premisses limit us to



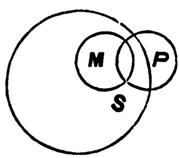
and the two remaining ones to



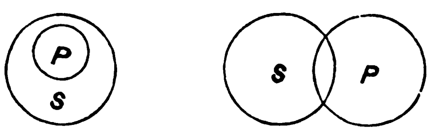
Combining these, we have



or



that is, so far as *S* and *P* are concerned, the premisses limit us to one or other of the following,—



These may be interpreted:

Some S is P, *Some S is not P*, and *Some P is S*.

We could of course obtain the same conclusions by the aid of the ordinary syllogistic moods.

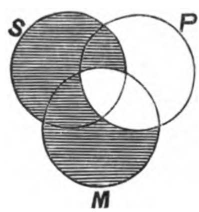
177. The application of Dr Venn's diagrammatic scheme to syllogistic reasonings.

We may take *Barbara* and *Camestres* to illustrate the above.

The premisses of *Barbara*,—

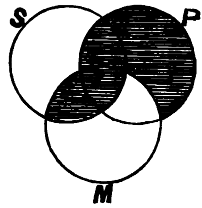
All M is P,
All S is M,

exclude certain compartments as shewn in the following diagram:



This yields at once the conclusion that *All S is P.*

Similarly for *Camestres* we have the following:



This scheme is especially adapted to illustrate the syllogistic processes when all the propositions involved are universal. A further device must be introduced when one of the premisses is particular. Compare section 94.

178. Lambert's scheme of diagrammatic notation.

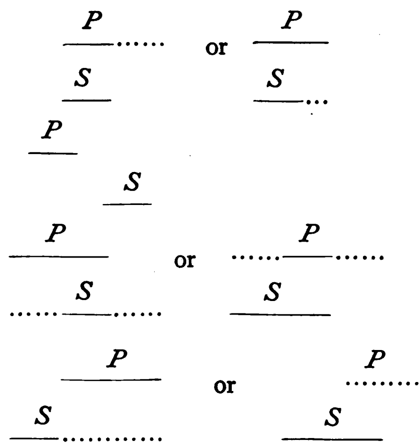
In a system based on that of Lambert[1], propositions may be represented as follows:

All S is P[2]

No S is P

Some S is P[2]

Some S is not P[2]



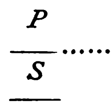
It will be observed that the extension of a term is represented by a horizontal straight line, and that so far as

[1] Lambert's own scheme was somewhat different. It involves difficulties from which the following modification of it is free.

[2] In the cases of **A**, **I**, and **O**, the diagrams given are alternative, in the sense that we may select which we please to represent our proposition, and either represents it completely.

Mr Johnson points out that by a further modification, Lambert's scheme becomes precisely analogous to Euler's. We now avoid dotted lines altogether, but each propositional form (except **E**) requires more

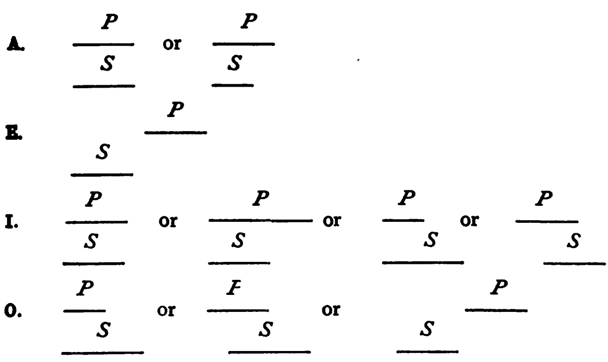
two such lines overlap each other the corresponding classes are coincident, while so far as they do not overlap the corresponding classes are mutually exclusive. The line is dotted in so far as we are uncertain with regard to a portion of the class. Thus, in the case of *All S is P*, the diagram



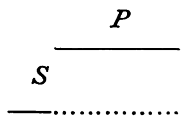
indicates that all *S* is contained under *P*, but that we are uncertain as to whether there is or is not any *P* which is not *S*.

In the case of *Some S is not P*, the diagram

than one combination of lines to cover all possible cases. Thus,—



These diagrams, it is obvious, are alternative in a different sense from those in the text. A comparison with section 94 will indicate their use more clearly. They take less room than Euler's circles. But they seem also to be less intuitively clear and less suggestive. The different cases too are less markedly distinct from one another. It is probable that one would in consequence be more liable to error in employing them.

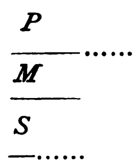


indicates that there is S which is not P, but that we are in ignorance as to the existence of any S that is P.

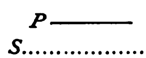
179. The application of Lambert's diagrammatic scheme to syllogistic reasonings.

As applied to syllogisms, the method indicated in the preceding section is much less cumbrous than the Eulerian diagrams[1]. We may take the following examples:—

Barbara[2]

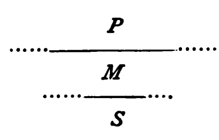


[1] Dr Venn (*Symbolic Logic*, p. 432) remarks,—"As a whole Lambert's scheme seems to me distinctly inferior to the scheme of Euler, and has in consequence been very little employed by other logicians." The criticism offered in support of this statement is chiefly directed against Lambert's own representation of the particular affirmative proposition, namely,—

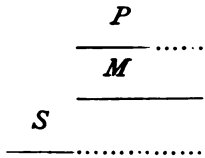


This diagram certainly seems as appropriate to **O** as it does to **I**; but the modification introduced in the text, and indeed suggested by Dr Venn himself, is not open to a similar objection.

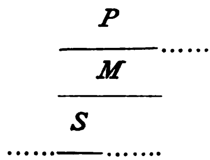
[2] The following representation of *Barbara*



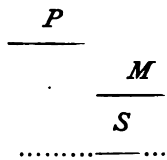
Baroco



Datisi



Fresison



EXERCISES.

180. Represent by means of the Eulerian diagrams the moods *Celarent*, *Festino*, *Datisi*, and *Bramantip*. Shew also by means of these diagrams that **IE** yields no conclusion in any figure.

181. Represent in Dr Venn's diagrammatic scheme the moods *Celarent*, *Cesare*, *Camenes*.

which was given in the first edition, is erroneous, for it would justify us in inferring *Some P is not S* as well as *All S is P*.

It is in general necessary in representing syllogisms that the line standing for the middle term should not be dotted over any part of its extent. This condition can be satisfied by selecting the appropriate alternative form in the case of **A**, **I**, and **O** propositions.

182. Represent in Lambert's scheme the moods *Darii*, *Cesare*, *Darapti*, *Bocardo*, *Fesapo*.

183. Taking the premisses of an ordinary syllogism in *Barbara*, *e.g.*, *all X is Y*, *all Y is Z*, determine precisely and exhaustively what these própositions affirm, what they deny, and what they leave in doubt, concerning the relations of the terms *X*, *Y*, *Z*. [L.]

[This question can be very well answered by the aid of any of the three diagrammatic schemes which we have just been discussing. Compare also Jevons, *Studies in Deductive Logic*, p. 216.]

CHAPTER VI.

IRREGULAR AND COMPOUND SYLLOGISMS.

184. The Enthymeme.

By the *Enthymeme*, Aristotle meant what has been called the "rhetorical syllogism" as opposed to the apodeictic, demonstrative, theoretical syllogism. The following is from Mansel's notes to *Aldrich* (pp. 209—211): "The Enthymeme is defined by Aristotle, *συλλογισμὸς ἐξ εἰκότων ἢ σημείων*. The *εἰκὸς* and *σημεῖον* themselves are Propositions; the former stating a *general probability*, the latter a *fact*, which is known to be an indication, more or less certain, of the truth of some further statement, whether of a single fact or of a general belief. The former is a proposition nearly, though not quite, *universal*; as 'Most men who envy hate': the latter is a *singular* proposition, which however is not regarded as a sign, except relatively to some other proposition, which it is supposed may be inferred from it. The *εἰκός*, when employed in an Enthymeme, will form the *major premiss* of a Syllogism such as the following:

Most men who envy hate,
This man envies,
therefore, This man (probably) hates.

The reasoning is logically faulty; for, the major premiss not being absolutely universal, the middle term is not distributed.

The σημεῖον will form one premiss of a Syllogism which may be in any of the three figures, as in the following examples:

Figure 1. All ambitious men are liberal,
Pittacus is ambitious,
Therefore, Pittacus is liberal.

Figure 2. All ambitious men are liberal,
Pittacus is liberal,
Therefore, Pittacus is ambitious.

Figure 3. Pittacus is liberal,
Pittacus is ambitious,
Therefore, All ambitious men are liberal.

The syllogism in the first figure alone is logically valid. In the second, there is an undistributed middle term; in the third, an illicit process of the minor."

On this subject the student may be referred to the remainder of the note from which the above extract is taken, and to Hamilton, *Discussions*, pp. 152—156.

An *enthymeme* is now usually defined as a syllogism incompletely stated, one of the premisses or the conclusion being understood but not expressed. As has been frequently pointed out, the arguments of everyday life are for the most part enthymematic. The same may be said of fallacious arguments, which are seldom completely stated, or their want of cogency would be more quickly recognised.

An enthymeme is said to be of the first order when the major premiss is suppressed; of the second order when the minor premiss is suppressed; and of the third order when the conclusion is suppressed.

Thus, "Balbus is avaricious, and therefore, he is unhappy," is an enthymeme of the first order; "All avaricious persons are unhappy, and therefore, Balbus is unhappy," is an enthymeme of the second order; "All avaricious persons are unhappy, and Balbus is avaricious," is an enthymeme of the third order.

185. The Polysyllogism; and the Epicheirema.

A chain of syllogisms, that is, a series of syllogisms so linked together that the conclusion of one becomes a premiss of another, is called a *polysyllogism*. In a polysyllogism, any individual syllogism the conclusion of which becomes the premiss of a succeeding one is called a *prosyllogism*; any individual syllogism one of the premisses of which is the conclusion of a preceding syllogism is called an *episyllogism*. Thus,—

$$\left.\begin{array}{r} \textit{All } C \textit{ is } D, \\ \textit{All } B \textit{ is } C, \\ \text{therefore, } \textit{All } B \textit{ is } D, \end{array}\right\} \text{ prosyllogism,}$$

$$\left.\begin{array}{r} \text{but, } \textit{All } A \textit{ is } B, \\ \text{therefore, } \textit{All } A \textit{ is } D. \end{array}\right\} \text{ episyllogism.}$$

The same syllogism may of course be both an episyllogism and a prosyllogism, as would be the case with the above episyllogism if the chain were continued further.

An *epicheirema* is a polysyllogism with one or more prosyllogisms briefly indicated only. That is, one or more of the syllogisms of which the polysyllogism is composed are enthymematic. Whately (*Logic*, p. 117) calls it accordingly an *enthymematic sentence*. The following is an example,

All B is D, because it is C,
All A is B,
therefore, *All A is D.*

186. The Sorites.

A *Sorites* is a polysyllogism in which all the conclusions are omitted except the final one; for example,

All A is B,
All B is C,
All C is D,
All D is E,
therefore, *All A is E.*

187. The ordinary Sorites, and the Goclenian Sorites.

In the ordinary Sorites, the premiss which contains the subject of the conclusion is stated first; in the Goclenian Sorites it is stated last. Thus,—

Ordinary Sorites,—*All A is B,*
All B is C,
All C is D,
All D is E,
therefore, *All A is E.*

Goclenian Sorites,—*All D is E,*
All C is D,
All B is C,
All A is B,
therefore, *All A is E.*

If, in the case of the ordinary Sorites, the argument were drawn out in full, the suppressed conclusions would appear as *minor* premisses in successive syllogisms. Thus, the

ordinary Sorites given above may be analysed into the three following syllogisms,—

(1) *All B is C,*
All A is B,
therefore, *All A is C*;

(2) *All C is D,*
All A is C,
therefore, *All A is D*;

(3) *All D is E,*
All A is D,
therefore, *All A is E.*

Here the suppressed conclusion of (1) is seen to be the minor premiss of (2), that of (2) the minor premiss of (3); and so it would go on if the number of propositions constituting the Sorites were increased.

In the Goclenian Sorites, the premisses are the same, but their order is reversed, and the result of this is that the suppressed conclusions become *major* premisses in successive syllogisms.

Thus the Sorites,—*All D is E,*
All C is D,
All B is C,
All A is B,
therefore, *All A is E,*

may be analysed into the following three syllogisms,—

(1) *All D is E,*
All C is D,
therefore, *All C is E*;

(2) *All C is E,*
All B is C,
therefore, *All B is E;*

(3) *All B is E,*
All A is B,
therefore, *All A is E.*

Here the conclusion of (1) becomes the major premiss of (2); and so on.

The ordinary Sorites[1] is that which is most usually discussed; but it may be noted that the order of premisses in the Goclenian form is that which really corresponds to the customary order of premisses in a simple syllogism.

[1] What is here called the ordinary Sorites is frequently spoken of as the *Aristotelian* Sorites; for example, by Archbishop Thomson (*Laws of Thought*, p. 201), and Spalding (*Logic*, p. 302). Hamilton however remarks,—"The name Sorites does not occur in any logical treatise of Aristotle; nor, as far as I have been able to discover, is there, except in one vague and cursory allusion, any reference to what the name is now employed to express" (*Lectures on Logic*, I. p. 375). The term Sorites (from σωρὸς, a heap) as used by ancient writers was employed to designate a particular sophism, based on the difficulty which is sometimes found in assigning an exact limit to a notion. "It was asked,—was a man bald who had so many thousand hairs; you answer, No: the antagonist goes on diminishing and diminishing the number, till either you admit that he who was not bald with a certain number of hairs, becomes bald when that complement is diminished by a single hair; or you go on denying him to be bald, until his head be hypothetically denuded." A similar puzzle is involved in the question,—On what day does a lamb become a sheep? Sorites in this sense is also called *sophisma polyzeteseos* or *fallacy of continuous questioning*. See Hamilton, *Lectures on Logic*, i. p. 464.

The distinct exposition of the kind of reasoning which is now known as the Sorites is attributed to the Stoics, and it is called by this name by Cicero; but it was not till much later that the name came into general use amongst logicians in this sense. The form of Sorites called the Goclenian was first given by Professor Goclenius of Marburg (1547 to 1628) in his *Isagoge in Organum Aristotelis*, 1598.

188. The special rules of the ordinary Sorites.

The special rules of the ordinary sorites are the following:

(1) Only one premiss can be negative; and if one is negative, it must be the last.

(2) Only one premiss can be particular; and if one is particular, it must be the first.

Any ordinary sorites may be represented in skeleton form, the quantity and quality of the premisses being left undetermined, as follows:—

$$\begin{array}{cc}
S & M_1 \\
M_1 & M_2 \\
M_2 & M_3 \\
\cdots & \cdots \\
\cdots & \cdots \\
M_{n-2} & M_{n-1} \\
M_{n-1} & M_n \\
M_n & P \\
\hline
S & P
\end{array}$$

(1) There cannot be more than one negative premiss, for if there were, (since a negative premiss in any syllogism necessitates a negative conclusion), we should in analysing the sorites somewhere come upon a syllogism containing two negative premisses.

Again, if one premiss is negative, the final conclusion must be negative. Therefore, P must be distributed in this conclusion. Therefore, it must be distributed in its premiss, *i.e.*, the last premiss, which must therefore be negative.

If any premiss then is negative, this is the one.

(2) Since it has been shewn that all the premisses, except the last, must be affirmative, it is clear that if any, except the first, were particular, we should somewhere commit the fallacy of undistributed middle.

189. The possibility of a Sorites in a Figure other than the First.

It will have been noticed that in analysing both the (so-called) Aristotelian and the Goclenian Sorites all the resultant syllogisms are in Figure 1. Such sorites therefore may themselves be said to be in Figure 1. The question arises whether a sorites is possible in any other figure.

Sir William Hamilton (*Lectures on Logic*, vol. 2, p. 403) remarks that "all logicians have overlooked the Sorites of Second and Third Figures." Reading on, we find that by a Sorites in the Second Figure he means such a reasoning as the following:—*No B is A*, *No C is A*, *No D is A*, *No E is A*, *All F is A*, therefore, *No B*, *or C*, *or D or E*, *is F*; and by a Sorites in the Third Figure such as the following:—*A is B*, *A is C*, *A is D*, *A is E*, *A is F*, therefore, *Some B*, *and C*, *and D*, *and E*, *are F*. (He does not himself give these examples; but that this is what he means may be deduced from his not very lucid statement,—"In Second and Third Figures, there being no subordination of terms, the only Sorites competent is that by repetition of the same middle. In First Figure, there is a new middle term for every new progress of the Sorites; in Second and Third, only one middle term for any number of extremes. In First Figure, a syllogism only between every second term of the Sorites, the intermediate term constituting the middle term. In the others, every two propositions of the common middle term form a syllogism.")

But it is clear that in the accepted sense of the term these are not sorites at all. In neither of them have we any chain argument, but our conclusion is a mere summation of the conclusions of a number of syllogisms having a common premiss[1].

[1] Hamilton's own definition of sorites, involved as it is, might have

In the above criticism I have followed J. S. Mill[1]. His own treatment of the question, however, seems open to refutation by the simple method of constructing examples. He considers that the first or last syllogism of a sorites may be in Figure 2 or 3, (*e.g.*, in Figure 2 we may have *A is B*, *B is C*, *C is D*, *D is E*, *No F is E*, therefore, *A is not F*), but that it is impossible that all the steps should be in either of these figures. "Every one who understands the laws of the second and third figures (or even the general laws of the syllogism) can see that no more than one step in either of them is admissible in a sorites, and that it must either be the first or the last."

But take the following (the suppressed conclusions being inserted in square brackets):—

All A is B,
No C is B,
[therefore, No *A* is *C*],
All D is C,
[therefore, No *A* is *D*],
All E is D,
[therefore, No *A* is *E*],

saved him from this error. He gives for his definition,—"When, on the common principle of all reasoning,—that the part of a part is a part of the whole,—we do not stop at the second gradation, or at the part of the highest part, and conclude that part of the whole, but proceed to some indefinitely remoter part, as *D*, *E*, *F*, *G*, *H*, &c., which, on the general principle, we connect in the conclusion with its remotest whole,—this complex reasoning is called a *Chain-Syllogism* or *Sorites*" (*Lectures on Logic*, vol. 1, p. 366).

[1] In connexion with it, Mill very justly remarks,—"If Sir W. Hamilton had found in any other writer such a misuse of logical language as he is here guilty of, he would have roundly accused him of total ignorance of logical writers" (*Examination of Hamilton*, p. 515).

All F is E,
therefore, *No A is F*[1].

All the syllogisms involved here are in Figure 2, and the Sorites itself may I think fairly be said to be in Figure 2.

The following again may be called a Sorites in Figure 3:—

Some D is E,
All D is C,
[therefore, Some *C* is *E*],
All C is B,
[therefore, Some *B* is *E*],
All B is A,
therefore, *Some A is E*[2].

190. Is there any case in which a conclusion can be obtained from two premisses, although the middle term is distributed in neither of them?

The ordinary syllogistic rule relating to the distribution of the middle term does not contemplate the recognition of any signs of quantity other than *all* and *some*. For example, the admission of the sign *most* yields the following valid reasoning:

[1] This Sorites is analogous to the so-called Aristotelian Sorites, the subject of the conclusion appearing in the premiss stated first, and the suppressed premisses being *minors* in their respective syllogisms. It is to be observed that the rules given in section 188 do not apply unless the Sorites is in Figure 1. For Sorites in Figures 2 and 3, however, other rules may be framed corresponding to the special rules of Figures 2 and 3 in the case of the simple Syllogism.

[2] This Sorites is analogous to the Goclenian Sorites, the subject of the conclusion appearing in the premiss stated last, and the suppressed premisses being *majors* in their respective syllogisms. I do not of course maintain that such Sorites as the above are likely to be found in common use.

Most M is P,
Most M is S,
therefore, *Some S is P.*

We understand *most* in the sense of *more than half*, and it clearly follows from the above premisses that there must be some *M* which is both *S* and *P*. But we cannot say that in either premiss the term *M* is distributed. Hence in order to meet cases of this kind which arise when other signs of quantity besides *all* and *some* are recognised, it becomes necessary to amend the ordinary rule with regard to the distribution of the middle term.

Sir W. Hamilton (*Logic*, vol. 2, p. 362) gives the following: "The quantifications of the middle term, whether as subject or predicate, taken together, must exceed the quantity of that term taken in its whole extent"; in other words, we must have an *ultra-total* distribution of the middle term in the two premisses taken together. Hamilton then continues somewhat too dogmatically,—"The rule of the logicians, that the middle term should be once at least distributed, is untrue. For it is sufficient if, in both the premisses together, its quantification be more than its quantity as a whole, (ultra-total). Therefore, a *major part*, (a *more* or *most*), in one premiss, and a *half* in the other, are sufficient to make it effective."

De Morgan (*Formal Logic*, p. 127) writes as follows: "It is said that in every syllogism the middle term must be universal in one of the premisses, in order that we may be sure that the affirmation or denial in the other premiss may be made of some or all of the things about which affirmation or denial has been made in the first. This law, as we shall see, is only a particular case of the truth: it is enough that the two premisses together affirm or deny of

more than all the instances of the middle term. If there be a hundred boxes, into which a hundred *and one* articles of two different kinds are to be put, not more than one of each kind into any one box, some one box, if not more, will have two articles, one of each kind, put into it. The common doctrine has it, that an article of one particular kind must be put into every box, and then some one or more of another kind into one or more of the boxes, before it may be affirmed that one or more of different kinds are found together." De Morgan himself works the question out in detail in his treatment of *the numerically definite syllogism* (*Formal Logic*, pp. 141—170). The following may be taken as an example of numerically definite reasoning:— If 70 per cent. of M are P, and 60 per cent. are S, then at least 30 per cent. are both S and P[1]. The argument may be put as follows,—On the average, of 100 M's 70 are P and 60 are S; suppose that the 30 M's which are not P are S, still 30 S's are to be found in the remaining 70 M's which are P's; and this is the desired conclusion. Problems of this kind constitute a borderland between Formal Logic and Algebra.

191. The Argument *a fortiori* and other deductive inferences that are not reducible to the ordinary syllogistic form.

We may take as an example of the argument *a fortiori*:

B is greater than C,
A is greater than B,
therefore, *A is greater than C.*

[1] Using other letters, this is the example given by Mill, *Logic*, Book 2, Chapter 2, § 1, *note*, and quoted by Herbert Spencer, *Principles of Psychology*, vol. 2, p. 88.

As this stands, it is clearly not in the ordinary syllogistic form since it contains four terms; some logicians however profess to reduce it to the ordinary syllogistic form as follows:

B is greater than C,
therefore, *a greater than B is greater than C,*
but, *A is greater than B,*
therefore, *A is greater than C.*

With De Morgan, we may treat this as a mere evasion, or as a *petitio principii.* The principle of the argument *a fortiori* is really assumed in passing from *B is greater than C* to *a greater than B is greater than C.*

The following attempted resolution[1] must I think be disposed of similarly:

Whatever is greater than a greater than C is greater than C,
A is greater than a greater than C,
therefore, *A is greater than C.*

At any rate, it is clear that this cannot be the whole of the reasoning, since *B* no longer appears in the premisses at all.

Mansel (*Aldrich*, pp. 199, 200) treats the argument *a fortiori* as a *material consequence*, and by this he means, "one in which the conclusion follows from the premisses solely by the force of the terms," *i.e.*, "from some understood proposition or propositions, connecting the terms, by the addition of which the mind is enabled to reduce the consequence to logical form." He would reduce the argument *a fortiori* in one of the ways already referred to. This however begs the question that the syllogistic is the only *logical* form. As a matter of fact the cogency of the argu-

[1] Cf. Mansel's *Aldrich*, p. 200.

ment *a fortiori* is just as intuitively evident as that of a syllogism in *Barbara* itself. Why should no relation be regarded as *formal* unless it can be expressed by the word *is*? Touching on this case, De Morgan remarks that the formal logician has a right to confine himself to any part of his subject that he pleases; "but he has no right except the right of fallacy to call that part the whole" (*Syllabus*, p. 42).

There are an indefinite number of other arguments which for similar reasons cannot be reduced to syllogistic form. For example,—*A* equals *B*, *B* equals *C*, therefore *A* equals *C*[1]; *X* is a contemporary of *Y*, and *Y* of *Z*, therefore *X* is a contemporary of *Z*; *A* is the brother of *B*, *B* is the brother of *C*, therefore *A* is the brother of *C*; *A* is to the right of *B*, *B* is to the right of *C*, therefore *A* is to the right of *C*; *A* is in tune with *B*, and *B* with *C*, therefore *A* is in tune with *C*.

We must then reject the claims that have been put for-

[1] "This is not an instance of common syllogism: the premisses are '*A is* an equal of *B*; *B is* an equal of *C*.' So far as common syllogism is concerned, that 'an equal of *B*' is as good for the argument as '*B*' is a *material* accident of the meaning of 'equal.' The logicians accordingly, to reduce this to a common syllogism, state the effect of composition of relation in a major premiss, and declare that the case before them is an example of that composition in a minor premiss. As in, *A* is *an equal of an equal* (of *C*); Every *equal of an equal* is an *equal*; therefore, *A* is an *equal* of *C*. This I treat as a mere evasion. Among various sufficient answers this one is enough: *men do not think as above.* When $A=B$, $B=C$, is made to give $A=C$, the word *equals* is a *copula* in thought, and not a *notion attached to a predicate.* There are processes which are not those of common syllogism in the logician's major premiss above: but waiving this, logic is an analysis of the form of thought, possible and actual, and the logician has no right to declare that other than the actual is actual." (De Morgan, *Syllabus*, pp. 31, 2.)

ward on behalf of the syllogism as the exclusive form of all deductive reasoning.

Such claims have been made, for example, by Whately. Syllogism, he says, is "the form to which *all* correct reasoning may be ultimately reduced" (*Logic*, p. 12). Again, he remarks, "An argument thus stated regularly and at full length, is called a Syllogism; which therefore is evidently not a peculiar *kind of argument*, but only a peculiar *form* of expression, in which every argument may be stated" (*Logic*, p. 26)[1].

Spalding seems to have the same thing in view when he says,—"An inference, whose antecedent is constituted by more propositions than one, is a Mediate Inference. The simplest case, that in which the antecedent propositions are two, is the Syllogism. The syllogism is the norm of all inferences whose antecedent is more complex; and all such inferences may, by those who think it worth while, be resolved into a series of syllogisms" (*Logic*, p. 158).

J. S. Mill endorses these claims. He remarks,—"All valid ratiocination; all reasoning by which from general propositions previously admitted, other propositions equally or less general are inferred; may be exhibited in some of the above forms," *i.e.*, the syllogistic moods (*Logic*, I. p. 191).

What is required to fill the logical gap which is created by the admission that the syllogism is *not* the norm of all

[1] Cf. also Whately, *Logic*, pp. 24, 5, and p. 34. Professor Ray expresses himself equally strongly. He says,—"The syllogism is the type of all valid reasoning; for no reasoning will be valid, unless it can be thrown into the form of a syllogism. As a matter of fact, in daily life, men draw inferences in many different ways, but only those among them will be valid, and properly deserving of the name, which are capable of being ultimately reduced to the syllogistic form, the rest being nothing but suggestions of association, fancy, imagination, &c., wrongly called inferences" (*Text-book of Deductive Logic*, p. 254).

valid formal inference has been called the *Logic of Relatives*. The function of the Logic of Relatives is to "take account of relations generally, instead of those merely which are indicated by the ordinary logical copula *is*" (Venn, *Symbolic Logic*, p. 400). The line which this new Logic is likely to take, if it is ever fully worked out, is indicated by the following passage from De Morgan (*Syllabus*, pp. 30, 31):—

"A *convertible* copula is one in which the copular relation exists between two names *both ways*: thus 'is fastened to,' 'is joined by a road with,' 'is equal to,' &c. are *convertible* copulæ. If '*X* is equal to *Y*' then '*Y* is equal to *X*,' &c. A *transitive* copula is one in which the copular relation joins *X* with *Z* whenever it joins *X* with *Y* and *Y* with *Z*. Thus 'is fastened to' is usually understood as a transitive copula: '*X* is fastened to *Y*' and '*Y* is fastened to *Z*' give '*X* is fastened to *Z*.' All the copulæ used in this syllabus are *transitive*. The intransitive copula cannot be treated without more extensive consideration of the combination of relations than I have now opportunity to give: a second part of this syllabus or an augmented edition, may contain something on this subject." The student may further be referred to Venn, *Symbolic Logic*, pp. 399—404.

Exercises.

192. Find and prove the special rules of the Goclenian Sorites.

193. Take any Enthymeme (in the modern sense) and supply premisses so as to expand it into (*a*) a syllogism, (*b*) an epicheirema, (*c*) a sorites; and name the mood, order, or variety of each product. [C.]

CHAPTER VII.

CONDITIONAL AND HYPOTHETICAL SYLLOGISMS.

194. The Conditional Syllogism, the Hypothetical Syllogism, and the Hypothetico-Categorical Syllogism.

The forms of reasoning in which conditional or hypothetical conclusions are inferred from two conditional or two hypothetical premisses are apparently overlooked by some logicians; at any rate, they frequently receive no distinct recognition, the term "hypothetical syllogism" being limited to the case in which one premiss only is hypothetical.

We may give the following definitions:

(1) A *Conditional Syllogism* is a mediate reasoning consisting of three propositions in which both the premisses and the conclusion are conditional in form;

e.g.,— *Whenever C is D, E is F,*
Whenever A is B, C is D,
therefore, *Whenever A is B, E is F.*

(2) A *Hypothetical Syllogism* is a mediate reasoning consisting of three propositions in which both the premisses and the conclusion are hypothetical in form;

e.g.,—If Q is true, R is true,
If P is true, Q is true,
therefore, *If P is true, R is true.*

(3) A *Hypothetico-Categorical Syllogism*[1] is a mediate reasoning consisting of three propositions in which one of the premisses is hypothetical in form, while the other premiss and the conclusion are categorical;

e.g.,—If P is true, Q is true,
P is true,
therefore, *Q is true.*

This nomenclature, so far as the distinction between the hypothetical and the hypothetico-categorical syllogism is concerned, is adopted by Spalding and Ueberweg. Some logicians (*e.g.*, Fowler) give the name "hypothetical syllogism" to all the above forms of reasoning without distinction. Others (*e. g.*, Jevons) define the hypothetical syllogism so as

[1] It seems unnecessary to discuss separately the case in which a conditional and a categorical premiss are combined: *e.g.*, All selfish people are unhappy; If a child is spoilt, he is sure to be selfish; therefore, If a child is spoilt he will be unhappy. Such a syllogism as this is obviously resolvable into an ordinary categorical syllogism by reducing the conditional premiss to categorical form—All spoilt children are selfish. Or it may be resolved into a conditional syllogism by transforming the categorical premiss into the corresponding conditional—If any one is selfish, he is sure to be unhappy. The following is another example: If water is salt, it will not boil at 212°; Sea water is salt; therefore, Sea water will not boil at 212°. Cf. Mr F. B. Tarbell in *Mind*, 1883, p. 578.

The hypothetico-categorical syllogism as above defined cannot be so summarily disposed of.

to include the last form only, the others not being recognised as distinct forms of reasoning at all. This view may be to some extent justified by the very close analogy that exists between the syllogism with two conditional or two hypothetical premisses and the categorical syllogism; but the difference in form is worth at least a brief discussion.

195. Distinctions of Mood and Figure in the case of Conditional and Hypothetical Syllogisms.

In the Conditional Syllogism (as defined in the preceding section) the antecedent of the conclusion is equivalent to the minor term of the categorical syllogism, the consequent of the conclusion to the major term, and the element which does not appear in the conclusion at all to the middle term. Distinctions of mood and figure may be recognised in precisely the same way as in the case of the categorical syllogism. For example,

Barbara,— *Whenever C is D, E is F,*
Whenever A is B, C is D,
therefore, *Whenever A is B, E is F.*

Festino,—*Never when E is F, is it the case that C is D,*
Sometimes when A is B, C is D,
therefore, *Sometimes when A is B, it is not the case that E is F.*

Darapti,— *Whenever C is D, E is F,*
Whenever C is D, A is B,
therefore, *Sometimes when A is B, E is F.*

Camenes,— *Whenever E is F, C is D,*
Never when C is D, is it the case that A is B,
therefore, *Never when A is B, is it the case that E is F.*

It has been already shewn that all true hypothetical propositions are singular. We cannot therefore have a

hypothetical syllogism corresponding to any categorical syllogism that contains a particular premiss or a particular conclusion. Hypothetical syllogisms may however be constructed which are analogous to *Barbara*, *Celarent*, *Cesare*, *Camestres*, and *Camenes*. Thus the syllogism given in the preceding section corresponds to *Barbara*; the following corresponds to *Cesare*:

If R is true, Q is not true,
If P is true, Q is true,
therefore, *If P is true, R is not true.*

196. The Application of the Doctrine of Reduction to Conditional and Hypothetical Syllogisms.

Conditional Syllogisms in Figures 2, 3, 4 may be reduced to Figure 1 just as in the case of Categorical Syllogisms. Thus, the conditional syllogism in *Camenes* given in the preceding section is reduced as follows to *Celarent*:

Never when C is D, is it the case that A is B,
Whenever E is F, C is D,
therefore, *Never when E is F, is it the case that A is B,*
therefore, *Never when A is B, is it the case that E is F.*

According to the ordinary rule as indicated in the mnemonic, the premisses have here been transposed, and the conclusion of the new syllogism is converted in order to obtain the original conclusion.

Hypothetical Syllogisms in *Cesare*, *Camestres*, and *Camenes* may also be reduced to *Celarent*. For example, the hypothetical syllogism in *Cesare* given in the preceding section is reduced by simply converting the major premiss, which then becomes *If Q is true, R is not true.*

197. The Moods of the Hypothetico-Categorical Syllogism.

It is usual to distinguish two moods of the hypothetico-categorical syllogism:

(1) The *modus ponens* (also called the *constructive* hypothetical syllogism) in which the categorical premiss affirms the antecedent of the hypothetical premiss, thereby justifying as a conclusion the affirmation of its consequent. For example,

If P is true, Q is true,
P is true,
therefore, *Q is true.*

(2) The *modus tollens* (also called the *destructive* hypothetical syllogism) in which the categorical premiss denies the consequent of the hypothetical premiss, thereby justifying as a conclusion the denial of its antecedent. For example,—

If A is B, C is D,
C is not D,
therefore, *A is not B.*

These moods may be considered to correspond respectively to Figures 1 and 2 of the categorical syllogism.

An attempt may be made to reduce the *modus ponens* to pure categorical form as follows:

The case of P being true is the case of Q being true,
The actual case is the case of P being true,
therefore, *The actual case is the case of Q being true.*

This is not really satisfactory; but it is worth giving in order to bring out the analogy between the above example of the *modus ponens* and the categorical syllogism in *Barbara.*

The following corresponds to *Celarent*:

If A is B, C is not D,
A is B,
therefore, *C is not D.*

The example of the *modus tollens* given above corresponds to *Camestres*. The following corresponds to *Cesare*:

If A is not B, C is not D,
C is D,
therefore, *A is B*[1].

198. Reduction of the *modus tollens* to the *modus ponens.*

Any case of the *modus tollens* may be reduced to the *modus ponens* and *vice versa.*

Thus, *If P is true, Q is true,*
Q is not true,
therefore, *P is not true,*

becomes, by contraposition of the hypothetical premiss,

[1] Lotze in his classification of hypothetico-categorical syllogisms (*Logic*, § 93) omits this case and one other. He would however regard it as what he calls a *modus ponendo ponens*, because both the categorical premiss and the conclusion appear as affirmatives. It will be observed that the distinction as drawn in the text between the *modus ponens* and the *modus tollens* turns not on the absolute quality of the conclusion, but on whether it affirms the consequent or denies the antecedent of the hypothetical premiss. This is the really important distinction. The above syllogism might be written,—

If A is not B, it is not the case that C is D,
but *C is D,*
therefore, *It is not the case that A is not B.*

The analogy with *Cesare* is then more obvious.

If Q is not true, P is not true,
Q is not true,
therefore, *P is not true*;

and this is the *modus ponens*.

It may be worth noticing here that a categorical syllogism in *Camestres* may similarly be reduced to *Celarent* without transposing the premisses:

All P is M,
No S is M,
therefore, *No S is P,*

becomes, by contraposition of the major and obversion of the minor premiss,

No not-M is P,
All S is not-M,
therefore, *No S is P.*

199. Fallacious modes of arguing from a hypothetical major premiss.

There are two principal fallacies to which we are liable in arguing from a hypothetical major premiss:—

(1) It is a fallacy if we regard the affirmation of the consequent as justifying the affirmation of the antecedent. For example,

If A is B, C is D,
C is D,
therefore, *A is B.*

(2) It is a fallacy if we regard the denial of the antecedent as justifying the denial of the consequent. For example,

If P is true, Q is true,
P is not true,
therefore, *Q is not true.*

These fallacies may be regarded as corresponding respectively to ***undistributed middle*** and ***illicit major*** in the case of categorical syllogisms[1].

200. The claims of the Hypothetico-Categorical Syllogism to be regarded as Mediate Inference.

Kant, Hamilton[2], Bain, and others argue that inferences of the kind that we have just been considering are not properly to be regarded as mediate but as immediate inferences.

Now, taking the syllogism,—

If A is B, C is D,
but *A is B,*
therefore, *C is D,*

the conclusion is at any rate apparently obtained by a combination of two premisses, and the process is moreover one of elimination. The burden of proof, therefore, seems to lie with those who deny the claims of such an inference as this to be called mediate.

Professor Bain's arguments upon this point (*Logic, Deduction*, p. 117) are not quite easy to formulate; but they resolve themselves into one or other or both of the following:—

[1] From what has been said above it follows that we may lay down the following canon for the hypothetico-categorical syllogism: Given a hypothetical premiss, then the affirmation of the antecedent justifies the affirmation of the consequent; and the denial of the consequent justifies the denial of the antecedent; but not conversely in either case.

[2] *Logic*, vol. 2, p. 383. On p. 378 however Hamilton seems to take the other view.

(1) He seems to argue that the so-called hypothetical syllogism is not really mediate inference, *because* it is "a pure instance of the Law of Consistency"; in other words, because "the conclusion is implied in what has already been stated." But is not this the case in all formal mediate inference? It cannot be maintained that the categorical syllogism is more than a pure instance of the Law of Consistency; or that the conclusion in such a syllogism is not implied in what has been already stated.

(2) But he may mean that the conclusion is implied in the hypothetical premiss alone. Indeed he goes on to say, "'If the weather continues fine, we shall go into the country' is transformable into the equivalent form 'The weather continues fine, and so we shall go into the country.' Any person affirming the one, does not, in affirming the other, declare a new fact, but the same fact." Surely this is not intended to be understood literally. Take the following:— If war is declared, I must return home; If the sun moves round the earth, modern astronomy is a delusion. Are these respectively equivalent to,—War has been declared, and so I must return home; The sun moves round the earth, and so modern astronomy is a delusion? Besides, if the proposition *If A is B, C is D* implies that *A is B*, what becomes of the possible reasoning, "But *C is not D*, therefore, *A is not B*"?

Further arguments that have been adduced on the same side are as follows:—

(1) "There is no middle term in the so-called hypothetical syllogism".[1] The answer is that there is something

[1] This is Kant's argument. A more plausible argument would be that there is no *minor* term. But if this is granted, it only shews a want of complete analogy between the hypothetico-categorical syllogism and the categorical syllogism. In attempting to reduce the one to the

in the premisses which does not appear in the conclusion, and that this corresponds to the middle term of the categorical syllogism.

(2) "In the so-called hypothetical syllogism, the minor and the conclusion indifferently change places"[1]. This statement is erroneous. Taking the valid syllogism given at the commencement of this section and transposing the so-called minor and the conclusion, we have a fallacy. Compare the preceding section.

(3) "The major in a so-called hypothetical syllogism consists of two propositions, the categorical major of two terms." This merely tells us that a hypothetical syllogism is not the same in form as a categorical syllogism, but seems to have no bearing on the question whether the so-called hypothetical syllogism is a case of mediate or of immediate inference.

other, a minor term of the form "the actual case" or something equivalent thereto is made to appear. Compare section 197.

[1] This argument is Hamilton's. He remarks that in hypothetical syllogisms, "*the same proposition* is reciprocally medium or conclusion" (*Logic*, vol. 2, p. 379). Dr Ray (*Text-Book of Deductive Logic*, Note C) holds that in what I have said above, Hamilton is interpreted wrongly; and that he meant no more than that with a hypothetical premiss *If A is B, C is D*, a relation between *A* and *B* may be either the other premiss (as in the *modus ponens*) or the conclusion (as in the *modus tollens*). Dr Ray is possibly right. But if so, Hamilton expresses himself very clumsily. For *A is B* (the premiss of the *modus ponens*) is certainly not *the same proposition* as *A is not B* (the conclusion of the *modus tollens*). It may be added that the argument in its new form is irrelevant. In the categorical syllogism we have something precisely analogous. For given a major premiss *All M is P*, a relation between *M* and *S* may be the minor premiss (in which case *M* will be the middle term), or it may be the conclusion (in which case *M* will be the major term). Compare the syllogisms: *All M is P, All S is M*, therefore, *All S is P*; *All M is P, No S is P*, therefore, *No S is M*.

Turning now to the other side of the question, I do not see what satisfactory answers can be given to the following arguments in favour of regarding the hypothetico-categorical syllogism as a case of mediate inference. In any such syllogism, the two premisses are quite distinct, neither can be inferred from the other, but both are necessary in order that the conclusion may be obtained. Again, if we compare with it the inferences which are on all sides admitted to be immediate inferences from the hypothetical proposition, the difference between the two cases is apparent. From *If P is true, Q is true*, I can infer immediately *If Q is not true, P is not true*; but I require also to know that *Q is not true* in order to be able to infer that *P is not true.*

And whether the hypothetico-categorical syllogism can or cannot be actually reduced to pure categorical form, it can at least be shewn to be analogous to the ordinary categorical syllogism, which is admitted to be a case of mediate reasoning. Moreover there are distinct forms,—the *modus ponens* and the *modus tollens*,—which are analogous to distinct forms of the categorical syllogism; and fallacies in the hypothetical syllogism correspond to certain fallacies in the categorical syllogism.

Professor Bowen indeed remarks (*Logic*, p. 265):—"The reduction of a Hypothetical Judgment to a Categorical shews very clearly the Immediacy of the reasoning in what is called a Hypothetical Syllogism. Thus, *If A is B, C is D*, is equivalent to *All cases of A is B are cases of C is D*, therefore,

$$\left\{\begin{array}{l}\textit{Some cases of A is B are cases of}\\ \textit{This case of A is B is a case of}\end{array}\right\} \textit{C is D.}"$$

But does not this overlook the fact that a new judgment is required to tell me that this *is* a case of *A* being *B*? The

mere statement that *some cases of A is B are cases of C is D* is clearly not equivalent to the conclusion of the hypothetical syllogism[1]. By analogy we should have to argue that the following categorical syllogism in *Barbara* is an immediate inference: *All M is P, This is M*, therefore, *This is P.* Thus the argument again proves too much.

In the case of the *modus tollens,—If P is true, Q is true; but Q is not true*; therefore, *P is not true*,—the argument in favour of regarding it as mediate inference is still more forcible; but of course the *modus ponens* and the *modus tollens* stand and fall together[2].

Professor Croom Robertson (*Mind*, 1877, p. 264) has suggested an explanation as to the manner in which this controversy may have arisen. He distinguishes the *hypothetical* "if" from the *inferential* "if," the latter being equivalent to *since, seeing that, because.* No doubt by the aid of a certain accentuation the word "if" may be made to carry with it this force. Professor Robertson quotes a passage from *Clarissa Harlowe* in which the remark "If you have the value for my cousin that you say you have, you must needs think her worthy to be your wife," is explained by the speaker to mean, "*Since* you have, &c." Using the word in this sense, the conclusion *C is D* certainly follows immediately from the bare statement *If A is B, C is D*; or rather this statement itself affirms the conclusion. When, however, the word "if" carries with it this inferential implication, we cannot regard the proposition in which it occurs as strictly hypothetical. We have rather a condensed mode

[1] Professor Bowen clearly has in view a conditional as distinguished from a pure hypothetical major premiss. But this distinction does not materially affect the present argument.

[2] In section 207 it is shewn further that the Hypothetical Syllogism and the Disjunctive Syllogism also stand and fall together.

of expression including two statements in one; I should indeed turn the argument the other way by saying that in the single statement thus interpreted we have a hypothetical syllogism expressed elliptically[1].

EXERCISES.

201. Shew how the *modus ponens* may be reduced to the *modus tollens*.

202. Test the following: "If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it." [V.]

203. Let *X*, *Y*, *Z*, *P*, *Q*, *R*, be six propositions: given, (1) Of *X*, *Y*, *Z*, one and only one is true;
(2) Of *P*, *Q*, *R*, one and only one is true;
(3) If *X* is true, *P* is true;
(4) If *Y* is true, *Q* is true;
(5) If *Z* is true, *R* is true;

prove *syllogistically*,

(i) If *X* is false, *P* is false;
(ii) If *Y* is false, *Q* is false;
(iii) If *Z* is false, *R* is false.

204. Construct Conditional Syllogisms in *Cesare*, *Bocardo*, *Fesapo*, and reduce them to Figure 1.

205. Name the mood and figure of the following, and shew that either one may be reduced to the other form:

[1] Compare Mansel's *Aldrich*, p. 103.

(1) *If R is true, Q is true,*
If P is true, Q is not true,
therefore, *If P is true, R is not true.*

(2) *If Z is true, Y is true,*
If Y is true, X is not true,
therefore, *If X is true, Z is not true.*

CHAPTER VIII.

DISJUNCTIVE SYLLOGISMS.

206. The Disjunctive Syllogism.

A *Disjunctive Syllogism* may be defined as a formal reasoning consisting of two premisses and a conclusion, of which one premiss is disjunctive while the other premiss and the conclusion are categorical[1].

For example,

A is either B or C,
A is not B,
therefore, *A is C.*

[1] Archbishop Thomson's definition of the disjunctive syllogism—"An argument in which there is a disjunctive judgment" (*Laws of Thought*, p. 197)—must I think be regarded as too wide. It would include such a syllogism as the following,—

B is either C or D,
A is B,
therefore, *A is either C or D.*

The argument here in no way turns upon the disjunction, and the reasoning may be regarded as an ordinary categorical syllogism in *Barbara*, the major term being complex.

A more general treatment of reasonings involving disjunctive judgments is given in Part IV.

The categorical premiss in this example denies one of the alternatives stated in the disjunctive premiss, and we are consequently enabled to affirm the other alternative as our conclusion. This is called the *modus tollendo ponens.*

Some logicians also recognise as valid a *modus ponendo tollens*, in which the categorical premiss affirms one of the alternatives stated in the disjunctive premiss, and the conclusion denies the other alternative. Thus,

A is either B or C,
A is B,
therefore, *A is not C.*

This proceeds on the assumption that the elements of the disjunction are mutually exclusive, which however on the view adopted in section 110 is not necessarily the case. The recognition or denial of the validity of the *modus ponendo tollens* depends then upon our interpretation of the disjunctive proposition itself.

No doubt exclusiveness is often intended to be implied and is understood to be implied. For example, "He was either first or second in the race, He was second, therefore, He was not first." This reasoning would ordinarily be understood to be valid. But I hold that its validity depends not on the expressed major premiss, but on the understood premiss, "No one can be both first and second in a race." I should say that the following reasoning is equally valid with the one stated above, "He was second in the race, therefore, He was not first." The disjunctive premiss is therefore quite immaterial to the reasoning; we could do just as well without it, for the really vital premiss, "No one can be both first and second in a race," is true, and would be accepted as such, whether the disjunctive proposition "He was either first or second" be true or not.

But independently of this particular argument, it will be allowed that a special implication of the kind above contemplated cannot be recognised when we are dealing with purely symbolic forms. If we regard the *modus ponendo tollens* as formally valid, we must be prepared to interpret the alternatives in a disjunctive proposition as *in every case* mutually exclusive.

207. The force of a Disjunctive Proposition as a premiss in an argument is equivalent either to that of a Conditional or to that of a Hypothetical Proposition.

This follows from the resolution of Disjunctives given in Part II, chapter 9. Thus,

Either A is B or C is D,
A is not B,
therefore, *C is D*;

may be resolved into—

If A is not B, C is D,
A is not B,
therefore, *C is D*;

or into—

If C is not D, A is B,
A is not B,
therefore, *C is D*[1].

A corollary from the above is that those who deny the character of mediate reasoning to the hypothetical syllogism must also deny it to the disjunctive syllogism, or else they

[1] Again, the *modus ponendo tollens* is (on the interpretation here adopted) equivalent to one of the fallacies mentioned in section 199.

must refuse to recognise the resolution of the disjunctive proposition into one or more hypothetical propositions.

[In the above example it is not quite clear from its form whether the so-called hypothetical is a true hypothetical or a conditional. But in the following examples, which are added to illustrate the distinction, it is evident that the disjunctives are equivalent to a true hypothetical and to a conditional respectively:

Either all A's are B's or all A's are C's,
This A is not B,
therefore, *All A's are C's*;

All A's are either B or C,
This A is not B,
therefore, *This A is C.*]

208. The Dilemma.

The proper place of the Dilemma among Conditional Arguments is difficult to determine, inasmuch as conflicting definitions are given by different logical writers. It may be useful to comment briefly upon some of these definitions.

(1) Mansel (*Aldrich*, p. 108) defines the Dilemma as "a syllogism having a conditional (hypothetical) major premiss *with more than one antecedent*, and a disjunctive minor." Equivalent definitions are given by Whately and Jevons.

Three forms of dilemma are recognised by these writers:—

i. The *Simple Constructive* Dilemma.

If A is B, C is D; and if E is F, C is D;
But *either A is B or E is F;*
Therefore, *C is D.*

ii. The *Complex Constructive* Dilemma.

If A is B, C is D; and if E is F, G is H;
But *either A is B or E is F;*
Therefore, *Either C is D or G is H.*

iii. The *Destructive* Dilemma (always *Complex*).

If A is B, C is D; and if E is F, G is H:
But *either C is not D or G is not H;*
Therefore, *Either A is not B or E is not F.*

The Destructive Dilemma is said to be always complex; and the simple form corresponding to the third of the above is certainly excluded by the definition given. It would run,—

If A is B, C is D; and if A is B, E is F;
But *either C is not D or E is not F;*
Therefore, *A is not B;*

and here there is *only one antecedent* in the major.

But the question arises whether such exclusion is not arbitrary, and whether this definition ought not therefore to be rejected.

Whately regards the name Dilemma as necessarily implying two *antecedents*; but does it not rather imply two *alternatives*, each of which is equally distasteful? He goes on to assert that the excluded form is merely a destructive hypothetical syllogism, similar to the following,—

If A is B, C is D;
C is not D;
therefore, *A is not B.*

But the two really differ precisely as the simple constructive dilemma,

If A is B, C is D; and if E is F, C is D;
But *either A is B or E is F;*
therefore, *C is D;*

differs from the constructive hypothetical syllogism,

If A is B, C is D;
A is B;
therefore, *C is D.*

Besides, it is clear that it is not merely a destructive hypothetical syllogism such as has been already discussed, since the premiss which is combined with the hypothetical premiss is not categorical but disjunctive[1].

(2) Professor Fowler (*Deductive Logic*, p. 116) gives the following:—"There remains the case in which one premiss of the complex syllogism is a conjunctive (*i.e.*, a hypothetical), and the other a disjunctive proposition, it being of course understood that the disjunctive proposition

[1] The argument,—

If A is B, C is D and E is F;
But *either C is not D or E is not F;*
Therefore, *A is not B;*

must be distinguished from the following,—

If A is B, C is D and E is F;
But *C is not D, and E is not F;*
Therefore, *A is not B.*

In the latter of these there is no alternative given at all, and the reasoning is equivalent to two simple hypothetical syllogisms, yielding the same conclusion, namely,—

(1) *If A is B, C is D;*
But *C is not D;*
Therefore, *A is not B.*

(2) *If A is B, E is F;*
But *E is not F;*
Therefore, *A is not B.*

deals only with expressions which have already occurred in the conjunctive proposition. This is called a *Dilemma*."

Under this definition, it is no longer required that there shall be at least two antecedents in the hypothetical premiss; and hence, four forms are included, namely, the two constructive dilemmas, and a simple as well as a complex destructive.

(3) The following definition is sometimes given:—"The Dilemma (or Trilemma or Polylemma) is an argument in which two (or three or more) alternatives are given, but it is shewn that whichever alternative is taken the same conclusion follows"[1].

This includes the simple constructive dilemma and the simple destructive dilemma (as already given); but it does not allow that either of the complex dilemmas is properly so-called, since in each case we are left with the same number of alternatives in the conclusion as are contained in the disjunctive premiss.

This definition, however, embraces forms that are excluded by both the preceding definitions. For example,

> *If A is, either B or C is;*
> But *neither B nor C is;*
> Therefore, *A is not*[2].

(4) Hamilton (*Logic*, I. p. 350) defines the Dilemma as "a syllogism in which the sumption (major) is at once hypothetical and disjunctive, and the subsumption (minor) sublates the whole disjunction, as a consequent, so that the antecedent is sublated in the conclusion." This involved

[1] This definition gives point to the expression "the horns of a dilemma."

[2] Cf. Ueberweg, *System of Logic*, § 123.

definition appears to have chiefly in view the form last given, namely,—

If A is, either B or C is;
Neither B nor C is;
Therefore, *A is not;*

but it excludes the following,—

If A is, C is; and if B is, C is;
But *either A is or B is;*
Therefore, *C is.*

This however is one of the typical forms of Dilemma according to all the preceding definitions.

(5) Thomson (*Laws of Thought*, p. 203) gives the following,—"A dilemma is a syllogism with a conditional (hypothetical) premiss, in which either the antecedent or the consequent is disjunctive."

This definition is probably wider than Thomson himself intended. It would include such forms as the following:—

If A is B or E is F, then C is D;
But *C is not D;*
Therefore, *A is not B, and E is not F.*

If A is B, C is D or E is F;
But *A is B;*
Therefore, *C is D or E is F.*

Jevons (*Elements of Logic*, p. 168) remarks that "Dilemmatic arguments are more often fallacious than not, because it is seldom possible to find instances where two alternatives exhaust all the possible cases, unless indeed one of them be the simple negative of the other." In other words, many dilemmatic arguments will be found to contain a premiss involving a fallacy of incomplete disjunction. It should however be observed that in strictness a syllogistic argument

is not itself to be called fallacious because it contains a false premiss. The fallacy that Jevons has in view is a material rather than a formal fallacy.

Exercises.

209. Comment upon the following definitions of the disjunctive syllogism:—

"A disjunctive syllogism is a syllogism of which the major premiss is a disjunctive and the minor a simple proposition, the latter affirming or denying one of the alternatives stated in the former."

"A disjunctive syllogism is a syllogism whose major premiss is a disjunctive proposition."

210. Is it possible to apply distinctions of Figure either to Hypothetical or to Disjunctive Syllogisms? [c.]

211. Comment upon Jevons's statement:—"It will be noticed that the disjunctive syllogism is governed by totally different rules from the ordinary categorical syllogism, since a negative premiss gives an affirmative conclusion in the former, and a negative in the latter."

212. Examine the following:—

The cause must either precede the effect, or be simultaneous with it, or succeed it. The last supposition is absurd; and the second would render it impossible to distinguish the cause from the effect. On the first supposition the cause must cease before the effect comes into being; but, surely, that which is not cannot be a cause. Either,

then, there is no cause for any effect, or we are unable to discover it. [C.]

213. What can be inferred from the premisses, *Either A is B or C is D*, *Either C is not D or E is not F?* Exhibit the reasoning in the form of a dilemma.

CHAPTER IX.

THE QUANTIFICATION OF THE PREDICATE.

214. The eight propositional forms resulting from the explicit Quantification of the Predicate.

The fundamental postulate of Logic, according to Sir W. Hamilton, is "that we be allowed to state explicitly in language all that is implicitly contained in thought"; and since he also maintains that "in thought the predicate is always quantified," he makes it follow immediately from his postulate, that "in logic, the quantity of the predicate must be expressed, on demand, in language."

This doctrine of the explicit quantification of the predicate led Hamilton to recognise eight distinct propositional forms instead of the customary four:—

All S is all P,	**U.**
All S is some P,	**A.**
Some S is all P,	**Y.**
Some S is some P,	**I.**
No S is any P,	**E.**
No S is some P,	η.
Some S is not any P,	**O.**
Some S is not some P.	ω.

The new symbols here introduced are those employed by Archbishop Thomson[1], and they are the ones commonly recognised in connexion with the quantification of the predicate.

The symbols used by Hamilton were *Afa*, *Afi*, *Ifa*, *Ifi*, *Ana*, *Ani*, *Ina*, *Ini*. Here *f* indicates an affirmative proposition, *n* indicates a negative; *a* means that the corresponding term is distributed, *i* that it is undistributed.

Spalding's symbols (*Logic*, p. 83) are A^2, A, I^2, I, E, $\frac{1}{2}E$, O, $\frac{1}{2}O$. Mr Carveth Read (*Theory of Logic*, p. 193) suggests A^2, A, I^2, I, E, E_2, O, O_2.

The equivalence of these various symbols is shewn in the following table:—

	Thomson.	Hamilton.	Spalding.	Carveth Read.
All *S* is all *P*	U	*Afa*	A^2	A^2
All *S* is some *P*	A	*Afi*	A	A
Some *S* is all *P*	Y	*Ifa*	I^2	I^2
Some *S* is some *P*	I	*Ifi*	I	I
No *S* is any *P*	E	*Ana*	E	E
No *S* is some *P*	η	*Ani*	$\frac{1}{2}E$	E_2
Some *S* is not any *P*	O	*Ina*	O	O
Some *S* is not some *P*	ω	*Ini*	$\frac{1}{2}O$	O_2

[1] He himself however ultimately rejects the forms η and ω.

For the new forms we might also use the symbols *SuP*, *SyP*, *SηP*, *SωP*, on the principle explained in section 36.

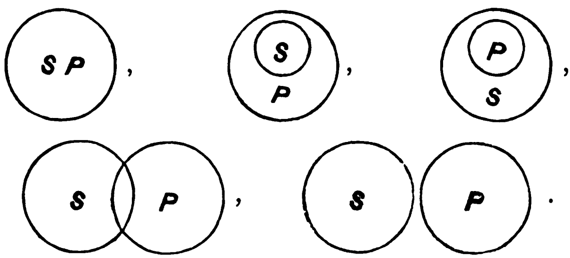
215. The meaning to be attached to the word *some* in the eight propositional forms recognised by Sir William Hamilton.

Professor Baynes, in his authorised exposition of Sir William Hamilton's new doctrine, would at the outset lead us to suppose that we have no longer to do with the indeterminate "some" of the Aristotelian Logic, but that this word is now to be used in the more definite sense of "*some, but not all.*" We have seen that the fundamental postulate of Logic on which Hamilton bases his doctrine is "that we be allowed to state explicitly in language, all that is implicitly contained in thought"; and applying this postulate, Dr Baynes (*New Analytic of Logical Forms*) remarks:—"Predication is nothing more or less than the expression of the relation of quantity in which a notion stands to an individual, or two notions to each other. If this relation were indeterminate—if we were uncertain whether it was of part, or whole, or none—there could be no predication. Since, therefore, the predicate is always quantified in thought, the postulate applies; *i.e.*, in logic, the quantity of the predicate must be expressed, on demand, in language. For example, if the objects comprised under the subject be some part, but not the whole, of those comprised under the predicate, we write *All X is some Y*, and similarly with other forms."

But if it is true that we know definitely the relative extent of subject and predicate, and if "some" is used strictly in the sense of "some but not all," we should have but *five* propositional forms instead of eight, namely—*All S*

is all P, *All S is some P*, *Some S is all P*, *Some S is some P*[1], *No S is any P*.

We have already shewn (section 94) that the only possible relations between two terms in respect to their extension are given by the five diagrams,—



These correspond respectively to the above five propositions[2]; and it is clear that on the view indicated by Dr Baynes the eight forms are redundant[3].

I am altogether doubtful whether writers who have adopted the eightfold scheme have themselves recognised the pitfalls that surround the use of the word *some*. Many passages might be quoted in which they distinctly adopt the meaning—"some, not all." Thus, Thomson (*Laws of Thought*, p. 150) makes **U** and **A** inconsistent. Bowen (*Logic*, pp. 169, 170) would pass from **I** to **O** by immediate inference[4]. Hamilton himself agrees with Thomson

[1] Using *some* in the sense here indicated, *Some S is some P* necessarily implies *Some S is not any P* and *No S is some P*.

[2] Namely **U**, **A**, **Y**, **I**, **E**. **O** and η cannot be interpreted as giving precisely determinate information. **O** gives an alternative between **Y** and **I**, and η between **A** and **I**. For the interpretation of ω see note 1 on p. 299.

[3] Cf. Venn, *Symbolic Logic*, chap. I.

[4] "This sort of Inference," he says, "Hamilton would call *Integration*, as its effect is, after determining one part, to reconstitute the whole by bringing into view the remaining part."

and Bowen on these points; but he is curiously indecisive on the general question here raised. He remarks (*Logic*, II. p. 282) that *some* "is held to be a definite *some* when the other term is definite," *i.e.*, in **A** and **Y**, η and **O**; but "on the other hand, when both terms are indefinite or particular the *some* of each is left wholly indefinite," *i.e.*, in **I** and ω[1]. This is very confusing, and it would be most difficult to apply the distinction consistently. Hamilton himself certainly does not so apply it. For example, on his view it should no longer be the case that two affirmative premisses necessitate an affirmative conclusion; or that two negative premisses invalidate a syllogism. Thus, the following should be regarded as valid:—

All P is some M,
All M is some S,
therefore, *Some S is not any P.*

No M is any P,
Some S is not any M,
therefore, *Some or all S is not any P.*

Such syllogisms as these, however, are not admitted by

[1] Mr Lindsay, however, in expounding Hamilton's doctrine (*Appendix to Ueberweg's System of Logic*, p. 580) says more decisively,—"Since the subject must be equal to the predicate, vagueness in the predesignations must be as far as possible removed. *Some* is taken as equivalent to *some but not all*."

Spalding (*Logic*, p. 184) definitely chooses the other alternative. He remarks that in his own treatise "the received interpretation *some at least* is steadily adhered to."

Mr Carveth Read (*Theory of Logic*, p. 196) distinguishes two schemes of what he calls Bidesignate Relationships (Quantified Predicates) in one of which the sign *Some* is understood to mean *Some only*, and in the other *Some at least*. In each case, however, he seems to retain eight distinct propositional forms.

Hamilton and Thomson[1]. Hamilton's supreme canon of the categorical syllogism (*Logic*, II. p. 357) is:—"What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related; that relation subsists between the two terms themselves." This clearly provides that one premiss at least shall be affirmative, and that an affirmative conclusion shall follow from two affirmative premisses. Thomson (*Laws of Thought*, p. 165) explicitly lays down the same rules; and his table of valid moods (given on p. 188) is (with the exception of one obvious misprint) correct and correct only if *some* means "some, it may be all."

It is clear that in the Hamiltonian doctrine there is a want of internal consistency. It is not going too far to say that the doctrine is essentially of an unscientific character.

216. What results would follow if we were to interpret 'Some *A*'s are *B*'s' as implying that 'Some other *A*'s are not *B*'s'?

[Jevons, *Studies in Deductive Logic*, p. 151.]

Professor Jevons himself answers this question by saying, "The proposition 'Some *A*'s are *B*'s' is in the form **I**, and according to the table of opposition **I** is true if **A** is true; but **A** is the contradictory of **O**, which would be the form of 'Some other *A*'s are not *B*'s.' Under such circumstances **A** could never be true at all, because its truth would involve the truth of its own contradictory, which is absurd."

This is turning the criticism the wrong way, and proves

[1] And on the other hand, Thomson admits as valid certain combinations which on the above interpretation are not valid.

too much. It is not true that we necessarily involve ourselves in self-contradiction if we use *some* in the sense of *some only*. What should be pointed out is that if we use the word in this sense, the truth of **I** no longer follows from the truth of **A**; but on the other hand these two propositions are inconsistent with each other.

Taking the five propositional forms which are obtained by attaching this meaning to *some*, namely,—*All S is all P*, *All S is some P*, *Some S is all P*, *Some S is some P*, *No S is P*,—it should be observed that each one of these propositions is inconsistent with each of the others, and also that no one is the contradictory of any one of the others. If, for example, on this scheme we wish to express the contradictory of **U**, we can do so only by affirming an alternative between **Y**, **A**, **I** and **E**.

Nothing of all this appears to have been noted by the Hamiltonian writers[1], even in the cases in which they explicitly profess to use *some* in the sense of *some only*.

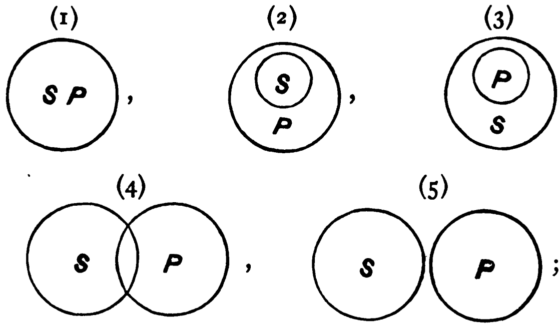
How the above five forms may be expressed by means of the ordinary Aristotelian four forms has been discussed in section 95.

217. If in the eight Hamiltonian forms of proposition *some* is used in the ordinary logical sense[2], what is the precise information given by each of these propositions?

[1] Thomson (*Laws of Thought*, p. 149) gives a scheme of opposition in which **I** and **E** appear as contradictories, but **A** and **O** as contraries. He appears to use *some* in the sense of *some but not all* in the case of **A** and **Y** only.

[2] If *some* is used in the sense of *some, but not all*, the answer to this question is given on page 291, note 2, and page 299, note 1.

Taking the five possible relations between two terms, and numbering them as folows,—



we may write against each of the propositional forms the relations which are compatible with it[1] :—

U	1
A	1, 2
Y	1, 3
I	1, 2, 3, 4
E	5
η	2, 4, 5
O	3, 4, 5
ω	1, 2, 3, 4, 5

[1] If the Hamiltonian writers had attempted to illustrate their doc-

We have then the following pairs of contradictories,—**A**, **O**; **Y**, η; **I**, **E**. The contradictory of **U** is obtained by affirming an alternative between η and **O**.

We may point out how each of the above would be expressed without the use of quantified predicates:—

$$\mathbf{U} = SaP,\ PaS;$$
$$\mathbf{A} = SaP;$$
$$\mathbf{Y} = PaS;$$
$$\mathbf{I} = SiP;$$
$$\mathbf{E} = SeP;$$
$$\eta = PoS;$$
$$\mathbf{O} = SoP.$$

What information, if any, is given by ω is discussed in section 220.

218. The propositions U and Y.

It must be admitted that these propositions are met with in ordinary discourse. All definitions are **U** propositions; so are all propositions of which both the subject and the predicate are singular terms. Take also such propositions as the following: Christianity and civilization are coextensive; Mercury, Venus, &c. are all the planets; Common salt is the same thing as sodium chloride[1]. Any exclusive proposition[2] may be given as an example of **Y**; *e.g.*,

trine by means of the Eulerian diagrams, they would I think either have found it to be unworkable, or they would have worked it out to a more distinct and consistent issue.

[1] **U** propositions are called by Jevons *simple identities* as distinguished from *partial identities*. See *Principles of Science*, Ch. 3.

[2] Compare section 38.

Graduates alone are eligible for the appointment[1]; Only *S is P*.

We cannot then agree with Professor Fowler that the additional forms "are not merely unusual, but are such as we never do use"[2]. **U** and **Y**[3] at any rate ought to receive some recognition in Logic. Still in treating the syllogism &c. on the traditional lines, it seems better to retain the traditional schedule of propositions. The addition of **U** and **Y** does not tend towards simplification, but the reverse; and their full force can be expressed in other ways. On this view, when we meet with a **U** proposition, *All S is all P*, we may resolve it into the two **A** propositions, *All S is P* and *All P is S*, which taken together are equivalent to it; and when we meet with a **Y** proposition, *Some S is all P* or *S alone is P*, we may replace it by the **A** proposition *All P is S*, which it yields by conversion.

219. The proposition η.

This proposition in the form *No S is some P* is not I think ever found in ordinary use. We may however recognise its possibility; and it must be pointed out that a form of proposition which we do meet with, namely, *Not only S is P* or *Not S alone is P*, is practically η, provided we do not regard this proposition as implying that any *S* is certainly *P*.

Archbishop Thomson remarks that η "has the semblance only, and not the power of a denial. True though it is, it does not prevent our making another judgment of

[1] This proposition is evidently equivalent to—Some graduates are all who are eligible for the appointment. It must however be added that the *some* is here indefinite as in the ordinary scheme of propositions.

[2] *Deductive Logic*, p. 31.

[3] The *some* of the subject being interpreted *some at least*.

the affirmative kind, from the same terms"[1]. This is erroneous. Undoubtedly **A** and η may be true together; but **U** and η cannot, and **Y** and η are strictly contradictories[2]. The relation of contradiction in which **Y** and η stand to each other is perhaps brought out more clearly if they are written in the forms *Only S is P*, *Not only S is P*, or *S alone is P*, *Not S alone is P*. It will be observed moreover that η is the converse of **O**, and *vice versa*. If therefore η has no power of denial, the same will be true of **O** also. But it certainly is not true of **O**.

The recognition of a six-fold schedule of propositions including **Y** and η (but not **U** on account of its composite character or ω) may be advocated on the ground that each propositional form will then have a simple converse. The doctrines of opposition, of immediate inference, and of the syllogism, if worked out on the basis of this schedule, would in some respects gain in symmetry and would present many interesting features.

220. The Hamiltonian proposition ω, "Some *S* is not some *P*."

The proposition ω, "Some *S* is not some *P*," is not inconsistent with any of the other propositional forms, not even with **U**, "All *S* is all *P*." For example, "all equilateral triangles are all equiangular triangles," yet nevertheless "this equilateral triangle is not that equiangular triangle," which is all that ω asserts. "Some *S* is not some *P*" is indeed always true except when both the subject and the predicate are the name of an individual and the

[1] *Laws of Thought*, § 79.

[2] I am here again interpreting *some* as indefinite. If it means *some at most*, then the power of denial possessed by η is increased.

same individual[1]. De Morgan[2] (*Syllabus*, p. 24) points out that its contradictory is,—"*S* and *P* are singular and identical; there is but one *S*, there is but one *P*, and *S* is *P*." It may be said without hesitation that the proposition ω is of absolutely no logical importance.

221. Test the validity of the following syllogisms, and shew that the reasoning contained in those that are valid can be expressed without the use of quantified predicates:—

In Figure 1, **UUU**, **IU**η.
In Figure 2, η**UO**, **AUA**.
In Figure 3, **YAI**, **Y**η**E**.

(1) **UUU** in Figure 1 is valid:—

All M is all P,
All S is all M,
therefore, *All S is all P*.

It will be observed that whenever one of the premisses is **U**, the conclusion may be obtained by substituting *S* or *P* (as the case may be) for *M* in the other premiss.

[1] *Some* being again interpreted in its ordinary logical sense. Mr Johnson points out that if *some* means *some but not all*, we are led to the paradoxical conclusion that ω is equivalent to **U**. *Some but not all S is not some but not all P* informs us that certain *S*'s are not to be found amongst a certain portion of the *P*'s but that they are to be found amongst the remainder of the *P*'s, while the remaining *S*'s are to be found amongst the first set of *P*'s. Hence *all S is P*; and it follows similarly that *all P is S*. *Some but not all S is not some but not all P* is therefore equivalent to *All S is all P*.

[2] De Morgan in several passages criticizes with great acuteness the Hamiltonian scheme of propositions.

Without the use of quantified predicates, the above reasoning may be expressed by means of the two syllogisms,—

All M is P,	*All M is S,*
All S is M,	*All P is M,*
therefore, *All S is P.*	therefore, *All P is S.*

(2) **IU**η in Figure 1 is invalid, if *some* is used in its ordinary logical sense. The premisses are *Some M is some P,* and *All S is all M.* We may therefore obtain the legitimate conclusion by substituting *S* for *M* in the major premiss. This yields *Some S is some P.*

If, however, *some* is here used in the sense of *some only, No S is some P* follows from *Some S is some P,* and the original syllogism is valid, although a negative conclusion is obtained from two affirmative premisses.

This syllogism is given valid by Thomson (*Laws of Thought,* § 103); but apparently only through a misprint for **IE**η. In his scheme of valid syllogisms Thomson seems consistently to interpret *some* in its ordinary logical sense. Using the word in the sense of *some only,* several other syllogisms would be valid that he does not give as such[1].

(3) η**UO** in Figure 2 is valid:—

No P is some M,
All S is all M,
therefore, *Some S is not any P.*

Without the use of quantified predicates, we can obtain an equivalent argument in *Bocardo,* thus—

Some M is not P,
All M is S,
therefore, *Some S is not P.*

[1] Compare section 215.

(4) **AUA** in Figure 2 runs as follows:—

All P is some M,
All S is all M,
therefore, *All S is some P.*

Here we have neither undistributed middle nor illicit process of major or minor, and yet the syllogism is invalid. Applying the rule given above that "whenever one of the premisses is **U**, the conclusion may be obtained by substituting *S* or *P* (as the case may be) for *M* in the other premiss," we find that the valid conclusion is *Some S is all P.* More generally, it follows from this rule of substitution that *if one premiss is* **U** *while in the other premiss the middle term is undistributed, then the term combined with the middle term in the* **U** *premiss must be undistributed in the conclusion.* This appears to be the one additional syllogistic rule required if we recognise **U** propositions in syllogistic reasonings.

All danger of fallacy is avoided by breaking up the **U** proposition into two **A** propositions. In the case before us we have,—*All P is M, All M is S; All P is M, All S is M.* From the first of these pairs of premisses we get the conclusion *All P is S*; in the second pair the middle term is undistributed, and therefore no conclusion is yielded at all.

(5) **YAI** in Figure 3 is valid:—

Some M is all P,
All M is some S,
therefore, *Some S is some P.*

The conclusion is however weakened, since we might infer *Some S is all P.* It will be observed that when we quantify the predicate, the conclusion of a syllogism may be weakened in respect to its predicate as well as in respect to

its subject. In the ordinary doctrine of the syllogism this is for obvious reasons not possible.

Without quantification of the predicate the above reasoning may be expressed in *Bramantip*, thus,—

All P is M,
All M is S,
therefore, *Some S is P.*

We could get the full conclusion, *All P is S*, in *Barbara.*

(6) YηE in Figure 3 is invalid :—

From *Some M is all P,*
and *No M is some S,*
we infer that *No S is any P;*
but this involves illicit process of the minor.

222. The Figured and the Unfigured Syllogism.

The distinction between the figured and the unfigured syllogism is due to Hamilton, and is connected with his doctrine of the Quantification of the Predicate.

By a rigid quantification of the predicate the distinction between subject and predicate may be dispensed with; and such being the case there is no ground left for distinction of figure (which depends upon the position of the middle term as subject or predicate in the premisses). This gives what Hamilton calls the *Unfigured Syllogism.* For example,—

Any bashfulness and any praiseworthy are not equivalent,
All modesty and some praiseworthy are equivalent,
therefore, Any bashfulness and any modesty are not equivalent.

All whales and some mammals are equal,

All whales and some water animals are equal,
therefore, Some mammals and some water animals are equal.

There is an approach here towards the Equational Logic.

Hamilton gives a distinct canon for the unfigured syllogism as follows:—"In as far as two notions either both agree, or one agreeing the other does not, with a common third notion; in so far these notions do or do not agree with each other."

EXERCISES.

223. Write out the various judgments, including **U** and **Y**, which are logically opposed to the judgment: No puns are admissible. State in the case of each judgment thus formed what is the kind of opposition in which it stands to the original judgment, and also the kind of opposition between each pair of the new judgments. [C.]

224. Explain precisely how it is that **O** admits of ordinary conversion if the principle of the Quantification of the Predicate is adopted, although not otherwise.

225. Examine the validity of the following moods:—

In Figure 1, **UAU, YOO, EYO**;
In Figure 2, **AAA, AYY, UO**ω;
In Figure 3, **YEE, OYO, A**ω**O**. [C.]

226. Enquire in what figures, if any, the following moods are valid, noting cases in which the conclusion is weakened:—**AUI**; **YAY**; **UO**η; **IU**η; **UEO**. [L.]

227. Is it possible that there should be three propositions such that each in turn is deducible from the other two? [V.]

CHAPTER X.

EXAMPLES OF ARGUMENTS AND FALLACIES.

228. Examine technically the following arguments :—

(1) Those who hold that the Insane should not be punished ought in consistency to admit also that they should not be threatened ; for it is clearly unjust to punish any one without previously threatening him.

(2) If he pleads that he did not steal the goods, why, I ask, did he hide them, as no thief ever fails to do? [v.]

229. Examine technically the following arguments :—

Knavery and folly always go together ; so, knowing him to be a fool I distrusted him.

If I deny that poverty and virtue are inconsistent, and you deny that they are inseparable, we can at least agree that some poor are virtuous.

How can you deny that the infliction of pain is justifiable if punishment is sometimes justifiable and yet always involves pain? [v.]

230. Examine the following argument :—

You cannot affirm that all his acts were virtuous, for you deny that they were all praiseworthy, and you allow that nothing that is not praiseworthy is virtuous.

If the argument is valid, reduce it to syllogistic form, naming the mood and figure.

231. Examine the following reasoning:—

How can you deny that any poor should be relieved, when you deny that sickness and poverty are inseparable, and also that any sick should not be relieved? [V.]

X

232. What conclusions, if any, can be drawn respectively from the following pairs of propositions?

(*a*) There is no Turk that is not brave,
None but the brave deserve the fair.

(*b*) All X is Y,
Only Z is Y.

(*c*) No X is Y unless it is Z,
No Z is both X and Y. [L.]

233. In how many different moods may the argument implied in the following proposition be stated?

"No one can maintain that all persecution is justifiable who admits that persecution is sometimes ineffective."

How would the formal correctness of the reasoning be affected by reading "deny" for "maintain"? [V.]

234. What conclusions (if any) can be drawn from each pair of the following sentences taken two and two together?

(1) None but gentlemen are members of the club;

(2) Some members of the club are not officers;

(3) All members of the club are invited to compete;

(4) All officers are invited to compete.

Point out the mood and figure in each case in which you make a valid syllogism; and state your reasons when you consider that no valid syllogism is possible. [V.]

235. Detect the fallacy in the following argument:—

"A vacuum is impossible, for if there is nothing between two bodies they must touch." [N.]

236. Write the following arguments in syllogistic form, and reduce them to Figure 1:—

(α) Falkland was a royalist and a patriot; therefore, some royalists were patriots.

(β) All who are punished should be responsible for their actions; therefore, if some lunatics are not responsible for their actions, they should not be punished.

(γ) All who have passed the Little-Go have a knowledge of Greek; hence *A. B.* cannot have passed the Little-Go, for he has no knowledge of Greek.

237. Whately says,—"'Every true patriot is disinterested, few men are disinterested, therefore few men are true patriots,' might appear at first sight to be in the second figure and faulty; whereas it is *Barbara* with the premisses transposed."

Do you consider this resolution of the above syllogism to be the correct one?

238. Examine the validity of the following arguments:—

(α) Old Parr, healthy as the wild animals, attained the age of 152 years; all men might be as healthy as the wild

animals; therefore, all men might attain to the age of 152 years.

(β) Most M is P,
Most S is M,
therefore, Some S is P.

239. Examine the validity of the following arguments:—

(i) Since the end of poetry is pleasure, that cannot be unpoetical with which all are pleased.

(ii) It is quite absurd to say "I would rather not exist than be unhappy," for he who says "I will this, rather than that," chooses something. Non-existence, however, is no something, but nothing, and it is impossible to choose rationally when the object to be chosen is nothing.

240. Examine the following arguments, stating them in syllogistic form, and pointing out fallacies, if any:—

(*a*) Some who are truly wise are not learned; but the virtuous alone are truly wise; the learned, therefore, are not always virtuous.

(*b*) If all the accused were innocent, some at least would have been acquitted; we may infer, then, that none were innocent, since none have been acquitted.

(*c*) Every statement of fact deserves belief; many statements, not unworthy of belief, are asserted in a manner which is anything but strong; may we infer, therefore, that some statements not strongly asserted are statements of fact?

(*d*) That many persons who commit errors are blameworthy is proved by numerous instances in which the commission of errors arises from gross carelessness.

[University of Melbourne.]

241. Can the following arguments be reduced to syllogistic form?

(1) The sun is a thing insensible;
The Persians worship the sun;
Therefore, the Persians worship a thing insensible.

(2) The Divine law commands us to honour kings;
Louis XIV. is a king;
Therefore, the Divine law commands us to honour Louis XIV. [*Port Royal Logic.*]

242. Examine the following arguments; where they are valid, reduce them if you can to syllogistic form; and where they are invalid, explain the nature of the fallacy:—

(1) We ought to believe the Scripture;
Tradition is not Scripture;
Therefore, we ought not to believe tradition.

(2) Every good pastor is ready to give his life for his sheep;
Now, there are few pastors in the present day who are ready to give their lives for their sheep;
Therefore, there are in the present day few good pastors.

(3) Those only who are friends of God are happy;
Now, there are rich men who are not friends of God;
Therefore, there are rich men who are not happy.

(4) The duty of a Christian is not to praise those who commit criminal actions;
Now, those who engage in a duel commit a criminal action;
Therefore, it is the duty of a Christian not to praise those who engage in duels.

(5) The gospel promises salvation to Christians;
Some wicked men are Christians;
Therefore, the gospel promises salvation to wicked men.

(6) He who says that you are an animal speaks truly;
He who says that you are a goose says that you are an animal;
Therefore, he who says that you are a goose speaks truly.

(7) You are not what I am;
I am a man;
Therefore, you are not a man.

(8) We can only be happy in this world by abandoning ourselves to our passions, or by combating them;
If we abandon ourselves to them, this is an unhappy state, since it is disgraceful, and we could never be content with it;
If we combat them, this is also an unhappy state, since there is nothing more painful than that inward war which we are continually obliged to carry on with ourselves;
Therefore, we cannot have in this life true happiness.

(9) Either our soul perishes with the body, and thus, having no feelings, we shall be incapable of any evil; or if the soul survives the body, it will be more happy than it was in the body;
Therefore, death is not to be feared.

[*Port Royal Logic.*]

243. Examine the following arguments:—

(1) "He that is of God heareth my words: ye therefore hear them not, because ye are not of God."

(2) All the fish that the net inclosed were an indiscriminate mixture of various kinds: those that were set aside

and saved as valuable, were fish that the net inclosed: therefore, those that were set aside and saved as valuable, were an indiscriminate mixture of various kinds.

(3) Testimony is a kind of evidence which is very likely to be false: the evidence on which most men believe that there are pyramids in Egypt is testimony: therefore, the evidence on which most men believe that there are pyramids in Egypt is very likely to be false.

(4) If Paley's system is to be received, one who has no knowledge of a future state has no means of distinguishing virtue and vice: now one who has no means of distinguishing virtue and vice can commit no sin: therefore, if Paley's system is to be received, one who has no knowledge of a future state can commit no sin.

(5) If Abraham were justified, it must have been either by faith or by works: now he was not justified by faith (according to James), nor by works (according to Paul): therefore, Abraham was not justified.

(6) For those who are bent on cultivating their minds by diligent study, the incitement of academical honours is unnecessary; and it is ineffectual, for the idle, and such as are indifferent to mental improvement: therefore, the incitement of academical honours is either unnecessary or ineffectual.

(7) He who is most hungry eats most; he who eats least is most hungry: therefore, he who eats least eats most.

(8) A monopoly of the sugar-refining business is beneficial to sugar-refiners: and of the corn-trade to corn-growers: and of the silk-manufacture to silk-weavers, &c., &c.; and thus each class of men are benefited by some restrictions. Now all these classes of men make up the whole community: therefore a system of restrictions is beneficial to the community. [Whately, *Logic.*]

244. The following are a few examples in which the reader can try his skill in detecting fallacies, determining the peculiar form of syllogisms, and supplying the suppressed premisses of enthymemes.

(1) None but those who are contented with their lot in life can justly be considered happy. But the truly wise man will always make himself contented with his lot in life, and therefore he may justly be considered happy.

(2) All intelligible propositions must be either true or false. The two propositions "Cæsar is living still," and "Cæsar is dead," are both intelligible propositions; therefore they are both true, or both false.

(3) Many things are more difficult than to do nothing. Nothing is more difficult to do than to walk on one's head. Therefore many things are more difficult than to walk on one's head.

(4) None but Whigs vote for Mr B. All who vote for Mr B. are ten-pound householders. Therefore none but Whigs are ten-pound householders.

(5) If the Mosaic account of the cosmogony is strictly correct, the sun was not created till the fourth day. And if the sun was not created till the fourth day, it could not have been the cause of the alternation of day and night for the first three days. But either the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now, or else the sun must have been the cause of the alternation of day and night for the first three days. Hence it follows that either the Mosaic account of the cosmogony is not strictly correct, or else the word "day" is used in Scripture in a different sense to that in which it is commonly accepted now.

(6) Suffering is a title to an excellent inheritance; for God chastens every son whom He receives.

(7) It will certainly rain, for the sky looks very black.

[Solly, *Syllabus of Logic.*]

245. Give the technical name of the following argument:—Payment by results sounds extremely promising; but payment by results necessarily means payment for a minimum of knowledge; payment for a minimum of knowledge means teaching in view of a minimum of knowledge; teaching in view of a minimum of knowledge means bad teaching.

246. State the following arguments in logical form, and examine their validity:—

(1) Poetry must be either true or false: if the latter, it is misleading; if the former, it is disguised history, and savours of imposture as trying to pass itself off for more than it is. Some philosophers have therefore wisely excluded poetry from the ideal commonwealth.

(2) If we never find skins except as the teguments of animals, we may safely conclude that animals cannot exist without skins. If colour cannot exist by itself, it follows that neither can anything that is coloured exist without colour. So if language without thought is unreal, thought without language must also be so.

(3) Had an armistice been beneficial to France and Germany, it would have been agreed upon by those powers; but such has not been the case; it is plain therefore that an armistice would not have been advantageous to either of the belligerents.

(4) If we are marked to die, we are enow
To do our country loss: and, if to live,
The fewer men, the greater share of honour.
[O.]

247. Dr Johnson remarked that "a man who sold a penknife was not necessarily an ironmonger." Against what logical fallacy was this remark directed? [C.]

248. Exhibit the following in syllogistic form; naming the mood and figure; when possible, reduce them to the first figure: (*a*) The disciples of Wagner overrate him, for he has caused a great reform in dramatic art, and all great reformers are over-estimated by their followers. (*b*) Some undergraduates are guilty of conduct to which no gentleman would stoop; so some undergraduates are not gentlemen. (*c*) Not all the things we neglect are worthless, for some truths are neglected and none are without value. [C.]

249. Examine on logical principles the following arguments; and, if you find any fallacies, name them:

(*a*) The existence of State-officials is unjustifiable: for since men are by nature equal, it is contrary to nature that one should govern another.

(*b*) Instinct and reason are opposed: so a good action, if instinctive, is the opposite of that which reason would dictate. [C.]

250. Put the following propositions into their simplest logical form; name the Syllogistic Moods in which they can be proved; and find premisses that in some Mood will prove them:

(1) Not all the unhappy are evildoers.
(2) Only the wise are free. [C.]

251. Examine the following arguments, pointing out any fallacies that they contain:

(*a*) The more correct the logic, the more certainly will the conclusion be wrong if the premisses are false. Therefore, where the premisses are wholly uncertain the best logician is the least safe guide.

(*b*) The spread of education among the lower orders will make them unfit for their work: for it has always had that effect on those among them who happen to have acquired it in previous times.

(*c*) This pamphlet contains seditious doctrines. The spread of seditious doctrines may be dangerous to the State. Therefore, this pamphlet must be suppressed. [C.]

252. "To prove that Dissent is wrong you must appeal to the authority of the Church, and this you must base on the Bible; and you must also deny the supremacy of Conscience. Moreover you, at least, as an Anglican, must ignore the Reformation."

How should you draw out fully the argument here implied? To what extent does it naturally fall into syllogistic form? [V.]

253. No one can maintain that all republics secure good government who bears in mind that good government is inconsistent with a licentious press.

What premisses must be supplied to express the above reasoning in *Ferio*, *Festino*, and *Ferison* respectively? [V.]

254. Using any of the forms of Immediate Inference, shew in how many moods the following argument can be expressed:—"Every law is not binding, for some laws are morally bad, and nothing which is so is binding." [L.]

255. State the following reasonings in strict logical form, and estimate their validity:—

(*a*) As thought is existence, what contains no element of thought must be non-existent.

(*b*) Since the laws allow everything that is innocent, and avarice is allowed, it is innocent.

(*c*) Timon being miserable is an evil-doer, as happiness springs from well-doing.

(*d*) No wise man is unhappy; for no dishonest man is wise, and no honest man is unhappy. [L.]

256. Comment carefully upon the following statements:—

"The most perfect Logic will not serve a man who starts from a false premiss."

"I am enough of a logician to know that from false premisses it is impossible to draw a true conclusion."

[L.]

257. Might I be satisfied that a particular war was a just one, assuming (what was the fact) that it was popular, and also (what is more doubtful) that all just wars are popular?

Are honours and rewards, public or private, to be pronounced useless, because they cannot influence the stupid, and men of genius rise above them?

Because some persons in the dark cannot help thinking of ghosts, though they do not believe in them, does it follow that it is absurd to maintain that, when we cannot avoid thinking or conceiving of a thing, it must be true? [L.]

CHAPTER XI.

PROBLEMS ON THE SYLLOGISM.

258. Given a valid syllogism, then in no case will the combination of either premiss with the conclusion establish the other premiss.

This proposition admits of proof by means of the syllogistic rules.

We have to shew that if one premiss and the conclusion of a valid syllogism be taken as a new pair of premisses they do not in any case suffice to establish the other premiss.

Were it possible for them to do so, then *the premiss given true would have to be affirmative*, for if it were negative, the original conclusion would be negative, and combining these we should have two negative premisses which could yield no conclusion.

Also, *the middle term would have to be distributed in the premiss given true.* This is clear if it is not distributed in the other premiss; and since the other premiss is the conclusion of the new syllogism, if it is distributed there, it must *also* be distributed in the premiss given true or we should have an illicit process in the new syllogism.

Therefore, the premiss given true, being affirmative, and distributing the middle term, cannot distribute the other term which it contains. Neither therefore can this term be distributed in the original conclusion. But this is the term which will be the middle term of the new syllogism, and *we shall therefore have undistributed middle.*

The given syllogism then being valid, we have shewn it to be impossible that a new syllogism, having one of the original premisses and the original conclusion for its premisses, and the other original premiss for its conclusion, can be valid also[1].

259. If for both the premisses of a valid syllogism we substitute their contradictories, this will not in any case enable us to establish the contradictory of the original conclusion.

The premisses of the original syllogism must be either (α) both affirmative, or (β) one affirmative and one negative.

In case (α), the contradictories of the original premisses will both be negative; and from two negatives nothing follows.

In case (β), the contradictories of the original premisses will be one negative and one affirmative; and if this combination yields any conclusion, it will be negative. But the original conclusion must also be negative, and therefore its contradictory will be affirmative.

In neither case then can we establish the contradictory of the original conclusion.

[1] Other methods of solution more or less distinct from the above might be given. A somewhat similar problem is discussed by Solly, *Syllabus of Logic*, pp. 123—126, 132—136.

260. If for one of the premisses of a valid syllogism we substitute its contradictory, this will not in any case enable us to establish the contradictory of the original conclusion.

This follows at once from the proposition established in section 258. Let the premisses of a valid syllogism be P and Q and the conclusion R. P and the contradictory of Q will not prove the contradictory of R; for if so it would follow that P and R would prove Q; but this has been shewn not to be the case.

We have now established by strictly formal reasoning Aristotle's theorem that although it is not possible syllogistically to get a false conclusion from true premisses, it is quite possible to get a true conclusion from false premisses[1]. In other words, the falsity of one or both of the premisses does not prove the falsity of the conclusion of a syllogism[2]. The preceding section deals with the case in which both the

[1] Hamilton (*Logic*, I. p. 450) considers the doctrine "that if the conclusion of a syllogism be true, the premisses may be either true or false, but that if the conclusion be false, one or both of the premisses must be false" to be extra-logical, if it is not absolutely erroneous. He is clearly wrong, since the doctrine in question admits of a purely formal proof.

[2] "In all cases where T is not given in direct perception, but deduced from premisses, what really depends on the correctness of those premisses is not the truth of T, but only our insight into that truth. Without correct premisses T cannot indeed be *proved*, but nevertheless it can be true and its truth is independent of any errors we may commit, when reflecting about it, and subsists even when conclusively deduced from premisses materially false. This point deserves notice, for it is a common mistake in reasoning to take the invalidity of the proof which is offered for T as a proof of the falsehood of T itself, and to confuse the refutation of an argument with the disproof of a fact." Lotze, *Logic*, § 240.

premisses are false; the present section with that in which one only of the premisses is false.

261. How far is the validity of the ordinarily recognised syllogistic moods dependent upon any particular supposition with regard to the existential implication of propositions?

We may as in Part II. Chapter VIII. take different suppositions with regard to the existential implication of propositions[1], and proceed to consider how far the validity of the various syllogistic moods is affected by each in turn. We may take the following suppositions:—

(1) Every proposition implies the existence both of its subject and of its predicate[2].

(2) Every proposition implies the existence of its subject.

(3) No proposition implies the existence either of its subject or of its predicate.

(4) A particular proposition implies the existence of its subject; a universal proposition does not.

First, let every proposition imply the existence both of its subject and of its predicate. In this case, the existence of the major, middle, and minor terms is in every case guaranteed by the premisses, and therefore no further assumption with regard to existence is required in order that the conclusion may be legitimately obtained[3]. We may regard

[1] By *existence* we mean as before *existence in the universe of discourse*, whatever that may be.

[2] It will be observed that this is not quite the same as supposition (1) in sections 102 and 103.

[3] If however we are to be allowed to proceed as in section 121, (where from all P is M, all S is M, we inferred that some not-S is not-P), we must posit the existence not merely of the terms directly involved, but also of their contradictories.

the above supposition as that which is tacitly made in the ordinary doctrine of the syllogism.

Secondly, let every proposition imply the existence of its subject. Under this supposition, as we have already seen, an affirmative proposition ensures the existence of its predicate also; but not so a negative proposition. It follows that any mood will be valid unless the minor term is in its premiss the predicate of a negative proposition. This cannot happen in Figure 1 or in Figure 2, since in these figures the minor is always subject in its premiss; nor in Figure 3, since in this figure the minor premiss is always affirmative. In Figure 4 the only moods with a negative minor are *Camenes* and its weakened form **AEO**. Our conclusion then is that on the given supposition every ordinarily recognised mood is valid except these two[1].

Thirdly, let no proposition imply the existence either of its subject or of its predicate. Let the major, middle and minor terms be respectively *P*, *M*, *S*. The conclusion will imply that if there is any *S* there is some *P* or not-*P* (according

[1] Reduction to Figure 1 appears to be affected by this supposition, since it makes the simple conversion of **E** in general invalid. The conversion of **E** is involved in the reduction of *Cesare*, *Camestres*, and *Festino* from Figure 2; and of *Camenes*, *Fesapo*, and *Fresison* from Figure 4. Since however one premiss must be affirmative the existence of the middle term is thereby guaranteed, and hence the simple conversion of **E** in the second figure, and in the major of the fourth becomes valid. Also the conversion of the conclusion resulting from the reduction of *Camestres* is legitimate, since the original minor term is subject in its premiss. Hence *Camenes* (and its weakened form) are the only moods whose reduction is rendered illegitimate by the supposition under consideration. This result agrees with that reached in the text. [It will be observed that in order to reduce the weakened form of *Camenes*, we get first *Celarent*, then convert the conclusion, and then take the subaltern of the proposition thus obtained.]

as it is affirmative or negative). Will the premisses also imply this? If not, then the syllogism is not valid.

It has been shewn in section 127 that a universal affirmative conclusion, *All S is P*, can be proved only by means of the premisses,—*All M is P*, *All S is M*; and it is clear that these premisses themselves imply that if there is any *S* there is some *P*. On our present supposition, then, a syllogism is valid if its conclusion is universal affirmative.

Again, as shewn in section 127, a universal negative conclusion, *No S is P*, can be proved only in the following ways,—

(i) *No M is P*, (or *No P is M*),
All S is M,

therefore, *No S is P*.

(ii) *All P is M*,
No S is M, (or *No M is S*),

therefore, *No S is P*.

In (i) the minor premiss implies that if *S* exists then *M* exists, and the major premiss that if *M* exists then not-*P* exists.

In (ii) the minor premiss implies that if *S* exists then not-*M* exists, and the major premiss that if not-*M* exists then not-*P* exists (as shewn in section 103).

It follows that a syllogism is valid if its conclusion is universal negative.

Next, let the conclusion be particular. In Figure 1, the implication of the conclusion with regard to existence is contained in the premisses themselves, since the minor term is the subject of an affirmative minor premiss, and the

middle term the subject of the major premiss. In Figure 2, the weakened moods may be regarded as disposed of in what has been already said with regard to universal conclusions; for under our present supposition subalternation is a valid process. The remaining moods with particular conclusions in Figure 2 are *Festino* and *Baroco*. In the former, the minor premiss implies that if S exists then M exists, and the major that if M exists then not-P exists; in the latter, the minor premiss implies that if S exists then not-M exists, and the major that if not-M exists then not-P exists.

All the ordinarily recognised moods, then, in Figures 1 and 2 are valid. But it is otherwise with moods yielding a particular conclusion in Figures 3 and 4, with the single exception of the weakened form of *Camenes*[1]. Subalternation being a valid process, the legitimacy of the latter follows from the legitimacy of *Camenes* itself. But in all other cases in Figures 3 and 4, the minor term is the predicate of an affirmative minor premiss. Its existence therefore carries no further implication of existence with it in the premisses. It does so in the conclusion. Hence all moods in Figures 3 and 4 with the exception of **AEE** and **AEO** in the latter figure are invalid. Take, as an example, a syllogism in *Darapti*,—

All M is P,
All M is S,

therefore, *Some S is P.*

The conclusion implies that if S exists P exists; but consistently with the premisses, S may be existent while

[1] And *Camenes* itself is the only mood with a universal conclusion in these figures.

M and *P* are both non-existent. An implication is therefore contained in the conclusion which is not justified by the premisses.

Our results may be summed up as follows:—On the supposition that no proposition implies the existence either of its subject or of its predicate, all the ordinarily recognised moods of Figures 1 and 2 are valid, but none of those of Figures 3 and 4 excepting *Camenes* and the weakened form of *Camenes*[1].

Fourthly, let particulars imply, while universals do not imply, the existence of their subjects. The legitimacy of moods with universal conclusions may be established as in the preceding case. Taking moods with particular conclusions, it is obvious that they will be valid if the minor premiss is particular having the minor term as its subject; or if the minor premiss is particular affirmative, whether the minor term is its subject or predicate. *Disamis*, *Bocardo*, and *Dimaris* are also valid since the major premiss in each case guarantees the existence of *M*, and the minor implies that if *M* exists then *S* exists. The above will be found to cover all moods in which one premiss is particular. There remain only the moods in which from two universals we infer a particular. It is clear that all these moods must be invalid, for their conclusions will imply the existence of the minor term, and this cannot be guaranteed by the premisses.

On the supposition then that particulars imply while universals do not imply the existence of their subjects, the

[1] An express statement concerning existence may however render the rejected moods legitimate. If, for instance, the existence of the middle term is expressly given, then *Darapti* becomes valid. Compare section 121.

invalid moods are all the weakened moods together with *Darapti*, *Felapton*, *Bramantip*, and *Fesapo*[1].

262. "Whatever P and Q may stand for, we may shew *a priori* that some P is Q. For All PQ is Q by the law of identity, and similarly All PQ is P; therefore, by a syllogism in *Darapti*, Some P is Q." How would you deal with this paradox?

A solution is afforded by the discussion contained in the preceding section; and this example seems to shew that the enquiry,—how far assumptions with regard to existence are involved in syllogistic processes,—is not irrelevant or unnecessary.

263. What conclusion can be drawn from the following propositions? The members of the board were all either bondholders or shareholders, but not both; and the bondholders, as it happened, were all on the board. [V.]

We have given,—

No member of the board is both a bondholder and a shareholder,

All bondholders are members of the board;

and these premisses yield a conclusion (in *Celarent*),

No bondholder is both a bondholder and a shareholder,

that is, No bondholder is a shareholder.

[1] It will be observed that the letter *p* occurs in the mnemonic for each of these moods, indicating that their reduction to Figure 1 involves *conversion per accidens*. On the supposition under discussion this process is invalid, and we may find here a ratification of the above result.

264. The following rules were drawn up for a club:—(i) The financial committee shall be chosen from amongst the general committee; (ii) No one shall be a member both of the general and library committees, unless he be also on the financial committee; (iii) No member of the library committee shall be on the financial committee.

Is there anything self-contradictory or superfluous in these rules? [VENN, *Symbolic Logic*, p. 261.]

Let F = member of the financial committee,
G = member of the general committee,
L = member of the library committee.

The above rules then become,—

(i) All F is G;
(ii) If L is G, it is F;
(iii) No L is F.

From (ii) and (iii) we obtain

(iv) No L is G.

The rules may therefore be written,

(1) All F is G,
(2) No L is G,
(3) No L is F.

But in this form (3) is deducible from (1) and (2).

All that is contained therefore in the rules as originally stated may be expressed by (1) and (2); that is, the rules as originally stated were partly superfluous, and they may be reduced to

(1) The financial committee shall be chosen from amongst the general committee;

(2) No one shall be a member both of the general and library committees.

If (ii) is interpreted as implying that there are individuals who are on both the general and library committees, then it follows that (ii) and (iii) are inconsistent with each other.

265. Given that the middle term is distributed twice in the premisses of a syllogism, determine *directly*, (*i.e.*, without any reference to the special rules of the figures, or the possible moods in each figure), in what different moods it might possibly be.

The premisses must be either both affirmative, or one affirmative and one negative.

In the first case, both premisses being affirmative can distribute their subjects only. The middle term must therefore be the subject in each, and both must be universal. This limits us to the one syllogism,—

All M is P,
All M is S,
therefore, *Some S is P*.

In the second case, one premiss being negative, the conclusion must be negative and will therefore distribute the major term. Hence, the major premiss must distribute the major term, and also (by hypothesis) the middle term. This condition can be fulfilled only by its being one or other of the following,—*No M is P*, or *No P is M*. The major being negative, the minor must be affirmative, and in order to distribute the middle term must be *All M is S*.

In this case we get two syllogisms, namely,—

No M is P,
All M is S,
therefore, *Some S is not P*.

No P is M,
All M is S,
therefore, *Some S is not P.*

The given condition limits us therefore to three syllogisms (one affirmative and two negative); and by reference to the mnemonic verses we may now identify these with *Darapti* and *Felapton* in Figure 3, and *Fesapo* in Figure 4.

266. Given a valid syllogism with two universal premisses and a particular conclusion, such that, if for either of the premisses its subaltern is substituted the same conclusion cannot be inferred, determine the mood and figure of the syllogism.

If there be such syllogism, let *S, M, P* be its minor, middle, and major terms respectively.

Since the conclusion is particular it must be either *Some S is P*, or *Some S is not P.*

First, if possible, let it be *Some S is P.*

The only term which we require to distribute in the premisses is *M.* But since we have two universal premisses, *two* terms must be distributed in them as subjects[1]. One of these must be superfluous; and therefore for one of the premisses we may substitute its subaltern, and still get the same conclusion.

The conclusion cannot then be *Some S is P.*

Secondly, if possible, let the conclusion be *Some S is not P.*

If the subject of the minor premiss is *S*, we may clearly substitute its subaltern without affecting the conclusion.

[1] We here include the case in which the middle term is itself twice distributed.

The subject of the minor premiss must therefore be *M*, which will thus be distributed in this premiss. *M* cannot also be distributed in the major, or else it is clear that its subaltern might be substituted for the minor and nevertheless the same conclusion inferred. The major premiss must therefore be affirmative with *M* for its predicate. This limits us to the syllogism,—

All P is *M*,
No M is S,
therefore, *Some S is not P*;

and this syllogism, which is **AEO** in Figure 4, does fulfil the given conditions, for if either premiss is made particular, it becomes invalid.

The above amounts to a general proof of the proposition laid down in section 147. *Every syllogism in which there are two universal premisses with a particular conclusion is a strengthened syllogism, with the single exception of* **AEO** *in Figure* 4.

267. Given two valid syllogisms in the same figure in which the major, middle, and minor terms are respectively the same, shew, without reference to the mnemonic verses, that if the minor premisses are subcontraries, the conclusions will be identical.

The minor premiss of one of the syllogisms must be **O**, and the major premiss of this syllogism must therefore be **A** and the conclusion **O**. The middle and the major terms having then to be distributed in the premisses, this syllogism is determined, namely,—

All P is M,
Some S is not M,
therefore, *Some S is not P*.

Since the other syllogism is to be in the same figure, its minor premiss must be *Some S is M*; the major must therefore be universal, and in order to distribute the middle term must be negative. This syllogism therefore is also determined, namely,—

No P is M,
Some S is M,
therefore, *Some S is not P*.

The conclusions of the two syllogisms are thus shewn to be identical.

268. Is it possible that there should be a valid syllogism such that, each of the premisses being converted, a new syllogism is obtainable giving a conclusion in which the old major and minor terms have changed places? Prove the correctness of your answer by general reasoning, and if it is in the affirmative, determine the syllogism or syllogisms fulfilling the given conditions.

If such a syllogism be possible, it cannot have two affirmative premisses, or (since **A** can only be converted *per accidens*) we should have two particular premisses in the new syllogism.

Therefore, *the original syllogism must have one negative premiss*. This cannot be **O**, since **O** is inconvertible.

Therefore, *one premiss of the original syllogism must be* **E**.

First, let this be the major premiss. Then the minor premiss must be affirmative, and its converse being a particular affirmative will not distribute either of its terms. But this converse will be the *major* premiss of the new syllogism, which also must have a negative conclusion. We should then have illicit major in the new syllogism, and this supposition will not give us the desired result.

Secondly, let the minor premiss of the original syllogism be **E**. The major premiss in order to distribute the old major term must be **A**, with the major term as subject. We get then the following, satisfying the given conditions:—

All P is M,
No M is S, or *No S is M*,
therefore, *No S is P*, or *Some S is not P*;

that is, we really have four syllogisms, such that both premisses being converted, thus,—

No S is M, or *No M is S*,
Some M is P,—

we have a new syllogism giving a conclusion in which the old major and minor terms have changed places, namely,

Some P is not S.

Symbolically,—

$$\begin{array}{ll} PaM, & \left.\begin{array}{r} SeM, \\ \text{or } MeS, \end{array}\right\} \\ \left.\begin{array}{r} MeS, \\ \text{or } SeM, \end{array}\right\} & MiP, \\ \hline \therefore \left.\begin{array}{l} SeP \\ \text{or } SoP \end{array}\right\} & \therefore PoS. \end{array}$$

If it be required to retain the *quantity* of the original conclusion, this must be *SoP*; in this case then we have only two syllogisms fulfilling the given conditions.

EXERCISES.

269. Given a valid syllogism in Figure 1, is there any case in which (i) the contradictories of its premisses, (ii) the contradictory of one premiss combined with the other premiss, will furnish premisses for another valid syllogism?

270. Given the two following statements *false*:—Either all M is all P, or some M is not P; Some S is not M;—what is all that you can infer (*a*) with regard to S in terms of P, (*b*) with regard to P in terms of S?

271. If (1) it is false that whenever X is found Y is found with it, and (2) not less untrue that X is sometimes found without the accompaniment of Z, are you justified in denying that (3) whenever Z is found there also you may be sure of finding Y? And however this may be, can you in the same circumstances judge anything about Y in terms of Z? [R.]

272. If whenever X is present, Z is not absent, and sometimes when Y is absent, X is present, but if it cannot be said that the absence of X determines anything about either Y or Z, can anything be determined as between Z and Y? [R.]

273. If B is always found to coexist with A, except when X is Y (which it commonly, though not always, is), and if, even in the few cases where X is not Y, C is never found absent without B being absent also, can you make any other assertion about C? [R.]

274. *Some M is not P; All S is all M.* What conclusion follows from the combination of these premisses?

Can you infer anything about either S or P from the knowledge that both the above propositions are false?

275. Every English peer is entitled to sit in the House of Lords, and every member of the House of Commons must be elected to Parliament by a constituency; but no one entitled to a seat in the House of Lords is thus elected

to Parliament. What can we conclude from these premisses about (1) an English peer, (2) any one entitled to a seat in the House of Lords? [University of Melbourne.]

276. From P follows Q; and from R follows S; but Q and S cannot both be true; shew that P and R cannot both be true. [De Morgan.]

277. An apparent syllogism of the second figure with a particular premiss is found to break the general rules of the syllogism in this particular only, that the middle term is undistributed. If the particular premiss is false and the other true, what do we know about the truth or falsity of the conclusion?

278. Can an apparent syllogism break all the rules of the syllogism at once?

279. On the supposition that no proposition implies the existence either of its subject or of its predicate, find in what cases the *Reduction* of Syllogisms to Figure 1 is invalid.

280. Given that **O** is the major premiss of a valid syllogism, determine, by the aid of the general rules of the syllogism, and without reference to the mnemonic verses, its mood and figure.

281. Given that **O** is the minor premiss of a valid syllogism, determine, by the aid of the general rules of the syllogism, and without reference to the mnemonic verses, its mood and figure.

282. If the major premiss of a valid syllogism is affirmative, and if the major term is distributed both in

premisses and conclusion, while the minor term is undistributed in both, determine *directly* the mood and figure. [N.]

283. If the major term be distributed in the premisses and undistributed in the conclusion of a valid syllogism, determine *directly* the mood and figure. [C.]

284. Given two valid syllogisms in the same figure in which the major, middle, and minor terms are respectively the same, shew, without reference to the mnemonic verses, that if the minor premisses are contradictories, the conclusions will not be contradictories.

285. Is it possible that there should be two syllogisms having a common premiss such that their conclusions, being combined as premisses in a new syllogism, may give a universal conclusion? If so, determine what the two syllogisms must be. [N.]

PART IV.

A GENERALIZATION OF LOGICAL PROCESSES IN THEIR APPLICATION TO COMPLEX PROPOSITIONS[1].

CHAPTER I.

THE COMBINATION OF SIMPLE TERMS.

286. Complex Terms.

A *simple term* may for our present purpose be defined as one which is represented by a single symbol; *e.g.*, *A*, *P*, *X*. The combination of simple terms yields a *complex term*.

Simple terms may be combined (1) conjunctively, or (2) disjunctively.

(1) "What is both *A* and *B*" is a complex term resulting from the *conjunctive* combination of the simple terms *A*

[1] The following pages deal with problems that have ordinarily been relegated to Symbolic Logic. They do not however directly treat of Symbolic Logic itself, if that term is understood in its ordinary sense, namely, as designating the branch of the science in which symbols of *operation* are used. Of course in one sense all Formal Logic is symbolic.

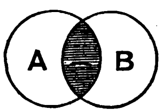
and B[1]. It is convenient to denote a complex term of this kind by a simple juxtaposition of the terms involved, thus—AB[2]. Accordingly the proposition "AB is CD" is to be interpreted "Anything that is both A and B is both C and D."

The conjunctive combination of terms is sometimes called *determination*[3]; and we may speak of the elements combined in a conjunctive term as the *determinants* of that term. Thus A and B are the determinants of the complex term AB.

(2) "What is either A or B" is a complex term resulting from the disjunctive combination of the simple terms A and B[4]. We may define an *alternative* as a term united disjunctively with others.

[1] This species of complex term is called by Jevons a *combined term* (*Pure Logic*, p. 15). So far as it requires a distinctive name I think I should prefer to call it a *conjunctive term*. The conjunctive combination of terms is in Symbolic Logic represented by the sign of *multiplication*.

[2] AB stands therefore for the class made up of all the individuals that belong both to the class A and to the class B. Thus if we represent A and B by circles, the shaded portion of the following diagram represents AB:



If the classes A and B lie entirely outside one another, then AB is a non-existent class.

[3] Compare Schröder, *Der Operationskreis des Logikkalkuls*, p. 6.

[4] This kind of complex term is called by Jevons a *plural term* (*Pure Logic*, p. 25). So far as it requires a distinctive name I think I should prefer to call it a *disjunctive term*. The disjunctive combination of terms is in Symbolic Logic represented by the sign of *addition*.

A complex term may of course involve both conjunctive and disjunctive combination: *e.g.*, *AB or CD*.

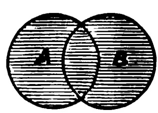
In the following pages, in accordance with the view indicated in section 110, the alternatives in a disjunctive term are not regarded as necessarily exclusive (except of course where they are formal contradictories). Thus, if we speak of anything as being *A or B* we do not intend to exclude the possibility of its being both *A and B*. In other words, *A or B* does not exclude *AB*[1].

One or two remarks may here be added with regard to the logical signification of the words *and*, *or*. In the predicate of a proposition their signification is clear; they indicate conjunctive and disjunctive combination respectively,—for example, *P is Q and R*, *P is Q or R*. But when they occur in the subject of a proposition there is in each case an ambiguity to which it is necessary to call attention.

Thus, *Anything that is either P or Q is R*, or *Whatever is either P or Q is R*, may sometimes for the sake of brevity be written *P or Q is R*. But the latter expression may also be interpreted to mean *One of the two P or Q is R, but we do not know which*; and in consequence of this ambiguity, the more definite mode of statement, *Whatever is either P or Q is R*, is to be preferred.

Again, the proposition *Whatever is either P or Q is R*

[1] If we represent *A* and *B* by circles then the shaded portion of the following diagram represents *A or B*:



It will be observed that if *A* and *a* are formal contradictories, then *A or a* stands for the entire universe of discourse.

may be expressed in the form *P and Q are R*; but here *and* has not a conjunctive force in the sense above defined. We get the true conjunctive sense of *and* in the subject of the proposition,— *Whatever is both P and Q is R*[1].

287. In a complex term the order of combination is indifferent[2].

This is true whether the combination be conjunctive or disjunctive.

Thus, *AB* and *BA* are precisely the same terms. It obviously comes to the same thing whether out of the class *A* we select the *B*'s or out of the class *B* select the *A*'s.

Again, *A or B* and *B or A* have precisely the same signification. It is obviously a matter of indifference whether we form a class by adding the *B*'s to the *A*'s or by adding the *A*'s to the *B*'s.

288. The Conjunctive Combination of Disjunctive Terms[3].

In order conjunctively to combine a simple term with a disjunctive term, we must conjunctively combine it with every alternative of the disjunctive. *A and* (*B or C*)[4]

[1] It will be observed that both in this case and in the case of *or*, we get rid of the ambiguity by making the words occur in the predicate of a subordinate sentence.

[2] This is sometimes spoken of as the law or property of *commutativeness*. Compare Boole, *Laws of Thought*, p. 31, and Jevons, *Principles of Science*, Chapter 2, § 8.

[3] Compare Jevons, *Principles of Science*, Chapter 5, § 7.

[4] In such a case as this the use of brackets is necessary in order to avoid ambiguity. Thus, *A and B or C* might mean *AB or C*, or as above *AB or AC*.

denotes whatever is *A* and at the same time either *B* or *C*, and it is clearly equivalent to *AB or AC*[1].

In order conjunctively to combine two disjunctive terms, we must conjunctively combine every alternative of the one with every alternative of the other. Thus, (*A or B*) *and* (*C or D*) denotes whatever is either *A* or *B* and at the same time either *C* or *D*, and it is clearly equivalent to *AC or AD or BC or BD*[2].

289. The Opposition of Complex Terms.

We shall find it convenient to denote the contradictory of any simple term by the corresponding small letter. Thus for not-*A* we write *a*, for not-*B* we write *b*. *A* and *a* therefore denote between them the whole universe of discourse (whatever that may be)[3], but they denote nothing in common. In other words, whatever *A* may designate, it is necessarily true that *Everything* (*in the universe of discourse*) *is A or a*; and that *A is not a*. It also follows that *Aa* necessarily represents a non-existent class; what is both *A* and not-*A* cannot have a place in any universe.

However complex a term may be, we can always find its contradictory by applying the criterion given in section 27. "A pair of contradictory terms are so related that between them they exhaust the entire universe to which reference

[1] This and the law that *A or BC* is equivalent to (*A or B*) *and* (*A or C*) are called by Schröder (*Der Operationskreis des Logikkalkuls*, pp. 9, 10) the *Laws of Distribution*. We return to the second of these laws in section 292.

[2] Whether or not we introduce algebraic symbols into Logic, there is here a very close analogy with algebraic multiplication which cannot be disguised.

[3] It is in consequence of this property that contradictory terms are sometimes spoken of as *complementary* terms.

is made, whilst there is no individual of which they can both be at the same time affirmed."

Now whatever is not *AB* must be either *a* or *b*, whilst nothing that is *AB* can be either of these; and *vice versa*.

$$\left\{\begin{array}{l} AB, \\ a \textit{ or } b, \end{array}\right.$$

are therefore a pair of contradictories.

Similarly,

$$\left\{\begin{array}{l} A \textit{ or } B, \\ ab, \end{array}\right.$$

are a pair of contradictories.

If, then, two simple terms are conjunctively combined into a complex term, the contradictory of this complex term is found by disjunctively combining the contradictories of the simple terms. And, conversely, if two simple terms are disjunctively combined into a complex term, the contradictory of this complex term is found by conjunctively combining the contradictories of the simple terms.

In each case, we substitute for the simple terms involved their contradictories, and (as the case may be) write *and* for *or*, or *or* for *and*.

But however complex a term may be, it will consist of a series of conjunctive and disjunctive combinations; and it may be successively resolved into the combination of pairs of relatively simple terms till it is at last shewn to result from the combination of absolutely simple terms. For example,—

$$ABC \textit{ or } DE \textit{ or } FG$$

results from the disjunctive combination of the pair,—

$$\left\{\begin{array}{c} ABC \textit{ or } DE, \\ FG; \end{array}\right.$$

ABC or DE results from the disjunctive combination of the pair,—

$$\begin{cases} ABC, \\ DE; \end{cases}$$

FG results from the conjunctive combination of the pair,—

$$\begin{cases} F, \\ G; \end{cases}$$

and similarly we may resolve *ABC*, *DE*.

We may hence deduce the following general rule for obtaining the contradictory of any complex term :—*For each simple term involved, substitute its contradictory; everywhere change* and *into* or, *and* or *into* and[1]. This rule is of simple application, and is of fundamental importance in the treatment of complex propositions adopted in the following pages.

Thus the contradictory of

A or BC

is *a and* (*b or c*),

i.e., *ab or ac.*

The contradictory of

ABC or ABD

is (*a or b or c*) *and* (*a or b or d*),

which, as we shall presently find[2], is resolvable into

a or b or cd.

Two terms may be formally *inconsistent* without being contradictories; *i.e.*, they cannot both be affirmed of any-

[1] Compare Schröder, *Der Operationskreis des Logikkalkuls*, p. 18.
[2] See section 292.

thing, but it may be that there are some things of which neither can be affirmed. Thus, we can say that, whatever A, B, and C may stand for, *AB is not bC*, (since if AB were bC it would involve something being at the same time both B and not-B); but we cannot say that, whatever A, B, and C may stand for, *Everything is AB or bC*, (since something may be Abc, which is neither AB nor bC). If a conjunctive term contains a determinant which is the contradictory of a determinant contained in another conjunctive term then it follows that these two conjunctive terms are formally inconsistent. Or, more briefly, two conjunctive terms are formally inconsistent if they contain contradictory determinants.

If two conjunctive terms are such that every determinant in one has corresponding to it in the other its contradictory, these two terms may be regarded as logical contraries[1]. Thus, AbC, aBc may be spoken of as contraries. A disjunctive term, such as *AB or ab*, does not seem to admit of a contrary distinct from its contradictory.

290. The Development of Terms by means of the Law of Excluded Middle.

By the Law of Excluded Middle[2], *A is AB or Ab*; and by the Law of Identity *Whatever is AB or Ab is A*. Hence the terms A and *AB or Ab* are equivalent. Similarly it follows that the term *ABC or ABc or AbC or Abc* is equivalent to A.

When for A is thus substituted *ABC or ABc or AbC or Abc*, it is called the development of a term with reference

[1] Compare section 28.

[2] This law is usually called by Jevons the *Law of Duality*. See *Principles of Science*, Chapter 5, § 5.

to other terms; accordingly in this particular instance A is developed with reference to B and C[1].

291. The Simplification of Complex Terms.

The following rules may be laid down for the simplification of complex terms:

(1) *The repetition of any given determinant is superfluous*[2].

Out of the class A to select the A's is obviously a process that leaves us just where we began. In other words, what is both A and A is identical with what is A. Thus AA merely denotes the class A, ABB merely denotes the class AB. Such terms in their original form are tautologous, and therefore if any determinant on its second appearance in a conjunctive term is struck out we are still left with an equivalent term.

(2) *The repetition of any given alternative is superfluous*[3].

[1] A disjunctive term consisting of two alternatives which are the same except as regards one determinant of each which are contradictories, *e.g.*, AB *or* Ab, is called by Jevons a *dual term*. See *Pure Logic*, p. 37.

[2] This is called by Jevons the *Law of Simplicity*. See *Pure Logic*, p. 15; and *Principles of Science*, Chapter 2, § 8. The corresponding equation $x^2 = x$ is in Boole's system fundamental. See *Laws of Thought*, p. 31.

[3] This is called by Jevons the *Law of Unity*. See *Pure Logic*, p. 26; and *Principles of Science*, Chapter 5, § 4. By aid of the rule for obtaining contradictories given in section 289 it may be shewn that the Law of Unity is deducible from the Law of Simplicity and *vice versa*. For if two terms are equivalent, their contradictories must be equivalent; but the contradictory of AA is a *or* a and the contradictory of A is a. Hence if AA and A are equivalent, a *or* a and a must be equivalent. And it is clear that we might with equal force argue from the Law of Unity to the Law of Simplicity.

To say that anything is *A or A* is obviously equivalent to saying simply that it is *A*; to say that anything is *A or BC or BC* is equivalent to saying that it is *A or BC*. Hence if any alternative on its second appearance in a disjunctive term is struck out we are still left with an equivalent term.

(3) *If two alternatives are the same except as regards one determinant of each which are contradictories, these two alternatives are together equivalent to a single alternative consisting only of the determinants common to them.*

The signification of this rule is that *AB or Ab* and *A* are equivalent terms; and this equivalence has been already established in the preceding section. The substitution of *A* for the dual term *AB or Ab* may be spoken of as the *reduction of dual terms*[1].

(4) *Any alternative containing contradictory determinants may be indifferently introduced or omitted.*

This rule follows at once from the fact that an alternative containing contradictory determinants cannot represent any existing class. It is obvious that the term *A or Bb* is equivalent to *A* simply.

(5) *Any alternative which is merely a subdivision of another alternative may be indifferently introduced or omitted*[2].

Thus, *AB* is a subdivision of *A*, and the signification of the above rule is that the terms *A or AB* and *A* are equivalent.

[1] Jevons (*Principles of Science*, Chapter 6, § 9) speaks of it as the "Abstraction of Indifferent Circumstances."

[2] This rule is called by Schröder the *Law of Absorption* (*Der Operationskreis des Logikkalkuls*, p. 12). It is equivalent to one of Boole's "Methods of Abbreviation" (*Laws of Thought*, p. 130). Compare, also, Jevons, *Pure Logic*, p. 26.

This may be established as follows: By the development of *A* with reference to *B*, *A or AB* becomes *AB or Ab or AB*; but by rule (2) above, this is equivalent to *AB or Ab*; which, again, by rule (3), is equivalent to *A*.

(6) *The contradictory of any alternative may be indifferently introduced or omitted as a determinant of any other alternative.*

The signification of the above rule is that the terms *A or aB* and *A or B* are equivalent.

This may be established as follows: By rule (5) *A or aB* may be replaced by *A or AB or aB*; and by rule (3) this is equivalent to *A or B*.

The above rules establish the following equivalences[1]:

(1) *AA* is equivalent to *A*;
(2) *A or A* is equivalent to *A*;
(3) *AB or Ab* is equivalent to *A*;
(4) *A or Bb* is equivalent to *A*;
(5) *A or AB* is equivalent to *A*;
(6) *A or aB* is equivalent to *A or B*.

[1] To every equivalence there corresponds (as is pointed out by Schröder, p. 3) another equivalence in which conjunctive combination is throughout substituted for disjunctive combination and *vice versa*. This indeed follows from the rule for obtaining contradictories given in section 289.

Of the above equivalences the first two correspond in this way to one another (see note 3 on page 342); the following correspond to the remaining four respectively:

(7) (*A or B*) *and* (*A or b*) is equivalent to *A*;
(8) *A and* (*B or b*) is equivalent to *A*;
(9) *A and* (*A or B*) is equivalent to *A*;
(10) *A and* (*a or B*) is equivalent to *AB*.

The proof of these equivalences is obvious. Making use of the Distributive Law given in section 288, it will be seen that (7) follows from (1), (4), (5); (8) from (3); (9) from (1), (5); (10) from (4).

By bearing these equivalences in mind, the labour of manipulating complex propositions may frequently be very much diminished.

292. (*A or B*) *and* (*A or C*) is equivalent to *A or BC*[1]; (*A or B*) *and* (*AC or D*) is equivalent to *AC or AD or BD*.

By the rule given in section 288, (*A or B*) *and* (*A or C*) is equivalent to *AA or AC or AB or BC*; and this, by rules (1) and (5) of the preceding section, is equivalent to *A or BC*. Similarly it follows that (*a or b or c*) *and* (*a or b or d*) is equivalent to *a or b or cd*.

Again (*A or B*) *and* (*AC or D*) is equivalent to *AAC or ABC or AD or BD*; and this, by rules (1) and (5) of the preceding section, is equivalent to *AC or AD or BD*.

From the above equivalences we obtain the two following practical rules which are of the greatest assistance in simplifying the process of conjunctively combining disjunctives:

(1) If two disjunctives which are to be conjunctively combined have an alternative in common, this alternative may be at once written down as one alternative of the result, and we need not go through the form of combining it with any of the remaining alternatives of either disjunctive;

(2) If two disjunctives are to be conjunctively combined and an alternative of one is a subdivision of an alternative of the other, then the former alternative may be at once written down as one alternative of the result, and we

[1] This is Schröder's second *Law of Distribution*. See note to section 288.

need not go through the form of combining it with the remaining alternatives of the other disjunctive[1].

293. Simplification of the term *AD or acD.*

Since *AcD* is a subdivision of *AD*, *AD or acD* is by section 291, rule (5), equivalent to *AD or AcD or acD*; and this by section 291, rule (3), is equivalent to *AD or cD.*

The above equivalence may also be established as follows: *AD or acD* is equivalent to (*A or ac*) *and D*; and this by section 291, rule (6), is equivalent to (*A or c*) *and D*; which again is equivalent to *AD or cD.*

294. Simplification of the term *aB or aC or bC or aE or bE or Ad or Ae or bd or be or cd or ce.*

bE or be is a dual and may be reduced to *b*; the above term therefore becomes *aB or aC or bC or aE or b or Ad or Ae or bd or cd or ce.* By section 291, rule (5), we may now omit all alternatives in which *b* occurs as a determinant, and by rule (6), *B* may be omitted wherever it occurs as a determinant; accordingly our term is reduced to *a or aC or aE or b or Ad or Ae or cd or ce.* Since *a* is now one alternative, a further application of the same rules leaves us with *a or b or d or e or cd or ce*; and this is immediately reducible to *a or b or d or e.*

EXERCISES.

295. Shew that *BC or bD or CD* is equivalent to *BC or bD.*

[1] These rules are equivalent to Boole's second Method of Abbreviation (*Laws of Thought*, p. 131).

296. Give the contradictories of the following terms in their simplest forms:—

AB or BC or CD;
AB or bC or cD;
ABC or aBc;
ABCD or Abcde or aBCDe or BCde.

297. Simplify the following terms:

(1) *Ab or aC or BCd or Bc or bD or CD*;

(2) *ACD or Ac or Ad or aB or bCD*;

(3) *aBC or aCD or aBe or aDe or AcD or abD or bcD or aDE or cDE*;

(4) (*A or b*) *and* (*A or c*) *and* (*a or B*) *and* (*a or C*) *and* (*b or C*).

298. Prove the following equivalences:

(1) *AB or AC or BC or abc or aB or C* is equivalent to *a or B or C*;

(2) *aBC or aBd or acd or bcd or ABd or Acd or abd or aCd or BCd* is equivalent to *Bd or cd or ad or aBC*;

(3) *Pqr or pQs or prs or qrs or pq or pS or qR* is equivalent to *p or q*.

CHAPTER II.

COMPLEX PROPOSITIONS.

299. Complex Propositions and Compound Propositions.

We speak of a *proposition* as being *complex* if either its subject or its predicate is a complex term.

Complex Propositions may be divided,—

First (as in the case of simple propositions), according as they are affirmative or negative:

e.g., *All AB is C or D*;
No AB is C or D.

Secondly (also as in the case of simple propositions), according as they are universal or particular:

e.g., *All AB is C or D*;
Some AB is C or D.

In the following pages propositions written in the indefinite form are interpreted as universal. Thus, by *AB is C or D* we understand *All AB is C or D*.

It must be added that in dealing with complex propositions we definitely adopt the view that universals do not

imply, while particulars do imply, the existence of their subjects[1].

Thirdly, according as only the subject or only the predicate or both subject and predicate are complex terms:

e.g., *AB is C*;
A is B or C,
AB is C or D.

Fourthly, according as there is or is not

(α) conjunctive combination in the subject:

e.g., *A is C or D*,
AB is C or D;

(β) conjunctive combination in the predicate:

e.g., *AB is C*;
AB is CD;

(γ) disjunctive combination in the subject:

e.g., *A is CD*,
Whatever is either A or B is CD;

(δ) disjunctive combination in the predicate:

e.g., *AB is C*,
AB is C or D.

Propositions, whether simple or complex, may themselves be combined conjunctively (*i.e.*, when two or more propositions are affirmed to be true together) or disjunctively (*i.e.*, when an alternative is given between two or more propositions). A *compound proposition* may be defined as one which consists of the conjunctive or disjunctive combination of other propositions. For example,

[1] See section 106.

All AB is C and some P is not either Q or R; Either all AB is C or some P is not either Q or R.

300. The Equivalence of Propositions.

Two propositions are said to be *equivalent* when each can be inferred from the other. Similarly, two sets of propositions[1] are said to be equivalent when every member of each set can be inferred from the other set.

When we infer a proposition or set of propositions from another proposition or set of propositions, there are therefore two cases to be distinguished.

First, where the force of the original statement is unaffected, so that we can pass back from the new proposition or propositions to the original proposition or propositions; in this case the process of inference leaves us with an equivalent proposition or set of propositions.

Secondly, where the force of the original statement is weakened, so that we cannot pass back from the new proposition or propositions to the original proposition or propositions; in this case the process of inference does not leave us with an equivalent proposition or set of propositions.

301. The Simplification of Complex Propositions.

The subjects or predicates of complex propositions may of course often be simplified by aid of the rules given in section 291, and the force of the assertion will remain unaffected.

The following special rules may be added:

[1] In other words, two compound conjunctive propositions.

(1) *In a universal negative or in a particular affirmative proposition any determinant of the subject may be indifferently introduced or omitted as a determinant of the predicate, and vice versa.*

To say that *No AB is AC* is precisely the same as to say that *No AB is C*, or that *No B is AC*. For to say that *No AB is AC* is the same thing as to deny that anything is *ABAC*; but, as shewn in section 291, the repetition of the determinant *A* is superfluous, and the statement may therefore be reduced to the denial that anything is *ABC*. And this may equally well be expressed by saying *No AB is C*, or *No B is AC*[1].

Similarly, *No AB is AC or AD* may be reduced to *No AB is C or D*, or to *No B is AC or AD*.

Again, *Some AB is AC* may be shewn to be equivalent to *Some AB is C*, or to *Some B is AC*; for it simply affirms that something is *ABAC*, and the proof follows as above.

(2) *In a universal affirmative or in a particular negative proposition any determinant of the subject may be indifferently introduced or omitted as a determinant of any alternative of the predicate.*

All A is AB may obviously be resolved into the two propositions *All A is A*, *All A is B*[2]. But the former of these is a merely identical proposition and gives no information. *All A is AB* is therefore equivalent to the simple proposition *All A is B*.

Similarly, *All AB is AC or BC* is equivalent to *All AB is C*.

[1] See also Chapter 4, On the Conversion of Propositions.

[2] The resolution of complex propositions into relatively simple ones is considered in detail in the following section.

Again, *Some A is not AB* affirms that *Some A is a or b*[1]; but by the Law of Contradiction *No A is a*; therefore, *Some A is not B*, and obviously we can also pass back from this proposition to the one from which we started.

Similarly, *Some AB is not either AC or BC* is equivalent to *Some AB is not C.*

(3) *In a universal affirmative or in a particular negative proposition any alternative of the predicate may be indifferently introduced or omitted as an alternative of the subject.*

If *all A is B or C*, then by the Law of Identity it follows that *whatever is A or B is B or C*; it is also obvious that we can pass back from this to the original proposition.

Again, if *some A or B is not either B or C*, then since by the Law of Identity *all B is B* it follows that *some A is not either B or C*; and it is also obvious that we can pass back from this to the original proposition[2].

(4) *In a universal affirmative or in a particular negative proposition the contradictory of any determinant of the subject may be indifferently introduced or omitted as an alternative of the predicate, and vice versa.*

By this rule the three following propositions are affirmed to be equivalent to one another:

$$\left\{\begin{array}{l}\textit{All AB is a or C};\\ \textit{All B is a or C};\\ \textit{All AB is C};\end{array}\right.$$

and also the three following:

$$\left\{\begin{array}{l}\textit{Some AB is not either a or C};\\ \textit{Some B is not either a or C};\\ \textit{Some AB is not C}.\end{array}\right.$$

[1] The process of Obversion is considered in detail in Chapter 3.

[2] What follows to the end of the section may be omitted till the chapters on Obversion and Contraposition have been read.

The rule follows directly from rule (1) by aid of the process of Obversion (see chapter 3).

(5) *In a universal negative or in a particular affirmative proposition the contradictory of any determinant of the subject may be indifferently introduced or omitted as an alternative of the predicate.*

By this rule the two following propositions are affirmed to be equivalent to one another:

$$\begin{cases} \textit{No } AB \textit{ is } a \textit{ or } C; \\ \textit{No } AB \textit{ is } C; \end{cases}$$

and also the two following:

$$\begin{cases} \textit{Some } AB \textit{ is } a \textit{ or } C; \\ \textit{Some } AB \textit{ is } C. \end{cases}$$

The rule follows directly from rule (2) by aid of the process of Obversion.

(6) *In a universal negative or in a particular affirmative proposition the contradictory of any determinant of the predicate may be indifferently introduced or omitted as an alternative of the subject.*

This rule follows from rule (3) by aid of the process of Obversion.

302. The Resolution of Universal Complex Propositions into relatively Simple Propositions[1].

Universal Affirmatives. Universal affirmative complex propositions may be immediately resolved into relatively simple ones, so far as there is conjunctive combination in the predicate, or disjunctive combination in the subject. Thus,—

[1] In other words, into *compound conjunctive propositions.*

(1) *X is AB*

is obviously resolvable into the two propositions,—

$$\begin{cases} X \text{ is } A, \\ X \text{ is } B. \end{cases}$$

And from these we can if we please pass back to the original proposition. In other words, the complex proposition is equivalent to the two propositions into which it is resolved.

(2) *Whatever is either X or Y is A*

is obviously resolvable into (and also equivalent to) the two propositions,—

$$\begin{cases} X \text{ is } A, \\ Y \text{ is } A. \end{cases}$$

Universal Negatives. Universal negative complex propositions may be immediately resolved into relatively simple ones, so far as there is disjunctive combination either in the subject or in the predicate. Thus,

(3) *Nothing that is either X or Y is A*

is obviously resolvable into (and also equivalent to) the two propositions,—

$$\begin{cases} No\ X \text{ is } A, \\ No\ Y \text{ is } A. \end{cases}$$

(4) *No X is either A or B*

is obviously resolvable into (and also equivalent to) the two propositions,—

$$\begin{cases} No\ X \text{ is } A, \\ No\ X \text{ is } B. \end{cases}$$

Attention must be paid to the difference here brought out between affirmative and negative propositions. So far as there is conjunctive combination in the subject or disjunc-

tive combination in the predicate of a universal affirmative proposition, or conjunctive combination either in the subject or in the predicate of a universal negative proposition, we cannot *immediately* resolve it into simpler propositions[1].

It may however be added that propositions falling into this latter category are themselves implied wherever we have two relatively simple universal propositions of the same quality disjunctively combined.

Thus,

(i) *No XY is A* is implied by *No X is A or no Y is A*;
(ii) *No X is AB* is implied by *No X is A or no X is B*;
(iii) *All XY is A* is implied by *All X is A or all Y is A*;
(iv) *All X is A or B* is implied by *All X is A or all X is B*.

303. Relatively Simple Propositions[2] implicated in Complex Particular Propositions; and the Resolution of Complex Particulars into Compound Disjunctives.

Particular Affirmatives. Particular affirmative complex propositions imply relatively simple ones, so far as there is conjunctive combination either in the subject or in the predicate.

Thus, (i) *Some XY is A* obviously implies *Some X is A* and *Some Y is A*;

(ii) *Some X is AB* obviously implies *Some X is A* and *Some X is B*.

[1] It will be shewn subsequently that even in these cases universal complex propositions may be resolved into relatively simple ones by the aid of obversion and contraposition. Compare especially chapter 5. It may be added that by obversion the rule relating to universal negatives may be deduced from that relating to universal affirmatives; or *vice versa*.

[2] In other words, *compound conjunctive propositions*.

Particular Negatives. Particular negative complex propositions imply relatively simple ones, so far as there is conjunctive combination in the subject or disjunctive combination in the predicate.

Thus, (iii) *Some XY is not A* obviously implies *Some X is not A* and *Some Y is not A*;

(iv) *Some X is not either A or B* obviously implies *Some X is not A* and *Some X is not B.*

Attention must be paid to the following very important difference between particulars and universals. In the resolution of the latter, the new propositions obtained are together equivalent to the original propositions. But the relatively simple propositions implied by a complex particular are not together equivalent to the original proposition, and we cannot pass back from them to it. For example, from *Some X is A* and *Some X is B* it does not follow that *Some X is AB* for we cannot be sure that the same *X*'s are referred to in the two cases. For this reason we have avoided saying that complex particular propositions can be *resolved into* relatively simple ones.

Complex particular propositions may however be resolved into equivalent *compound disjunctive propositions*, in which the alternatives are relatively simple. This is the case so far as there is disjunctive combination either in the subject or in the predicate of a particular affirmative, or disjunctive combination in the subject or conjunctive combination in the predicate of a particular negative.

Thus,

(1) *Some X is not AB* is equivalent to *Some X is not A or some X is not B*;

(2) *Some X or Y is not A* is equivalent to *Some X is not A or some Y is not A*;

(3) *Some X or Y is A* is equivalent to *Some X is A or some Y is A*;

(4) *Some X is A or B* is equivalent to *Some X is A or some X is B*.

All the results of this section follow from those of the preceding by the rules of contradiction and contraposition.

304. The Omission of Terms from a Complex Proposition.

From the two preceding sections we obtain immediately the following rules for inferring one proposition from another:

(1) *A determinant may at any time be omitted from an undistributed term*[1];

(2) *An alternative may at any time be omitted from a distributed term*[2].

For example,—

Whatever is A or B is XY, therefore, *All A is X*;

Some AB is XY, therefore, *Some A is X*;

Nothing that is A or B is X or Y, therefore, *No A is X*;

Some AB is not either X or Y, therefore, *Some A is not X*.

The above results must be distinguished from those obtained in section 301. In the cases discussed in that section, the terms omitted were superfluous in the sense that their omission left us with propositions equivalent to our original propositions; but in the above inferences we cannot pass back from conclusion to premiss. From *Some A is X*, for example, we cannot infer that *Some AB is X*.

[1] The subject of a particular or the predicate of an affirmative proposition.

[2] The subject of a universal or the predicate of a negative proposition.

305. The Introduction of Terms into a Complex Proposition.

Corresponding to the rules laid down in the preceding section, we have also the following:

(1) *A determinant may at any time be introduced into a distributed term;*

(2) *An alternative may at any time be introduced into an undistributed term.*

A distributed term is either the subject of a universal or the predicate of a negative proposition. But it is clear that if *All A is B*, then *All AX is B*; and that if *No A is B*, then *No AX is B*[1]. Also that if *No A is B*, then *No A is BX*; and that if *Some A is not B*, then *Some A is not BX.*

An undistributed term is either the subject of a particular or the predicate of an affirmative proposition. But it is clear that if *Some A is* (*or is not*) *B* then *Some A or X is* (*or is not*) *B.* Also that if *All* (or *some*) *A is B*, then *All* (or *some*) *A is B or X.*

From the above rules taken in connexion with the rules given in section 301 we may obtain the following corollaries:

(3) *In universal affirmatives, any determinant may be introduced into the predicate, if it is also introduced into the subject; and any alternative may be introduced into the subject if it is also introduced into the predicate.*

Given *All A is B*, then *All AX is B* by the above rule; and from this we obtain *All AX is BX* by rule (2) of section 301.

Again, given *All A is B*, then *All A is B or X*; and

[1] We cannot make a similar inference in the case of particulars. It does not follow from *Some A is B* that *Some AX is B*, since all the *A*'s that are *B* may happen also to be not-*x*.

therefore, by rule (3) of section 301, *Whatever is A or X is B or X.*

(4) *In universal negatives any alternative may be introduced into subject or predicate, if its contradictory is introduced into the other term as a determinant.*

Given *No A is B*, then *No AX is B*; and therefore, by rule (5) of section 301, *No AX is B or x.*

Again, given *No A is B*, then *No A is BX*; and therefore, by rule (6) of section 301, *No A or x is BX.*

In none of the inferences considered in this section can we pass back from our conclusion to the original proposition.

306. The Opposition of Complex Propositions.

We have already dealt with the opposition of complex terms, and the opposition of complex propositions in itself presents no special difficulty. It must however be borne in mind that since in dealing with complex propositions we definitely adopt the view that particulars imply existence while universals do not, we have the following divergences from the ordinary doctrine of opposition: (1) we cannot infer **I** from **A**, or **O** from **E**; (2) **A** and **E** are not necessarily inconsistent with one another; (3) **I** and **O** may both be false at the same time. The ordinary doctrine of *contradictory* opposition remains unaffected[1]. The following are examples of contradictory propositions: *All X is both A and B*, *Some X is not both A and B*; *Some X is Y and at the same time either P or Q or R*, *No X is Y and at the same time either P or Q or R.*

[1] See section 104.

307. The Interpretation of Propositions of the forms *No AB is B*, *All AB is a*, *All AB is Cc.*

Propositions of the above forms may sometimes result as a consequence of the manipulation of complex propositions; but they involve a contradiction in terms and are in direct contravention of the fundamental laws of thought. They must be interpreted as affirming the non-existence of the subject of the proposition. Thus, *All AB is a* is to be interpreted *No A is B*, or *All A is b.*

This must be taken in connexion with the discussion in section 106. The view was there adopted that no universal proposition implies the existence of its subject; but if it is affirmative it denies the existence of anything that is the subject while it is not the predicate. Thus *All AB is a* denies the existence of anything that is at the same time *AB* and not-*a*, *i.e.*, *A*. But *AB* is *AB* and *A*. The existence of *AB* is therefore denied.

Similarly, a universal negative proposition denies the existence of anything that is both its subject and its predicate. *No AB is B* therefore denies the existence of *ABB*, *i.e.*, of *AB*.

All AB is Cc affirms that *AB* is something that is non-existent, and therefore that it is itself non-existent.

On the view that a proposition does imply the existence of its subject, then if propositions of the above form are obtained, we are thrown back on the alternative that some inconsistency has already found place in the premisses. This applies to the case of particular propositions.

Exercises.

308. Shew that if *No A is bc or Cd*, then *No A is bd.*

309. Give the contradictory of each of the following propositions:—

(1) Flowering plants are either endogens or exogens, but not both;

(2) Flowering plants are vascular, and either endogens or exogens, but not both. [University of Melbourne.]

310. Simplify the following propositions:

(1) *All AB is BC or be or CD or cE or DE*;

(2) *Nothing that is either PQ or PR is Pqr or pQs or prs or qrs or pq or pS or qR.*

CHAPTER III.

THE OBVERSION OF COMPLEX PROPOSITIONS.

311. The Obversion of Complex Propositions.

The doctrine of Obversion is immediately applicable to Complex Propositions; and we require no modification of our former definition of Obversion. From any given proposition we may infer a new one by changing its quality and taking as a new predicate the contradictory of the original predicate. The proposition thus obtained is called the obverse of the original proposition.

The only difficulty connected with the obversion of complex propositions consists in finding the contradictory of a complex term; but a simple rule for performing this process has already been given in section 289:—*For each simple term involved, substitute its contradictory; write* and *for* or, *and* or *for* and.

Applying this rule to *AB or ab*, we have (*a or b*) *and* (*A or B*), *i.e.*, *Aa or Ab or aB or Bb*; but since *Aa* and *Bb* involve self-contradiction, they may by rule (4) of section 291, be omitted. The obverse, therefore, of *All X is AB or ab* is *No X is Ab or aB*.

As additional examples we may find the obverse of each of the following propositions: (1) *All A is BC or DE;* (2) *No A is BcE or BCF;* (3) *Some A is not either B or bcDEf or bcdEF.*

(1) *All A is BC or DE* gives *No A is* (*b or c*) *and at the same time* (*d or e*). In this proposition as it here stands it is necessary to use brackets in order to avoid ambiguity. The necessity of brackets is however obviated by combining the disjunctives in accordance with the rule given in section 288. This yields *No A is bd or be or cd or ce.*

(2) *No A is BcE or BCF.* Here by the application of the general rule we have as the contradictory of the predicate,—(*b or C or e*) *and at the same time* (*b or c or f*). Applying the rules given in sections 288 and 292, this becomes *b or Cc or Cf or ce or ef. Cc* may be omitted by rule (4) of section 291; also *ef* by rule (5) of the same section, for *ef* is either *Cef* or *cef.* Hence the required obverse is *All A is b or Cf or ce.*

(3) *Some A is not either B or bcDEf or bcdEF.* The obverse is,—*Some A is b and* (*B or C or d or e or F*) *and* (*B or C or D or e or f*); *i.e.*, applying rules given in chapter I, *Some A is bC or bDF or be or bdf.*

312. No citizen is at once a voter, a householder and a lodger; nor is there any citizen who is neither of the three.

Every citizen is either a voter but not a householder, or a householder and not a lodger, or a lodger without a vote.

Are these statements precisely equivalent? [V.]

It may be shewn that each of these statements is the

logical obverse of the other. They are therefore precisely equivalent.

Let V = voter,	v = not voter;
H = householder,	h = not householder;
L = lodger,	l = not lodger.

The first of the given statements is

No Citizen is VHL or vhl;

therefore (by obversion), *Every citizen is either v or h or l and is also either V or H or L;*

therefore (combining these possibilities), *Every citizen is either Hv or Lv or Vh or Lh or Vl or Hl.*

But (by the law of Excluded Middle), *Hv is either HLv or Hlv;*

therefore, *Hv is Lv or Hl.*

Similarly, *Lh is Vh or Lv;*
and *Vl is Hl or Vh.*

Therefore, *Every citizen is Vh or Hl or Lv,*

which is the second of the given statements.

Again, starting from this second statement, it follows (by obversion) that *No citizen is at the same time v or H, h or L, l or V;*

therefore, *No citizen is vh or vL or HL, and at the same time l or V;*

therefore, *No citizen is vhl or VHL,*

which brings us back to the first of the given statements.

EXERCISES.

313. Find the obverse of each of the following propositions:—

(1) *Nothing is X, Y or Z;*

(2) *All X is Ab or aC;*

(3) *All W is XZ or Yz or YZ or Xy or xZ;*

(4) *No Ab is CDEf or Cd or cDf or cdE;*

(5) *No De is ABC or Abc;*

(6) *Some A is not either Cd or cD or bcd.*

314. Shew that the two following propositions are equivalent to one another:—

No X is A or BC or BD or DE,

All X is aBcd or abDe or abd.

CHAPTER IV.

THE CONVERSION OF COMPLEX PROPOSITIONS.

315. The application of the term *Conversion* to Complex Propositions.

Generalising, we may say that we have a process of Conversion whenever from a given proposition we infer a new one in which a simple term that appeared in the predicate of the original proposition now appears in the subject, or *vice versa*.

Thus the inference from *No A is BC* to *No B is AC* is of the nature of Conversion. The process may be simply analysed as follows,—

No A is both B and C,

therefore, *Nothing is at the same time A, B, and C*,

therefore, *No B is both A and C*.

The reasoning may also be resolved into a series of ordinary conversions :—

No A is BC,

therefore (by conversion), *No BC is A*,

i.e., *within the sphere of C*, *No B is A*,

therefore (by conversion), *within the sphere of C, No A is B,*

i.e., No AC is B,

therefore (by conversion), *No B is AC.*

Or, it may be treated thus,—

No A is BC,

therefore, by section 301, rule (1), *No AC is BC,*

therefore, also by section 301, rule (1), *No AC is B,*

therefore (by conversion), *No B is AC.*

Similarly it may be shewn that from *Some A is BC* we may infer *Some B is AC.*

Hence we obtain the following rule: *In a universal negative or a particular affirmative proposition any determinant of the subject may be transferred to the predicate or* vice versa *without affecting the force of the assertion*[1].

We have just shewn how from

No A is BC,

we may obtain by conversion

No B is AC.

Similary, we may infer

No C is AB,
No AB is C,
No AC is B,
No BC is A.

[1] Also, of course, in a universal negative or a particular affirmative the subject and predicate may be transposed as wholes. For example, from *No A or B is CD* it follows that *No CD is A or B*; from *Some A or BD is CE or EF* it follows that *Some CE or EF is A or BD.* But in these inferences there is nothing that is in any way distinctive of complex propositions.

The proposition may also be written,—

There is no ABC,

or, *Nothing is at the same time A, B, and C.*

The last of these is an extremely useful form to which to bring universal negatives.

In the same way from *Some A is BC or BD* we may infer

Some AB is C or D,
Some B is AC or AD,
Something is ABC or ABD.

There is no inference by conversion from a universal affirmative or from a particular negative.

Exercises.

316. If *No De is ABc*, then *No ABcD is e*; if *No AbDF is K*, then *All AbcDE is f* or *k*; if *No ABC is EF*, then *Nothing is ABCEF.*

317. If *Some c is Bdk*, then *Some Bd is ck*; if *Some AbDE is bCE*, then *Something is AbCDE*; if *Some AB or AC is DE or DF*, then *Some BD or CD is not either a or ef.*

CHAPTER V.

THE CONTRAPOSITION OF COMPLEX PROPOSITIONS.

318. The application of the term Contraposition to Complex Propositions.

According to our original definition, we contraposit a proposition when we infer from it a new proposition which has the contradictory of the old predicate for its subject and the old subject for its predicate.

Thus, *No not-B is A* is the contrapositive of *All A is B*; *All not-B is not-A* is its obverted contrapositive. Adopting the same definition, the contrapositive of *All A is B or C* will be *No bc is A*; the obverted contrapositive *All bc is a*. The contrapositive of *All A is BC* will be *No b or c is A*.

The process can be applied to universal affirmatives and to particular negatives. By obversion, conversion, and then again obversion, it is clear that in each of these cases we may obtain a legitimate obverted contrapositive by *taking as a new subject the contradictory of the old predicate, and as a new predicate the contradictory of the old subject, the proposition retaining its original quality.* For example: *All A is BC*, therefore, *Whatever is b or c is a*; *Some A is not either B or C*, therefore, *Some bc is not a.*

The above may be called the full contrapositive of a complex proposition[1]. It should be observed that any proposition and its full contrapositive are equivalent to one another; in other words, we can pass back from a contrapositive (or an obverted contrapositive) to the original proposition.

We shall now find that in relation to complex propositions it is convenient to give to the term Contraposition an extended meaning. We may say that *we have a process of Contraposition when from a given proposition we infer a new one in which the contradictory of a simple term that appeared in the predicate of the original proposition now appears in the subject, or the contradictory of a simple term that appeared in the subject of the original proposition now appears in the predicate.*

We may distinguish three operations which will be included under this definition:

(1) The operation of obtaining the full contrapositive of a given proposition, as above described and defined[2].

(2) An operation which may be described as *the generalisation of the subject of a proposition by the addition of one or more alternatives in the predicate.* Thus, from *All AB is C* we may infer *All A is b or C*; from *Some AB is not either C or D* we may infer *Some A is not either b or C or D.*

[1] We may sometimes without any risk of confusion neglect the distinction between a contrapositive and an obverted contrapositive.

[2] In some cases we may desire to drop part of the information given by the complete contrapositive. Thus, from *All A is BC or E* we may infer *Whatever is be or ce is a*; but in a given application it may be sufficient for us to know that *All be is a.* This may be called a *partial* contrapositive of the original proposition.

For inferences of the above type the following general rule may be given: *Any determinant may be dropped from the subject of a universal affirmative or a particular negative proposition, if its contradictory is at the same time added as an alternative in the predicate.*

This rule follows immediately as a corollary from rule (4) of section 301, or we may establish it thus: Given *All AB is C* (or *Some AB is not C*),—and these may be taken, so far as the rule in question is concerned, as types of universal affirmatives and particular negatives respectively,—we have by obversion *No AB is c* (or *Some AB is c*), and thence, by the rule for conversion given in section 315, *No A is Bc* (or *Some A is Bc*); then again obverting we have *All A is either b or C* (or *Some A is not either b or C*), the required result.

It will be observed that these operations leave us with a proposition that is equivalent to our original proposition. There is, therefore, no loss of logical power.

By a repetition of the above process any universal affirmative proposition may be brought to the form *Everything is* X_1 *or* X_2*...or* X_n; and we shall find that by aid of this transformation, complex inferences are in many cases simplified and rendered easy.

(3) An operation which may be described as *the omission of one or more of a series of alternatives in the predicate by a further particularisation of the subject.* Thus, from *All A is B or C* we may infer *All Ab is C*; from *Some A is not either B or C* we may infer *Some Ab is not C.*

For inferences of the above type the following general rule may be given: *Any alternative may be dropped from the predicate of a universal affirmative or a particular negative*

proposition, if its contradictory is at the same time introduced as a determinant of the subject[1].

This rule is the converse of that given under the preceding head; and it follows from the fact that the application of that rule leaves us with an equivalent proposition. It also follows immediately from rule (4) of section 301.

The following may be taken as typical examples of the different operations included above under the name contraposition:—

All AB is CD or de;

therefore, *first*, *Anything that is either cD or cE or dE is a or b* (the full contrapositive[2], obverted, according to our original definition);

secondly, *All A is b or CD or de;*
thirdly, *Whatever is ABD or ABE is CD.*

[1] The application of this rule again leaves us with a proposition equivalent to our original proposition. The following corollary does not necessarily leave us with an equivalent: *If a new determinant is introduced into the subject of a universal affirmative proposition* (see section 305) *every alternative in the predicate which contains the contradictory of this determinant may be omitted.* Thus, from

Whatever is A or B is C or DX or Ex,

we may infer

Whatever is AX or BX is C or D.

The application of this rule may sometimes result in the disappearance of all the alternatives from the predicate; and the meaning of such a result is that we now have a non-existent subject.

Thus, given

P is ABCD or Abcd or aBCd,

if we particularise the subject by making it *PbC*, we find that all the alternatives in the predicate disappear. The interpretation is that the class *PbC* is non-existent, *i.e.*, *No P is bC*; a conclusion which of course might also have been obtained directly from the given proposition.

[2] From this, the partial contrapositives, *All cD is a or b*, *All cE is a or b*, &c. are immediately deducible.

Combinations of the second and third operations give

Anything that is Ac or Ad is b or de;
Anything that is BD or BE is a or CD;
&c.

In all the above cases some simple term disappears from the subject or from the predicate of the original proposition, and is replaced by its contradictory in the predicate or the subject accordingly. Only in the full contrapositive, however, is every simple term thus transposed.

No confusion need result from the nomenclature here proposed, since the extended use of the term Contraposition can be applied only to complex propositions. There is still only one kind of contraposition possible in the case of the simple categorical proposition.

The importance of Contraposition as we are now dealing with it in connexion with complex propositions is that by its means, *given a universal affirmative proposition of any complexity, we may obtain separate information with regard to any simple term that appears in the subject, or with regard to the contradictory of any simple term that appears in the predicate, or with regard to any combination of such terms.* Thus, given *All XY is P or Qr*, by the process described as the generalisation of the subject, we have

All X is y or P or Qr,
All Y is x or P or Qr,
Everything is x or y or P or Qr.

The particularisation of the subject gives

All XYp is Qr,
All XYq is P,
&c.;

and by the combination of these processes, we have

All Xp is y or Qr;
&c.

Again, the full contrapositive of the original proposition is

Whatever is pq or pR is x or y;

from which we have

All p is x or y or Qr,
All q is x or y or P,
&c.

319. Given "All *D* that is either *B* or *C* is *A*," shew that "Everything that is not-*A* is either not-*B* and not-*C* or else it is not-*D*." [De Morgan.]

This example and those given in section 325 are adapted from De Morgan, *Syllabus*, p. 42. They are also given by Jevons, *Studies*, p. 241, in connexion with his Equational Logic. They are all simple exercises in Contraposition.

We have, *What is either BD or CD is A*,
therefore, *All a is* (*b or d*) *and* (*c or d*),
therefore, *All a is bc or d.*

320. Infer all that you possibly can by way of Contraposition or otherwise, from the assertion, All *A* that is neither *B* nor *C* is *X*. [R.]

The given proposition may, by the second of the processes discussed above, be thrown into the form,—

Everything is either a or B or C or X;

and it is seen to be symmetrical with regard to the terms *a*, *B*, *C*, *X*, and therefore with regard to the terms *A*, *b*, *c*, *x*.

We are sure then that anything that is true of A is true *mutatis mutandis* of b, c, and x, that anything that is true of Ab is true *mutatis mutandis* of any pair of the terms, and similarly for combinations three and three together.

We have at once the four symmetrical propositions:

All A is B or C or X; (1)
All b is a or C or X; (2)
All c is a or B or X; (3)
All x is a or B or C. (4)

Then from (1) by particularisation of the subject:

All Ab is C or X; (i)

with the five corresponding propositions:

All Ac is B or X; (ii)
All Ax is B or C; (iii)
All bc is a or X; (iv)
All bx is a or C; (v)
All cx is a or B. (vi)

By a repetition of the same process, we have *All Abc is X* (which is the original proposition over again); (α)
and corresponding to this:

All Abx is C; (β)
All Acx is B; (γ)
All bcx is a. (δ)

It will be observed that the following are pairs of contrapositives,—

(1) (δ), (2) (γ), (3) (β), (4) (α), (i) (vi), (ii) (v), (iii) (iv).

321. If AB is either Cd or cDe, and also either eF or H, and if the same is true of BH, what do we know of that which is E?

We have given,—

Whatever is AB or BH is (Cd or cDe) and (eF or H);
therefore, *Whatever is AB or BH is CdeF or cDeF or CdH or cDeH*;

therefore, *Whatever is ABE or BHE is CdH*;

therefore, *All E is CdH or b or ah.*

322. Given *A* is *BC* or *BDE* or *BDF*, infer descriptions of the following terms *Ace*, *Acf*, *ABcD*.

[Jevons, *Studies*, pp. 237, 238.]

In accordance with rules already laid down, we have immediately,—

Ace is BDF;
Acf is BDE;
ABcD is E or F.

323. Summary of the results obtainable by Obversion, Conversion, and Contraposition.

(1) By *obversion* we can change any proposition from the affirmative to the negative form, or *vice versa.*

For example, *All AB is CD or EF*, therefore, *No AB is ce or cf or de or af*; *Some P is not QR*, therefore, *Some P is either q or r.*

(2) By the *conversion* of a universal negative we can obtain separate information with regard to any simple term that appears either in the subject or in the predicate, or with regard to any combination of these terms.

For example, *No AB is CD or EF*,
therefore, *No A is BCD or BEF*,
No C is ABD,
No BD is AC.

Also by conversion we can reduce any universal negative proposition to the form *Nothing is either* X_1 *or* X_2 ... *or* X_n.

For example, the above proposition is equivalent to the following: *Nothing is either ABCD or ABEF.*

(3) By the *conversion* of a particular affirmative we can obtain separate information with regard to any determinant of the subject or of the predicate, or with regard to any combination of such determinants.

For example, *Some AB or AC is DE or DF*,
therefore, *Some A is BDE or BDF or CDE or CDF*,
Some D is ABE or ABF or ACE or ACF,
Some AD is BE or BF or CE or CF.

Also by conversion we can reduce any particular affirmative proposition to the form *Something is either* X_1 *or* X_2... *or* X_n.

For example, the above proposition is equivalent to the following: *Something is either ABDE or ABDF or ACDE or ACDF.*

(4) By the *contraposition* of a universal affirmative we can obtain information with regard to any simple term that appears in the subject, or with regard to the contradictory of any simple term that appears in the predicate, or with regard to any combination of these terms.

For example, *All AB is CD or EF;*
therefore, *All A is b or CD or EF*,
All c is a or b or EF,
All Be is a or CD,
All ce is a or b,
All Adf is b,
&c.

Also by contraposition we can reduce any universal affirmative proposition to the form *Everything is either* X_1 *or* X_2*...or* X_n.

For example, the above proposition is equivalent to the following: *Everything is a or b or CD or EF.*

(5) By the *contraposition* of a particular negative we can obtain separate information with regard to any determinant of the subject or with regard to the contradictory of any alternative of the predicate or with regard to any combination of these.

For example, *Some AB or AC is not either D or EF,*
therefore, *Some A is not either bc or D or EF,*
Some d is not either a or bc or EF,
Some Ae or Af is not either bc or D.

Also by contraposition we can reduce any particular negative proposition to the form *Something is not either* X_1 *or* X_2 *... or* X_n.

For example, the above proposition is equivalent to the following: *Something is not either a or bc or D or EF.*

EXERCISES.

324. Contraposit the proposition: All *A* that is neither *B* nor *C* is both *X* and *Y*. [L.]

325. Find the contrapositive of each of the following propositions:

(1) All *A* is either *BC* or *BD*;
(2) Whatever is *B* or *CD* or *CE* is *A*;
(3) Whatever is either *B* or *C* and at the same time either *D* or *E* is *A*;

(4) Whatever is A or BC and at the same time either D or EF is X. [De Morgan.]

326. Find the full contrapositive of each of the following propositions :—

All A is $BCDe$ or $bcDe$;
Some AB is not either CD or cDE or de;
Whatever is AB or bC is aCd or Acd;
Where A is present along with either B or C, D is present and C absent or D and E are both absent ;
Some ABC or abc is not either DEF or def.

327. Compare the logical force of the following propositions :—

(1) All voters who are not lodgers are householders who pay rates ;

(2) No one who is not a lodger and who does not pay rates is a voter ;

(3) A voter who is a householder is not a lodger ;

(4) A householder who does not pay rates is not a voter ;

(5) All who pay rates or are householders are voters ;

(6) Anyone who is not a householder or who being a householder does not pay rates is either not a voter or else he is a lodger ;

(7) All who have a vote pay rates ;

(8) Anyone who has no vote is either not a ratepayer or not a householder.

328. Establish the following,—

(i) Where B is absent, either A and C are both present or A and D are both absent ; therefore, where C is absent, either B is present or D is absent.

(ii) Where A is present and also either B or E, either C is present and D absent or C is absent and D present; therefore, where C and D are either both present or both absent, either A is absent or B and E are both absent.

(iii) Where A is present, either B and C are both present or C is present D being absent or C is present F being absent or H is present; therefore, where C is absent, A cannot be present H being absent.

329. Given that whatever is PQ or AP is bCD or $abdE$ or $aBCdE$ or $Abcd$, shew that,—

(1) All abP is CD or dE or q;
(2) All DP is bC or aq;
(3) Whatever is B or Cd or cD is a or p;
(4) All B is C or p or aq;
(5) All Cd is a or p;
(6) All AB is p;
(7) If ae is c or d it is p or q;
(8) If BP is c or D or e it is aq.

330. Bring the following propositions to the form *Everything is either* X_1 *or* X_2 ... *or* X_n:—

Whatever is Ac or ab or aC is bdf or deF;

Nothing that is A and at the same time either B or C is D or dE.

CHAPTER VI.

THE COMBINATION OF COMPLEX PROPOSITIONS[1].

331. The Combination of Universal Affirmatives.

(1) Universal Affirmatives having the same subject.

$$\textit{All } X \textit{ is } P_1 \textit{ or } P_2 \ldots \textit{or } P_m,$$
$$\textit{All } X \textit{ is } Q_1 \textit{ or } Q_2 \ldots \textit{or } Q_n,$$

may be taken as types of two universal affirmative propositions having the same subject. It is obvious that in order to find the result of their combination we must conjunctively combine their predicates. Thus,

$$\begin{aligned} &\textit{All } X \textit{ is } (P_1 \textit{ or } P_2 \ldots \textit{or } P_m) \textit{ and also} \\ &\qquad (Q_1 \textit{ or } Q_2 \ldots \textit{or } Q_n); \\ &\textit{i.e., All } X \textit{ is } P_1Q_1 \textit{ or } P_1Q_2 \ldots \textit{or } P_1Q_n \\ &\qquad \textit{or } P_2Q_1 \textit{ or } P_2Q_2 \ldots \textit{or } P_2Q_n \\ &\qquad \textit{or } \ldots\ldots\ldots\ldots\ldots\ldots \\ &\qquad \ldots\ldots\ldots\ldots\ldots\ldots \\ &\qquad \textit{or } P_mQ_1 \textit{ or } P_mQ_2 \ldots \textit{or } P_mQ_n. \end{aligned}$$

The new proposition thus obtained is equivalent to the

[1] Throughout this chapter the proposition resulting from the combination is supposed to be non-compound.

two original propositions; it sums up all the information which they jointly contain, and we can pass back from it to them.

In almost all cases of conjunctive combination there are opportunities of simplification which it is important not to overlook. After a little practice it will not even in the first instance be found necessary to write out all the alternatives in full. The following are examples:—

(i) *All X is AB or bce,*
All X is aBC or DE;
therefore, *All X is ABDE.*

It will be found that all the other combinations in the predicate contain contradictories.

(ii) *All X is A or Bc or D,*
All X is aB or Bc or Cd;
therefore, *All X is Bc or aBD or ACd.*

(iii) *Everything is A or bd or cE,*
Everything is AC or aBe or d;
therefore, *Everything is AC or bd or Ad or cdE.*

(2) Universal Affirmatives having different subjects.

If we have two universal affirmative propositions with different subjects we may obtain from them a new proposition by conjunctively combining both their subjects and their predicates. Thus, if *All X is* P_1 *or* P_2, and *All Y is* Q_1 *or* Q_2, it follows that *All XY is* P_1Q_1 *or* P_1Q_2 *or* P_2Q_1 *or* P_2Q_2. But in this case the new proposition thus obtained is not equivalent to the premisses; and we cannot pass back from it to them.

We may of course combine the propositions without loss of information by first reducing each of them to the form: *Everything is* X_1 *or* X_2... *or* X_n.

332. The Combination of Universal Negatives.

(1) Universal Negatives having the same subject.

No X *is* P_1 *or* P_2 ...... *or* P_m,
No X *is* Q_1 *or* Q_2 ...... *or* Q_n,

may be taken as types of two universal negative propositions having the same subject. It is obvious that in order to find the result of their combination we must disjunctively combine their predicates. Thus,

No X *is* P_1 *or* P_2 ...... *or* P_m *or* Q_1 *or* Q_2 ...... *or* Q_n.

Again the new proposition is equivalent to the two original propositions; in other words, we can pass back from it to them. And again the process of combination is likely to give opportunities of simplification of which advantage should be taken. The following are examples:

(i) *No A is bc,*
No A is Cd;
therefore, *No A is bc or Cd.*

(ii) *No X is either aB or aC or bC or aE or bE,*
No X is either Ad or Ae or bd or be or cd or ce;
therefore, *No X is either a or b or d or e*[1].

(iii) *Nothing is aBC or aCD or aBe or aDe,*
Nothing is AcD or abD or bcD or aDE or cDE;
therefore, *Nothing is aBC or aD or cD or aBe.*

(2) Universal Negatives having different subjects.

If we have two universal negative propositions with different subjects we may obtain from them a new proposition by conjunctively combining their subjects and disjunctively combining their predicates. Thus, if *No X is*

[1] Compare section 294.

P_1 *or* P_2, and *No* Y *is* Q_1 *or* Q_2, it follows that *No* XY *is* P_1 *or* P_2 *or* Q_1 *or* Q_2. In this case, as in that considered in the preceding section, the inferred proposition is not equivalent to the premisses; and we cannot pass back from it to them.

333. The Combination of Universals with Particulars of the same Quality.

(1) Affirmatives.

Given a universal affirmative and a particular affirmative having the same subject, we may obtain from them a new particular affirmative proposition by conjunctively combining their predicates. If *All* X *is* P_1 *or* P_2, and *Some* X *is* Q_1 *or* Q_2, it follows that *Some* X *is* P_1Q_1 *or* P_1Q_2 *or* P_2Q_1 *or* P_2Q_2. It will be observed that the particular premiss affirms the existence of either XQ_1 *or* XQ_2, and therefore certainly guarantees the existence of X; and the universal premiss implies that if X exists then either XP_1 or XP_2 exists.

We can pass back from the conclusion to the particular premiss, but not to the universal premiss. The conclusion is therefore not equivalent to the two premisses taken together.

Universal Affirmatives and Particular Affirmatives having different subjects do not admit of being directly combined. We may however combine them by first reducing them respectively to the forms *Everything is* P_1 *or* P_2*...or* P_m, *Something is* Q_1 *or* Q_2*...or* Q_n, and then conjunctively combining their predicates in accordance with the above rule.

(2) Negatives.

Given a universal negative and a particular negative having the same subject we may obtain from them a new particular negative proposition by disjunctively combining

their predicates. If *No X is either* P_1 *or* P_2, and *Some X is not either* Q_1 *or* Q_2, it follows that *Some X is not either* P_1 *or* P_2 *or* Q_1 *or* Q_2. The validity of this process is obvious since the particular premiss guarantees the existence of *X*. By obversion it can also be exhibited as a corollary from the rule given on the preceding page. We can again pass back from the conclusion to the particular premiss, but not to the universal premiss.

With regard to the combination of universal negatives and particular negatives having different subjects, the remarks made concerning affirmatives apply *mutatis mutandis.*

334. The Combination of Affirmatives with Negatives.

By first obverting one of the propositions, an affirmative may be combined with a negative in accordance with the rules given in the preceding sections. For example,

(1) *All X is A or B,*
No X is aC,
therefore, *All X is A or Bc.*

(2) *Everything is P or Q,*
Nothing is Pq or pR,
therefore, *Nothing is pR or q.*

(3) *All X is AB or bce,*
Some X is not either aBC or DE,
therefore, *Some X is ABd or ABe or bce.*

335. The Combination of Particulars with Particulars.

Particulars cannot to any purpose be combined with particulars. It is true that from *Some X is* P_1 *or* P_2, *Some X is* Q_1 *or* Q_2, we can pass to *Some X is* P_1 *or* P_2 *or* Q_1 *or*

Q_2. But this is a mere weakening of the information given by either of the premisses singly; and by the rule that *an alternative may at any time be introduced into an undistributed term* (section 305), it could equally well be inferred from either premiss taken by itself. Again from *Some X is not either P_1 or P_2, Some X is not either Q_1 or Q_2*, we can pass to *Some X is not either P_1Q_1 or P_1Q_2 or P_2Q_1 or P_2Q_2*. But similar remarks again apply, since we have already found that *a determinant may at any time be introduced into a distributed term.*

It must however be added that if we are given two particular propositions *as alternatives* (*i.e.*, in a compound disjunctive proposition) we can combine them into a single equivalent (non-compound) proposition, whereas in the case of universals this is not possible[1]. For example, given that *Either some X is P_1 or P_2 or some X is Q_1 or Q_2*, we may combine the alternatives into the single equivalent proposition *Some X is P_1 or P_2 or Q_1 or Q_2*. But given that *Either all X is P_1 or P_2 or all X is Q_1 or Q_2*, we cannot obtain a single *equivalent* (non-compound) proposition.

EXERCISES.

336. Combine the following propositions into a single equivalent (non-compound) proposition: *All X is either a or B; No X is either AcD or AB or CD; All X is either C or d.*

337. Every voter is both a ratepayer and occupier, or not a ratepayer at all; If any voter who pays rates is an occupier, then he is on the list; No voter on the list is both a ratepayer and an occupier.

Examine the results of combining these three statements. [V.]

[1] Compare sections 302, 303.

CHAPTER VII.

INFERENCES FROM COMBINATIONS OF COMPLEX PROPOSITIONS.

338. *Problem.*—Given any universal proposition, and any term X, to discriminate between the cases in which the proposition does and those in which it does not afford information with regard to this term.

We may assume that the original proposition is not an identical proposition.

If it is negative, let it by obversion be made affirmative.

Then it may be written in the form

Whatever is P_1P_2...*or* Q_1Q_2...*or &c. is* A_1A_2...*or* B_1B_2...*or &c.*,
where P_1, Q_1, A_1, B_1, &c. are all simple terms[1].

As shewn in section 302, this may be resolved into the independent propositions :—

All P_1P_2...*is* A_1A_2...*or* B_1B_2...*or &c.*;
All Q_1Q_2...*is* A_1A_2...*or* B_1B_2...*or &c.*;
&c. *&c.* *&c.*;

in none of which is there any disjunction in the subject.

[1] So that both subject and predicate consist of a series of alternatives, in which all the determinants are simple; *i.e.*, we have no alternative of the form (A *or* B) *and at the same time* (C *or* D).

We may deal with these propositions separately, and if any one of them affords information with regard to X, then of course the original proposition does so.

We have then to consider a proposition of the form

$$All\ P_1P_2 \ldots P_n\ is\ A_1A_2 \ldots or\ B_1B_2 \ldots or\ \&c.$$

From this by contraposition we get,—

Everything is $A_1A_2 \ldots$ *or* $B_1B_2 \ldots$ *or &c. or* p_1 *or* $p_2 \ldots$ *or* p_n; and hence, X *is* $A_1A_2 \ldots$ *or* $B_1B_2 \ldots$ *or &c. or* p_1 *or* $p_2 \ldots$ *or* p_n.

We may now strike out all alternatives in the predicate which contain x.

If they all contain x, then the information afforded us with regard to X is that it is non-existent.

If some alternatives are left, then the proposition will afford information concerning X unless, when the predicate has been simplified to the fullest possible extent[1], one of the alternatives is itself X uncombined with any other term, in which case it is clear that we are left with a merely identical proposition.

Now one of these alternatives will be X in the following cases, and only in these cases;—

First, If one of the alternatives in the predicate of the original proposition, when reduced to the affirmative form, is X.

Secondly, If any set of alternatives in the predicate of the original proposition, when reduced to the affirmative form, constitute a development of X[2]; (since AX *or* aX is equivalent to X; ABX *or* AbX *or* aBX *or* abX is also equivalent to X; and so on).

[1] All superfluous terms being omitted, but the predicate still consisting of a series of alternatives in which all the determinants are simple.

[2] See section 290.

Thirdly, If one of the alternatives in the predicate of the original proposition, when reduced to the affirmative form, contains X in combination solely with some determinant of the subject; since in this case such alternative is equivalent to X simply[1].

For example,

AB is AX or D is equivalent to *AB is X or D*.

By contraposition of this proposition in its original form we have,—*Everything is AX or D or a or b*, but by section 291, rule (6), *AX or a* is equivalent to *X or a*.

Fourthly, If one of the determinants of the subject is x; since in that case we shall after contraposition have X as one of the alternatives in the predicate.

The above may now be summed up in the proposition:—

Any non-identical universal proposition will afford information with regard to any term X, unless (after it has been brought to the affirmative form) (1) one of the alternatives in the predicate is X, or (2) any set of alternatives in the predicate constitutes a development of X, or (3) any alternative in the predicate contains X in combination solely with some determinant that is also a determinant of the subject, or (4) x is a determinant of the subject.

If, after the proposition has been reduced to the affirmative form, all superfluous terms are omitted in accordance with the rules given in chapters 1 and 2, then the criterion becomes more simple,—

Any non-identical universal proposition will afford information with regard to any term X, unless (after it has been brought to the affirmative form and its predicate has been so simplified that it contains no superfluous terms) X is itself

[1] See section 301, rule (2).

an alternative of the predicate or x is a determinant of the subject[1].

If instead of X we have a complex term XYZ, then no determinant of this term must appear by itself as an alternative of the predicate, and there must be at least one alternative in the subject which does not contain as a determinant the contradictory of any determinant of this complex term; *i.e.*, no alternative in the predicate must be X, Y, or Z, or any combination of these, and some alternative in the subject must contain neither x, y, nor z.

The above criterion is of simple application.

339. *Problem.* Given any number of universal propositions involving any number of terms, to determine what is all the information that they jointly afford with regard to any given term or combination of terms that they contain.

The great majority of direct problems[2] involving complex propositions may be brought under the above general form. If the student will turn to Boole, Jevons, or Venn, he will find that it is by them treated as the central problem of Symbolic Logic[3].

[1] It may be added that every universal proposition, unless it be identical, will afford information *either with regard to X or with regard to x*. For if both X and x appear as alternatives of the predicate or as determinants of the subject of a universal affirmative, then the proposition will be identical.

[2] Inverse problems are discussed in chapter 12.

[3] "Boole," says Jevons, "first put forth the problem of Logical Science in its complete generality:—*Given certain logical premisses or conditions, to determine the description of any class of objects under those conditions.* Such was the general problem of which the ancient logic had solved but a few isolated cases—the nineteen moods of the syllogism,

A general method of solution is as follows:—

Let X be the term concerning which information is desired. Find what information each proposition gives separately with regard to X, thus obtaining a new set of propositions of the form

All X is P_1 or P_2 ... or P_n.

This is always possible by the aid of the rules for obversion and contraposition given in chapters 3 and 5. We have in the preceding section discriminated the cases in which any given proposition really affords information with regard to X. Those propositions which do not do so may of course be left altogether out of account.

Next combine the propositions thus obtained in the manner indicated in section 331. This will give the desired solution.

If information is desired with regard to several terms, it will be convenient to bring all the propositions to the form,—

Everything is P_1 or P_2 ... or P_n;

and to combine them at once, getting in a single proposition a summation of all the information given by the separate propositions taken together. From this we may immediately obtain all that is known concerning X by leaving out every alternative that contains x, all that is known concerning Y by leaving out every alternative that contains y, and so on.

the sorites, the dilemma, the disjunctive syllogism, and a few other forms. Boole shewed incontestably that it was possible, by the aid of a system of mathematical signs, to deduce the conclusions of all these ancient modes of reasoning, and an indefinite number of other conclusions. Any conclusion, in short, that it was possible to deduce from any set of premisses or conditions, however numerous and complicated, could be calculated by his method" (*Philosophical Transactions*, 1870). Compare also *Principles of Science*, Chapter 6, § 5.

The method may be varied by bringing the propositions to the form,— *No X is* Q_1 *or* Q_2 ... *or* Q_n,
or to the form,— *Nothing is* Q_1 *or* Q_2 ... *or* Q_n,
then combining as in section 332, and (if an affirmative solution is desired) finally obverting the result. It will depend on the form of the original propositions whether this variation is desirable[1].

In an equational system of Symbolic Logic, a solution with regard to any term X generally involves a partial solution with regard to x also. In employing the above methods, x must be found separately. It may be added that the complete solution for X and that for x sum up between them the whole of the information given by the original data; in other words, they are, taken together, equivalent to the given premisses. [Having determined that *All X is P* and that *All x is q*, we may by contraposition bring the latter proposition to the form *All Q is X*, when it may be found that P and Q have some alternatives in common. These alternatives are the terms which (in Boole's system) are taken in their whole extent in the equation giving X; and the solution thus obtained is closely analogous to that given by any equational system of Symbolic Logic.]

The following may be taken as a simple example of the first of the above methods. It is adapted from Boole (*Laws of Thought*, p. 118).

"Given 1st, that wherever the properties A and B are combined, either the property C, or the property D, is present also; but they are not jointly present; 2nd, that

[1] This second method is analogous to that which is usually employed by Dr Venn in his *Symbolic Logic*. Both methods bear a certain resemblance to Jevons's Indirect Method; but neither of them is identical with that method.

wherever the properties B and C are combined, the properties A and D are either both present with them, or both absent; 3rd, that wherever the properties A and B are both absent, the properties C and D are both absent also; and *vice versa*, where the properties C and D are both absent, A and B are both absent also. Find what can be inferred from the presence of A with regard to the presence or absence of B, C, and D".

The premisses may be written,—(1) *All AB is Cd or cD*; (2) *All BC is AD or ad*; (3) *All ab is cd*; (4) *All cd is ab*.

Then, from (1), *All A is b or Cd or cD*;
and from (2), *All A is b or c or D*;
therefore (by combining these), *All A is b or cD*;
(3) gives no information regarding A (see the preceding section); but by (4), *All A is C or D*;
therefore, *All A is bC or bD or cD*;
and, since *bD* is by development either *bCD* or *bcD*, this becomes

All A is bC or cD[1].

[This solves the problem as set. Proceeding also to determine *a*, we find that (1) gives no information with regard to this term; but by (2), *All a is b or c or d*; and by (3), *All a is B or cd*; therefore, *All a is Bc or Bd or cd*. Again by (4), *All a is b or C or D*. Therefore, *All a is BCd or BcD or bcd*; and by contraposition, *Whatever is Bcd or bC or bD or CD is A*[2].]

[1] We shall find that eliminating D by the rule given in the next section but one, we have *All A is bC or c*. This is Boole's result.

[2] Taking into account the result arrived at above with regard to A, it will be seen that this may be resolved into *Whatever is bC or bD is A* and *Nothing is BCD or Bcd*. These two propositions taken together with the solution for A are equivalent to the original premisses.

340. The Problem of Elimination.

By *elimination* in Logic is meant the omission of certain elements from a proposition or set of propositions with the object of expressing more directly and concisely the connexion between the elements which remain. We have an example of the process in the ordinary categorical syllogism, where the so-called *middle term* is eliminated. Thus, given the premisses *All M is P*, *All S is M*, we may infer *All S is MP*; but if we desire to know the relation between *S* and *P* independently of *M* we are content with the less precise but sufficient statement *All S is P*; in other words, we eliminate *M*.

Elimination has by some been considered to be absolutely essential to logical reasoning. It is not however necessarily involved either in the process of contraposition or in the process discussed in the preceding section; and from the point of view of the formal logician it is difficult to see how the name of inference can be denied to these processes. We must therefore refuse to regard elimination as of the essence of reasoning, although it may usually be involved therein[1].

341. Elimination from Universal Affirmatives.

Any universal affirmative proposition (or, by combination, any set of universal affirmative propositions) involving the term *X* and its contradictory *x* may by contraposition be reduced to the form *Everything is PX or Qx or R*, where *P*, *Q*, *R* are themselves simple or complex terms. We must however here admit the possibility of *P*, *Q*, *R* being of the forms *A or a*, *Aa*. These are equivalent respectively to *the entire universe of discourse* and to *nothing*. Thus, if

[1] Compare sections 122, 123.

P is of the form *A or a*, and Q is of the form *Aa*, our proposition will more naturally be written *Everything is X or R*; if Q is of the form *A or a* and R of the form *Aa*, it will more naturally be written *Everything is PX or x*.

Applying the rule given in section 304 that a determinant may at any time be omitted from an undistributed term we may eliminate X (and x) from the proposition *Everything is PX or Qx or R* by simply omitting them, and reducing the proposition to the form *Everything is P or Q or R*. If however either P or Q is of the form *A or a* (*i.e.* if either P or Q is equivalent to the entire universe of discourse), the resulting proposition will not afford any real information, since it is always true *a priori* that *Everything is A or a or &c.* Thus we are practically unable to eliminate X from such a proposition as *All A is X or BC*.

The following may be given as an example of elimination from universal affirmatives.

Let it be required to eliminate X (and x) from the propositions—*All P is XQ or xR, Whatever is X or R is p or XQR*. Combining these propositions, we have *Everything is XQR or p*; therefore, by elimination, *Everything is QR or p*, *i.e.*, *All P is QR*. It will be observed that we cannot eliminate P from the above propositions.

342. Elimination from Universal Negatives.

Any universal negative proposition (or, by combination, any set of universal negative propositions) containing the term X and its contradictory x may by conversion be reduced to the form *Nothing is PX or Qx or R*, where P, Q, R are interpreted as in the preceding section. Here we might, in accordance with the rule given in section 304, simply omit the alternatives *PX*, *Qx*, leaving us with the

proposition *Nothing is R*. This however is but part of the information obtainable by the elimination of *X*. We have also *No X is P*, and *No Q is x*, *i.e.*, *All Q is X*; whence by a syllogism in *Celarent* we may infer *No Q is P*. The full result of the elimination is therefore given by the proposition *Nothing is PQ or R*[1].

The following is an example: Let it be required to eliminate *X* from the propositions,—*No P is Xq or xr*, *No X or R is xP or Pq or Pr*. Combining these propositions we have *Nothing is XPq or XPr or xP or PqR*; therefore, by elimination in accordance with the above rule, *Nothing is Pq or Pr*, *i.e.*, *No P is q or r*.

[1] Compare Miss Christine Ladd's Essay on *The Algebra of Logic* (*Studies in Logic by Members of the Johns Hopkins University*). The same conclusion may be deduced by obversion from the result obtained in the preceding section. *Nothing is PX or Qx or R* becomes by obversion *Everything is prX or qrx or pqr*. Therefore, by elimination of *X*, *Everything is pr or qr;* and this proposition becomes by obversion, *Nothing is PQ or R*.

Another method by which the same result may be obtained is as follows: By developing the first alternative with reference to *Q* and the second with reference to *P*, *Nothing is PX or Qx or R* becomes *Nothing is PQX or PqX or PQx or pQx* or *R*. But *PQX or PQx* is reducible to *PQ*, and omitting *PqX* and *pQx*, we have *Nothing is PQ or R*.

It is interesting to observe that the above rule for elimination from negatives is equivalent to Boole's famous rule for elimination. In order to eliminate X from the equation $F(X) = 0$, he gives the formula $F(1)\,F(0) = 0$. Now any equation containing X can be brought to the form $AX + Bx + C = 0$, where A, B, C are independent of X. Applying Boole's rule we have $(A + C)\,(B + C) = 0$, *i.e.*, $AB + C = 0$; and this is precisely equivalent to the rule given in the text.

[Dr Venn (*Symbolic Logic*, p. 304) incidentally indicates how Boole's rule for elimination may be obtained by means of a syllogism (as above); yet in a note on the same page he seems to deny the possibility of such a transformation.]

343. Elimination from Particular Affirmatives.

Any particular affirmative proposition involving the term X may by conversion be reduced to the form *Something is either PX or Qx or R*, where P, Q, R are interpreted as in section 341. We may here immediately apply the rule given in section 304 that a determinant may at any time be omitted from an undistributed term; and the result of eliminating X is accordingly *Something is either P or Q or R*[1].

344. Elimination from Particular Negatives.

Any particular negative proposition involving the term X may by contraposition be reduced to the form *Something is not either PX or Qx or R*. By the development of the first alternative with reference to Q and that of the second alternative with reference to P, this proposition becomes *Something is not either PQX or PqX or PQx or pQx or R*. But *PQX or PQx* is by section 291 rule (3) reducible to PQ, and the alternatives *PqX*, *pQx* may by the rule given in section 304 be omitted. Hence we get the proposition *Something is not either PQ or R*, from which X has been eliminated[2].

345. In general if a term X is eliminated from several isolated propositions the combined results give

[1] Thus the rule for elimination from particular affirmatives is practically identical with the rule for elimination from universal affirmatives.

[2] Thus the rule for elimination from particular negatives is practically identical with the rule for elimination from universal negatives. The same rule may be obtained by obversion from the result obtained in the preceding section. *Something is not either PX or Qx or R;* therefore, *Something is either prX or qrx;* therefore, *Something is either pr or qr;* therefore, *Something is not either PQ or R.*

less information than is afforded by first combining the given propositions and then effecting the required elimination.

Schröder (*Der Operationskreis des Logikkalkuls*, p. 23) points out that first to eliminate and then combine is not the same thing as first to combine and then eliminate. There are indeed many cases in which we cannot eliminate at all unless we first combine the given propositions. This is of course obvious in syllogisms; and we have a similar case if we take the premisses *Everything is A or X*, *Everything is B or x*. We cannot eliminate X from either of these propositions taken by itself, since in each of them X (or x) appears as an isolated alternative. But by combination we have *Everything is AB or Ax or BX*; and this by the elimination of X becomes *Everything is A or B*[1].

There are other cases in which elimination from the separate propositions is possible, but where this order of procedure leads to a weakened conclusion. Take the propositions *Everything is AX or Bx*, *Everything is CX or Dx*. By first eliminating X and then combining, we have *Everything is AC or AD or BC or BD*. But by first combining and then eliminating X, our conclusion becomes *Everything is AC or BD* which obviously gives more information than is afforded by the previous conclusion.

EXERCISES.

346. Say by inspection, which of the following propositions give information concerning A, aB, b, bCd, respectively:

[1] Working with negatives we get the same result. Taking the propositions *Nothing is ax*, *Nothing is bX*, separately, we cannot eliminate X from either of them. But combining them in the proposition *Nothing is ax or bX*, we are able to infer *Nothing is ab*.

All Ab is bCd or c; All bd is A or bC or abc; Whatever is a or B is c or D; Whatever is Ab or bc is bE or cE or e; All X is AX or ab or Bc or Cd.

347. Eliminate *X* and *Y* from the following propositions: *All aX is BcY or bcy; No AX is BY; All AB is Y; No ABCD is xY.* Shew also that it follows from these propositions that *All XY is Ab or aBc.*

CHAPTER VIII.

PROBLEMS INVOLVING THREE TERMS.

348. Given that *everything is either Q or R*, and that *all R is Q, unless it is not P*, prove that *all P is Q*.

The premisses may be written,—(1) *r is Q*, (2) *PR is Q*.

By (1), *Pr is Q*, and by (2), *PR is Q*; but *P is Pr or PR*; therefore, *P is Q*.

349. *Where A is present, B and C are either both present at once or absent at once; and where C is present, A is present.* Describe the class *not-B* under these conditions. [Jevons, *Studies*, p. 204.]

The premisses are,—(1) *A is BC or bc*, (2) *C is A*.

By (1), *b is a or c*, and by (2), *b is A or c*; therefore, *b is c*.

The solution is, therefore, *Where B is absent, C also is absent.*

350. (1) Wherever there is smoke there is also fire or light; (2) Wherever there is light and smoke

there is also fire; (3) There is no fire without either smoke or light.

Given the truth of the above propositions, what is all that you can infer with regard to (i) circumstances where there is smoke; (ii) circumstances where there is not smoke; (iii) circumstances where there is not light? [W.]

Let A = circumstances where there is smoke, B = circumstances where there is light, C = circumstances where there is fire.

The premisses are,—(1) *A is B or C*, (2) *AB is C*, (3) *C is A or B*.

(1) and (2) yield *A is C*.

By (3), *a is B or c*.

By (1) and (3), *b is a or C, and also A or c*; therefore, *b is AC or ac*.

We have then,—

(i) Where there is smoke, there is fire;

(ii) Where there is not smoke, there is either light or there is no fire;

(iii) Where there is not light, there is either both fire and smoke or neither fire nor smoke.

351. Shew what may be inferred as a possible description of warm-blooded vertebrates from the following, and state whether any of the information there given is superfluous for the purpose:—(1) All vertebrates may be divided into warm-blooded, and cold-blooded, and all produce their young in but one of two ways, *i.e.*, are either viviparous or oviparous; (2) No feathered vertebrate is both viviparous and

warm-blooded; (3) No oviparous vertebrate that is cold-blooded has feathers; (4) Every viviparous vertebrate is either feathered or warm-blooded. [L.]

Taking vertebrate as the universe of discourse, let P = warm-blooded, Q = viviparous, R = feathered. Then by (1), p = cold-blooded, q = oviparous. The remaining premisses are as follows: (2) *No R is PQ*, (3) *No pq is R*, (4) *All Q is P or R*. Required a description of P.

By the obversion of (2), *All R is p or q*; therefore, *All P is q or r*. (3) and (4) give no information with regard to P, and are therefore superfluous for our purpose. Hence the required description is that *All warm-blooded vertebrates are either oviparous or featherless.*

Exercises.

352. Given (1) P is QR, (2) p is qr; shew that (3) Q is PR, (4) R is PQ.

353. Given (1) R is P or pq, (2) q is R or Pr, (3) qR is P; shew that p is Qr.

354. Shew the equivalence between the following sets of propositions:—(1) *a is BC; b is AC; C is Ab or aB*; (2) *a is BC; B is Ac or aC; c is AB*; (3) *A is Bc or bC; b is AC; c is AB.*

355. *A is Bc or bC; b is AC; c is AB.* Shew that all the information given by the combination of these propositions is also given by the propositions: *A is b or c; b is A; c is AB or ab.*

CHAPTER IX.

PROBLEMS INVOLVING FOUR TERMS.

356. Suppose that an analysis of the properties of a particular class of substances has led to the following general conclusions, viz.:

1st, That wherever the properties *A* and *B* are combined, either the property *C*, or the property *D*, is present also: but they are not jointly present.

2nd, That wherever the properties *B* and *C* are combined, the properties *A* and *D* are either both present with them, or both absent.

3rd, That wherever the properties *A* and *B* are both absent, the properties *C* and *D* are both absent also; and *vice versa*, where the properties *C* and *D* are both absent, *A* and *B* are both absent also.

Shew that wherever the property *A* is present, the properties *B* and *C* are not both present; also that wherever *B* is absent while *C* is present, *A* is present.

[Boole, *Laws of Thought*, pp. 118—120; compare also Venn, *Symbolic Logic*, pp. 276—278.]

One solution has already been given in section 339. We may also proceed as follows. The premisses are:

All AB is Cd or cD, (i)
All BC is AD or ad, (ii)
All ab is cd, (iii)
All cd is ab. (iv)

By (i), *No AB is CD*, therefore, *No A is BCD*. (1)
By (ii), *No BC is Ad*, therefore, *No A is BCd*. (2)

Combining (1) and (2), it follows immediately that *No A is BC*.

Boole also shews that *All bC is A*. This is a partial contrapositive of (iii). We have so far not required to make use of (iv) at all.

357. Given the same premisses as in the preceding section, prove that:—

(1) Wherever the property *C* is found, either the property *A* or the property *B* will be found with it, but not both of them together.

(2) If the property *B* is absent, either *A* and *C* will be jointly present, or *C* will be absent.

(3) If *A* and *C* are jointly present, *B* will be absent. [Boole, *Laws of Thought*, p. 129.]

First, By (i), *All C is a or b or d;* by (ii), *All C is a or b or D;* therefore, *All C is a or b*.

Also, by (iii), *All C is A or B*;
therefore, *All C is Ab or aB*. (1)

Secondly, By (iii), *All b is A or c*,
therefore, by section 291, rule (6), *All b is AC or c*. (2)

Thirdly, from (1) it follows immediately that

All AC is b. (3)

The given premisses may all be summed up in the proposition: *Everything is AbC or AbD or aBCd or abcd or BcD.* From this, the above special results and others follow immediately.

358. It is known of certain things that (1) where the quality *A* is, *B* is not; (2) where *B* is, and only where *B* is, *C* and *D* are. Derive from these conditions a description of the class of things in which *A* is not present, but *C* is. [Jevons, *Studies*, p. 200.]

The premisses are: (1) *All A is b*; (2) *All B is CD*; (3) *All CD is B.*

No information regarding *aC* is given by (1). But by (2), *All aC is b or D*; and by (3), *All aC is B or d.*

Therefore, *All aC is BD or bd.*

359. Taking the same premisses as in the previous section, draw descriptions of the classes *Ac*, *ab*, and *cD*. [Jevons, *Studies*, p. 244.]

By (1), *Everything is a or b*; and by (2), *Everything is b or CD*. Therefore, *Everything is aCD or b*; and by (3), *Everything is B or c or d.* Therefore, *Everything is aBCD or bc or bd.*

Hence we infer immediately *All Ac is b*, *All ab is c or d*, *All cD is b.*

360. Every *A* is one only of the two *B* or *C*; *D* is both *B* and *C* except when *B* is *E* and then it is neither; therefore no *A* is *D*.

[De Morgan, *Formal Logic*, p. 124.]

This example, originally given by De Morgan, (using however different letters), and taken by Professor Jevons to illustrate his symbolic method, (*Principles of Science*, chapter 6, § 10; *Studies in Deductive Logic*, p. 203), is chosen by Professor Croom Robertson to shew that "the most complex problems can, as special logical questions, be more easily and shortly dealt with upon the principles and with the recognised methods of the traditional logic" than by Jevons's system.

"The mention of *E as E* has no bearing on the special conclusion *A* is not *D* and may be dropt, while the implication is kept in view; otherwise, for simplification, let *BC* stand for 'both *B* and *C*,' and *bc* for 'neither *B* nor *C*.' The premisses then are,—

(1) *D* is either *BC* or *bc*,

(2) *A* is neither *BC* nor *bc*,

which is a well-recognised form of Dilemma with the conclusion *A* is not *D*. Or, by expressing (2) as *A* is not either *BC* or *bc*, the conclusion may be got in *Camestres*. If it be objected that we have here by the traditional processes got only a special conclusion, it is a sufficient reply that any conclusion by itself must be special. What other conclusion from these premisses is the common logic powerless to obtain?" (*Mind*, 1876, p. 222.)

The solution is also obtainable as follows,—

By the first premiss, *A is Bc or bC*, and by the second, *A is BC or bc or d*;

therefore, *A is Bcd or bCd*,

therefore, *A is d*.

361. There is a certain class of things from which *A* picks out the '*X* that is *Z*, and the *Y* that is

not *Z*,' and *B* picks out from the remainder 'the *Z* which is *Y* and the *X* that is not *Y*.' It is then found that nothing is left but the class '*Z* which is not *X*.' The whole of this class is however left. What can be determined about the class originally?

[Venn, *Symbolic Logic*, pp. 267, 8.]

The chief difficulty in this problem consists in the accurate statement of the premisses. Call the original class *W*. We then have,—*All W is XZ or Yz or YZ or Xy or xZ,*

i.e., All W is X or Y or Z. (1)

All xZ is W. (2)

No xZ is WXZ or WYz or WYZ or WXy,

i.e., (leaving out such part of this statement as is merely identical). *No xZ is WYZ.* (3)

We may now proceed as follows:—By (1) *All W is X or Y or Z*; and by (3), *All W is X or y or z.* Therefore, *All W is X or Yz or yZ.*

(2) affords no information regarding the class *W*, except that everything that is *Z* but not *X* is contained within it.

362. At a certain town where an examination is held, it is known that (1) Every candidate is either a junior who does not take Latin or a Senior who takes Composition; (2) Every junior candidate takes either Latin or Composition; (3) All candidates who take Composition also take Latin and are juniors. Shew that if this be so there can be no candidates there.

[Venn, *Symbolic Logic*, pp. 270, 1.]

Let *X* = candidate; *A* = junior, so that *a* = senior; *B* = taking Latin; *C* = taking Composition.

Our premisses are as follows: (1) *All X is Ab or aC*; (2) *All XA is B or C*; (3) *All XC is AB.*

(2) and (3) give *All XA is B*;
therefore, *No X is Ab*:
also by (3), *No X is aC.*

It therefore follows from (1) that there can be no such thing as *X.*

363. A given class is made up of those who are not either male guardians, or female ratepayers, or lodgers who are neither guardians nor ratepayers. How can we simplify the description of this class if we know that all guardians are ratepayers, that every person who is not a lodger is either a guardian or a ratepayer, and that all male ratepayers are guardians? [V.]

Let *X* = the given class; *A* = male; *B* = guardian; *C* = ratepayer; *D* = lodger.

Then *X* is made of those who are not either *AB* or *aC* or *bcD*; that is, *X* is made up of those who are

aBc or *AbC* or *acd* or *Abd* or *bcd.*

But, by development, *acd is aBcd or abcd*, and *Abd is AbCd or Abcd*;

therefore, *X is either aBc or AbC or bcd.*

Now we are told that (1) *All B is C*; (2) *All d is B or C*; (3) *All AC is B.*

From (1), it follows that there is no *aBc*; from (3) that there is no *AbC*; from (2) that there is no *bcd.*

It therefore follows that the given class is itself non-existent.

364. In a certain town the old buildings are either ecclesiastical and built entirely of stone, or, if not ecclesiastical are built entirely of brick; the brick-and-stone buildings are all modern as well as secular or they are neither; but there are no modern buildings at once secular and built entirely of stone. State what assumptions you make in interpreting the above, and determine (*a*) in what cases brick may be found in the buildings of this town and in what cases it cannot be, (*b*) what old buildings it would be useless to look for. [L.]

Let A = old, B = ecclesiastical, P = containing brick, Q = containing stone. Then assuming that old and modern, ecclesiastical and secular, are respectively contradictories, a = modern, b = secular.

The premisses are (1) *All A is BpQ or bPq*, (2) *All PQ is AB or ab*, (3) *No a is bpQ*.

We may interpret the question as follows: (*a*) after eliminating Q, determine P both positively and negatively; (*b*) determine A negatively.

(3) gives no information with regard to P[1]; but by (1) *All P is a or bq*, and by (2) *All P is AB or ab or q*; therefore, *All P is ab or aq or bq*. Eliminating Q, *All P is a or b*. Therefore, *Brick is to be found only in secular or modern buildings;* and by obversion, *No brick is to be found in old ecclesiastical buildings.*

(3) gives no information with regard to A[1], and (2) adds no information to that contained in (1)[2]. Hence the second part of the question is answered by simply obverting (1).

[1] See section 338.

[2] It merely tells us that *All A is B or p or q*.

No old buildings are ecclesiastical and built of brick, or ecclesiastical and not built of stone, or secular and built of stone, or secular and not built of brick.

EXERCISES.

365. Given that everything that is Q but not S is either both P and R or neither P nor R and that neither R nor S is both P and Q, shew that no P is Q.

366. Where C is present, A, B, and D are all present; where D is present, A, B, and C are either all three present or all three absent. Shew that when either A or B is present, C and D are either both present or both absent. How much of the given information is superfluous so far as the desired conclusion is concerned?

367. Given that no X that is Z is either Y or W, and that no Y that is W is either X or Z; what are the least additional data required in order to secure that no X is Y and that no Z is W? [V.]

368. If thriftlessness and poverty are inseparable, and virtue and misery are incompatible, and if thrift be a virtue, can any relation be proved to exist between misery and poverty? If moreover all thriftless people are either virtuous or not miserable, what follows? [V.]

369. At a certain examination, all the candidates who were entered for Latin were also entered for either Greek, French, or German, but not for more than one of these languages; all the candidates who were not entered for German were entered for two at least of the other languages; no candidate who was entered for both Greek and French

was entered for German, but all candidates who were entered for neither Greek nor French were entered for Latin. Shew that all the candidates were entered for two of the four languages but none for more than two.

370. Shew that the two following sets of propositions are equivalent to one another:

(1) *AB* is *D*; *ab* is *cd*; *c* is *ABD* or *abd*; *D* is *AB*.
(2) *ABC* is *D*; *abd* is *c*; *c* is *AD* or *abd*; *D* is *AB* or *ac* or *Bc*.

371. Shew that the following sets of propositions are equivalent to one another:—

(1) *a* is *b* or *c*; *b* is *aCd*; *c* is *aB*; *D* is *c*.
(2) *A* is *BC*; *b* is *aC*; *C* is *ABd* or *abd*.
(3) *A* is *B*; *B* is *A* or *c*; *c* is *aB*; *D* is *c*.
(4) *b* is *aC*; *A* is *C*; *C* is *d*; *aC* is *b*.
(5) *c* is *aB*; *D* is *aB*; *A* is *B*; *aB* is *c*.
(6) *A* is *BC*; *BC* is *A*; *D* is *Bc*; *b* is *C*.

372. Shew that a certain set of four properties must be found somewhere together, if the following facts are known: "Everything that has the first property or is without the last has the two others; and if everything that has both the first and last has one or other but not both of the two others, then something that has the first two must be without the last two." [J.]

CHAPTER X.

PROBLEMS INVOLVING FIVE TERMS.

373. Let the observation of a class of natural productions be supposed to have led to the following general results.

1st. That in whichsoever of these productions the properties A and C are missing, the property E is found, together with one of the properties B and D, but not with both.

2nd. That wherever the properties A and D are found while E is missing, the properties B and C will either both be found, or both be missing.

3rd. That wherever the property A is found in conjunction with either B or E, or both of them, there either the property C or the property D will be found, but not both of them. And conversely, wherever the property C or D is found singly, there the property A will be found in conjunction with either B or E, or both of them.

Shew that it follows that *In whatever substances the property A is found, there will also be found either*

the property C or the property D, but not both, or else the properties B, C, and D will all be wanting. And conversely, *Where either the property C or the property D is found singly, or the properties B, C, and D are together missing, there the property A will be found.* Shew also that *If the property A is absent and C present, D is present.*

[Boole, *Laws of Thought*, pp. 146—148. Venn, *Symbolic Logic*, pp. 280, 281. *Johns Hopkins Studies in Logic*, pp. 57, 58, 82, 83.]

The premisses are as follows:—

1st, *All ac is BdE or bDE*; (i)
2nd, *All ADe is BC or bc*; (ii)
3rd, *Whatever is AB or AE is Cd or cD*; (iii)
Whatever is Cd or cD is AB or AE; (iv)

We are required to prove:—

All A is Cd or cD or bcd; (α)
All Cd is A; (β)
All cD is A; (γ)
All bcd is A; (δ)
All aC is D. (ε)

First, By (iii), *A is Cd or cD or be.* (v)
By (ii), *Abe is c or d*;
by (iv), *Abe is CD or cd*;
therefore, *Abe is cd.*

Hence, by (v), *A is Cd or cD or bcd.* (α)

Secondly, (β) and (γ) follow immediately from (iv).

Thirdly, from (i), we have directly, *No ac is bd*;
therefore (by conversion), *No bcd is a*;
therefore, *All bcd is A.* (δ)

Lastly, by (iv), *Cd is A*;
therefore, by contraposition, *aC is D*. (ϵ)

We may obtain a complete solution so far as *A* is concerned as follows:

(i) gives no information whatever with regard to *A*[1].

But by (ii), *A is BC or bc or d or E*;
by (iii), *A is be or Cd or cD*;
therefore, *A is Cd or cDE or bcD or bce or bde*;
by (iv), *A is B or E or CD or cd*;
therefore, *A is cDE or bcde or BCd or CdE.*

This includes the partial solution with regard to *A*,—

A is Cd or cD or bcd.

Boole contents himself with this because he has started with the intention of eliminating *E* from his conclusion.

We may now solve for *a*. (ii) and (iii) give no information with regard to this term. But by (i), *All a is BdE or bDE or C*; and by (iv), *All a is CD or cd.* Therefore, *All a is BcdE or CD.* And this yields by contraposition,

Whatever is bc or Cd or cD or ce is A.

374. Given the same premisses as in the preceding section, shew that,—

1st. *If the property B be present in one of the productions, either the properties A, C, and D are all absent, or some one alone of them is absent.* And conversely, *if they are all absent it may be concluded that the property B is present.*

[1] Since *a* appears as a determinant of the subject. See section 338.

2nd. *If A and C are both present or both absent, D will be absent, quite independently of the presence or absence of B.* [Boole, *Laws of Thought*, p. 149.]

We may proceed here by combining all the given premisses in the manner indicated in section 339. From the result thus obtained the above conclusions as well as those contained in the preceding section will immediately follow.

By (iii), *Everything is a or be or Cd or cD*;
and by (iv), *Everything is AB or AE or CD or cd*;
therefore, *Everything is ABCd or ABcD or ACdE or AcDE or aCD or acd or bCDe or bcde*;
therefore by (i), *Everything is ABCd or ABcD or Abcde or ACdE or AcDE or aBcdE or aCD or bCDe*;
therefore by (ii), *Everything is ABCd or Abcde or ACdE or AcDE or aBcdE or aCD.* (v).

Hence, *All B is ACd or AcDE or acdE or aCD*;
All acd is BE;
All AC is Bd or dE;
All ac is BdE.

Eliminating *E* from each of the above we have the results arrived at by Boole.

Eliminating both *A* and *E* from (v) we have
Everything is BCd or bcd or Cd or cD or Bcd or CD;
i.e., *Everything is C or D or cd*, which is an identity. This is equivalent to Boole's conclusion that "there is no independent relation among the properties *B*, *C*, and *D*."

Any further results that may be desired are obtainable immediately from (v).

375. Given $XY = A$, $YZ = C$, find XZ in terms of A and C. [Venn, *Symbolic Logic*, pp. 279, 310—312. *Johns Hopkins Studies in Logic*, pp. 53, 54].

The premisses may be written as follows:

Everything is AXY or ax or ay; (1)

Everything is CYZ or cy or cz. (2)

By (1), *All XZ is AY or ay*;

and by (2), *All XZ is CY or cy*;

therefore, *All XZ is ACY or acy.*

Eliminating *Y*, *All XZ is AC or ac.*

This solves the problem as set. But in order to get a complete solution equivalent to that which would be obtained by Boole, the following may be added:

Solving as above for *x or z*, and eliminating *Y*, we have *All that is either x or z is AcXz or aCxZ or ac.*

Whence, by contraposition, *Whatever is AC or Ax or AZ or CX or Cz is XZ.*

In other words, *Whatever is AC or AZ or CX is XZ*; and *Nothing is Ax or Cz.*

376. Shew the equivalence between the three following systems of propositions: (1) *All Ab is cd*; *All aB is Ce*; *All D is E*; (2) *All A is B or c or D*; *All BE is A*; *All Be is Ad or Cd*; *All bD is aE*; (3) *Whatever is A or e is B or d*; *All a is bE or bd or BCe*; *All bC is a*; *All D is E.*

By obversion, the first set of propositions become

No Ab is C or D;

No aB is c or E;

No D is e;

and these propositions are combined in the statement

Nothing is either AbC or AbD or aBc or aBE or De. (1)

By obverting and combining the second set of propositions, we have *Nothing is AbCd or aBE or aBce or BDe or AbD or bDe.* (2)

But *AbCd or AbD* is equivalent to *AbC or AbD*; *aBE or aBce* to *aBE or aBc*; *BDe or bDe* to *De.* Hence (1) and (2) are equivalent.

Again, by obverting and combining the third set of propositions, we have *Nothing is AbD or bDe or aBc or aBE or abDe or acDe or AbC or De.* (3)

But since *bDe*, *abDe*, *acDe* are all subdivisions of *De*, (3) immediately resolves itself into (1).

377. Given (i) *All Pqr is ST*; (ii) *Q and R are always present or absent together*; (iii) *All QRS is PT or pt*; (iv) *All QRs is Pt*: (v) *All pqrS is T*; then it follows that (1) *All Pq is rST*; (2) *All Ps is QRt*; (3) *All pQ is RSt*; (4) *All pT is qr*; (5) *All Qs is PRt*; (6) *All QT is PRS*; (7) *All qS is rT*; (8) *All qs is pr*; (9) *All qt is prs*; (10) *All sT is pqr.*

By (i), *Everything is p or Q or R or ST*;

by (ii), *Everything is QR or qr*;

therefore, *Everything is QR or pqr or qrST*;

by (iii), *Everything is q or r or s or PT or pt*;

therefore, *Everything is pqr or qrST or QRs or PQRT or pQRt*;

by (iv), *Everything is q or r or S or Pt*;

therefore, *Everything is pqr or qrST or PQRst or PQRST or pQRSt*;

by (v), *Everything is s or P or Q or R or T*;

therefore, *Everything is pqrs or pqrT or qrST or PQRst or PQRST or pQRSt.*

The desired results follow from this immediately.

Exercises.

378. Given that,—

ad is *b* or *e*;
b is *AcD* or *ad* or *cde*;
c is *AbD* or *bde*;
e is *aBCd* or *bcd*;

shew that, *A* is *BCE* or *bcDE* or *bcde*;
a is *BCDE* or *BCde* or *bCdE* or *bcde*;
B is *ACE* or *aCde* or *CDE*;
C is *ABE* or *aBde* or *abdE* or *BDE*.

379. Given that,—

A is *Bc* or *bC*;
B is *DE* or *de*;
C is *De*;

shew that,—

A is *BcDE* or *Bcde* or *bCDe*;
BcD is *E*;
abd is *c*;
cd is *Be* or *ab*;
bCD is *e*.

[Jevons, *Pure Logic*, p. 66.]

380. At a certain examination it was observed that,—(i) all candidates who were entered for Greek were entered also for Latin; (ii) all candidates who were not entered for Greek were entered for English and French, and if they were also entered for Latin, they were entered for German;

(iii) all candidates who were entered for Latin and Greek while they were not entered for English were not entered for French; (iv) all candidates who were entered for Latin and Greek while they were not entered for French were not entered for German.

Shew that,—(1) Every candidate was either entered for English or else for both Latin and Greek; (2) Every candidate was entered either for Latin or else for both English and French; (3) All candidates entered for French were entered also for English; (4) All candidates entered for German were also entered for both English and French; (5) If a candidate was not entered for English, he was not entered for either French or German, but he was entered for both Latin and Greek; (6) If a candidate was not entered for French, he was entered for both Latin and Greek but not for German; (7) If a candidate was entered for Latin and also either entered for German or not entered for Greek, he was entered for English, French, and German; (8) If a candidate was entered both for Greek and German, he was also entered for English, Latin, and French; (9) If a candidate was entered neither for Greek nor German, he was entered for English and French but not for Latin; (10) Every candidate was entered for at least two languages; and no candidate who was entered for only two languages was entered for German.

381. Shew the equivalence between the two following sets of propositions:—

(1) A is BC or BE or CE or D;
B is $ACDE$ or $ACde$ or cdE;
C is AB or AE or aD;
D is $ABCE$ or Ace or aC;
E is AC or aCD or Bc.

(2) *a* is *BcdE* or *bcde* or *bD*;
b is *a* or *ce* or *dE*;
c is *AbDe* or *abde* or *BdE*;
d is *abce* or *BcE* or *Be* or *bE*;
e is *ab* or *bc* or *d*.

382. Find which of the following propositions may be omitted without any limitation of the information given,—All *Pq* is *rST*; All *Pr* is *qST*; All *Ps* is *QRt*: All *Pt* is *QRs*; No *pT* is *Q* or *R*; No *Q* is *r*; All *Qs* is *PRt*; No *qS* is *R* or *t*; No *qT* is *Ps* or *R*; All *R* is *Q*; All *Rs* is *PQt*; All *rS* is *qT*; All *rT* is *PqS* or *pq*; All *st* is *PQR* or *pqr*.

CHAPTER XI.

PROBLEMS INVOLVING SIX OR MORE TERMS.

383. From the premisses (1) *No Ax is cd or cy*, (2) *No BX is cde or cey*, (3) *No ab is cdx or cEx*, (4) *No A or B or C is xy*, deduce a proposition containing neither *X* nor *Y*. [*Johns Hopkins Studies*, p. 53.]

By (2), *No X is Bcde*,
and by (1) and (3), *No x is Acd or abcd or abcE*;
therefore, by section 342, *No Acd or abcd or abcE is Bcde*;
therefore, *No Acd is Be.*

It will be observed that since *Y* does not appear in the premisses, *y* can be eliminated only by omitting all the terms containing it.

384. The members of a scientific society are divided into three sections, which are denoted by *A*, *B*, *C*. Every member must join one, at least, of these sections, subject to the following conditions: (1) any one who is a member of *A* but not of *B*, of *B* but not of *C*, or of *C* but not of *A*, may deliver a lecture to the members if he has paid his subscription,

but otherwise not; (2) one who is a member of A but not of C, of C but not of A, or of B but not of A, may exhibit an experiment to the members if he has paid his subscription, but otherwise not; but (3) every member must either deliver a lecture or perform an experiment annually before the other members. Find the least addition to these rules which will compel every member to pay his subscription or forfeit his membership. [*Johns Hopkins Studies*, p. 54.]

Let A = member of section A, &c.; X = one who gives a lecture; Y = one who performs an experiment; Z = one who has paid his subscription.

The premisses are

(1) *Ab or aC or Bc is x or Z*;

(2) *Ac or aB or aC is y or Z*;

(3) *Every member is X or Y*;

and (4) *Every member is A or B or C.*

The problem is to find what is the least addition to these rules which will result in the conclusion that *Every member is Z.*

By (1), *z is either x or else* (*a or B*) *and* (*A or c*) *and* (*b or C*);

therefore, *z is x or ABC or abc.*

Similarly, by (2), *z is y or AC or abc*;

therefore, *z is xy or xAC or ABC or abc.*

By (3), *z is X or Y*;

therefore, *z is XABC or Xabc or xYAC or YABC or Yabc.*

By (4), *z is A or B or C*;

therefore, *z is XABC or xYAC or YABC*;

but *YABC is either XYABC or xYABC*;

therefore, *z is XABC or xYAC.*

Hence, we gain the desired result if we add to the premisses *No z is XABC or xYAC.* The required rule is therefore as follows: *No one who has not paid his subscription may join all three sections and deliver a lecture, nor may he join A and C and exhibit an experiment without delivering a lecture.*

385. What may be inferred independently of *X* and *Y* from the premisses: (1) *Either some A that is X is not Y, or all D is both X and Y*; (2) *Either some Y is both B and X, or all X is either not Y or C and not B?* [*Johns Hopkins Studies*, p. 85.]

The premisses may be written as follows: (1) *Either something is AXy, or everything is XY or d*; (2) *Either something is BXY, or everything is x or y or bC.*

By combining these premisses as in chapter VI, *Either something is AXy and something is BXY, or something is AXy and everything is x or y or bC, or something is BXY and everything is XY or d, or everything is bCXY or bCd or dx or dy*[1].

Therefore, eliminating *X* and *Y* (see sections 341 and 343), *Either something is A and something is B, or something is A, or something is B, or everything is bC or d*; and by combining the first three alternatives as in section 335, this becomes

Either something is A or B or everything is bC or d.

386. Six children, *A*, *B*, *C*, *D*, *E*, *F*, are required to obey the following rules: (1) on Monday and Tuesday no four can go out together; (2) on

[1] We cannot, if we are to be left with an equivalent proposition, express the first three of these alternatives in a non-compound form. See sections 333, 335.

Thursday, Friday, and Saturday, no three can stay in together; (3) on Tuesday, Wednesday, and Saturday, if *B* and *C* are together, then *A*, *B*, *E*, and *F* must be together; (4) on Monday and Saturday *B* cannot go out unless either *D*, or *A*, *C*, and *E* stay at home. *A* and *B* are first to decide what they will do, and *C* makes his decision before *D*, *E*, and *F*. Find (α) when *C* must go out, (β) when he must stay in, and (γ) when he may do as he pleases.

[*Johns Hopkins Studies*, p. 58.]

Let A = case in which A goes out, a = that in which he stays in, &c.

Then the premisses are as follows:

(1) On Monday and Tuesday,—*three at least must stay in*;

(2) On Thursday, Friday and Saturday,—*no three can stay in together*;

(3) On Tuesday, Wednesday and Saturday,—*Every case is ABEF or abef or Bc or bC*;

(4) On Monday and Saturday,—*Every case is ace or b or d.*

In order to solve the problem, we must combine the possibilities for each day, then eliminate *D*, *E*, and *F*, and find in what ways the movements of *A* and *B* determine those of *C*.

(i) On Monday,—we have *Every case is ace or b or d*, combined with the condition that three at least must stay in. One alternative therefore is *def* without further condition, and it follows that we can determine no independent relation between *A*, *B*, and *C*.

Hence *on Monday C may do as he pleases.*

(ii) On Tuesday,—we have *Every case is ABEF or abef or Bc or bC*, combined with the condition that three at least must stay in. Therefore, *Every case is abef or Bc or bC*[1]; and eliminating *D*, *E*, and *F*, *Every case is ab or Bc or bC.*

Hence it follows that *on Tuesday* (α) *if A goes out while B stays in, C must go out, and* (β) *if B goes out, C must stay in.*

(iii) On Wednesday,—*Every case is ABEF or abef or Bc or bC*; or eliminating *D*, *E*, and *F*, *Every case is AB or ab or Bc or bC.* Therefore, *Ab is C* and *aB is c.*

Hence *on Wednesday* (α) *if A goes out while B stays in, C must go out, and* (β) *if A stays in while B goes out, C must stay in.*

(iv) On Thursday and Friday,—the only condition is that no three can stay in together.

Hence *on Thursday and Friday if A and B both stay in, C must go out.*

(v) On Saturday,—*Every case is ABEF or abef or Bc or bC; also Every case is ace or b or d.* Combining these premisses, *Every case is ABdEF or abef or aBce or Bcd or bC.* But we have the further condition that no three can stay in together. Therefore, *Every case is ABdEF or ABcdEF or AbCDE or AbCDF or AbCEF or bCDEF.* Therefore eliminating *D*, *E*, and *F*, *Every case is AB or bC.*

Hence *on Saturday if B stays in C must go out.*

[1] The two alternatives *Bc* and *bC* might here be made more determinate, thus, *aBcd or aBce or aBcf or Bcde or Bcdf or Bcef* and *abCd or abCe or abCf or bCde or bCdf or bCef.* But since we know that we are going on immediately to eliminate *d*, *e*, and *f*, it is obvious, even without writing them out in full, that these more determinate expressions will at once be reduced again to *Bc* and *bC* simply.

387. Given,—

(1) *bc* is *DE* or *Df* or *hi*,
(2) *C* is *aB* or *DEFG* or *BFH*,
(3) *Bcd* is *eK* or *hi*,
(4) *Acf* is *d*,
(5) *i* is *BC* or *Cd* or *Cf* or *H*,
(6) *ABCDEFG* is *H* or *I*,
(7) *DEFGH* is *B*,
(8) *ABk* is *f* or *h*,
(9) *ADFIk* is *H*,
(10) *ADEFH* is *B* or *C* or *G* or *K*;

shew that,—*A* is *K*.

This problem may be solved in a straightforward manner by the general method formulated in section 339:—

By (1), *A is B or C or DE or Df or hi*;
By (2), *A is BFH or c or DEFG*;
therefore, *A is Bc or BFH or cDE or cDf or chi or DEFG*;
By (3), *A is b or C or D or hi or K*;
therefore, *A is BCFH or BcD or BDFH or cDE or cDf or chi or DEFG or K*;
By (4), *A is C or d or F*;
therefore, *A is BCFH or BcDF or BDFH or cDEF or cdhi or cFhi or DEFG or K*;
By (5), *A is BC or Cd or Cf or H or I*;
therefore, *A is BCDEFG or BCFH or BcDFI or BDFH or cDEFH or cDEFI or DEFGH or DEFGI or K*;
By (6), *A is b or c or d or e or f or g or H or I*;
therefore, *A is BCFH or BcDFI or BDFH or cDEFH or cDEFI or DEFGH or DEFGI or K*;

By (7), *A is B or d or e or f or g or h*;
therefore, *A is BCFH or BcDFI or BDEFGI or BDFH or cDEFgH or cDEFgI or cDEFhI or DEFGhI or K*;

By (8), *A is b or f or h or K*;
therefore, *A is BcDFhI or bcDEFgH or bcDEFgI or cDEFhI or DEFGhI or K*;

By (9), *A is d or f or H or i or K*;
therefore, *A is bcDEFgH or K*;

By (10), *A is B or C or d or e or f or G or h or K*;
therefore, *A is K.*

EXERCISES.

388. Given,—

(1) *aB* is *c* or *D*;
(2) *BE* is *DF* or *cdF*;
(3) *C* is *aB* or *BE* or *D*;
(4) *bD* is *e* or *F*;
(5) *bf* is *a* or *C* or *DE*;
(6) *bcdE* is *Af* or *aF*;
(7) *A* is *B* or *CDEf* or *cDf* or *cdE*;

it follows that,—(i) *A* is *B*; (ii) *C* is *D*; (iii) *E* is *F*.

389. Shew the equivalence between the two following sets of propositions:—(1) *AB* is *CD* or *EF*; *Cd* is *Ab* or *Ef*; *eF* is *aB* or *cD*; *ab* is *cd*; *cd* is *ef*; *ef* is *ab*; (2) *a* is *BC* or *BD* or *bcdef*; *AB* is *CDE* or *cDEF*; *e* is *abcdf* or *F*; *Abcd* is *ef*; *aBCd* is *f*; *AbCF* is *E*.

CHAPTER XII.

INVERSE PROBLEMS.

390. Nature of the Inverse Problem.

By the *Inverse Problem* I mean a certain problem so-called by Professor Jevons. Its nature will be indicated by the following extracts, which are from the *Principles of Science* and the *Studies in Deductive Logic* respectively.

"In the Indirect process of Inference we found that from certain propositions we could infallibly determine the combinations of terms agreeing with those premisses. The inductive problem is just the inverse. Having given certain combinations of terms, we need to ascertain the propositions with which they are consistent, and from which they may have proceeded. Now if the reader contemplates the following combinations,—

$$\begin{array}{ll} ABC & abC \\ aBC & abc, \end{array}$$

he will probably remember at once that they belong to the premisses $A = AB$, $B = BC$. If not, he will require a few trials before he meets with the right answer, and every trial will consist in assuming certain laws and observing whether

the deduced results agree with the data. To test the facility with which he can solve this inductive problem, let him casually strike out any of the possible combinations involving three terms, and say what laws the remaining combinations obey. Let him say, for instance, what laws are embodied in the combinations,—

$$\begin{array}{ll} ABC & aBC \\ Abc & abC. \end{array}$$

"The difficulty becomes much greater when more terms enter into the combinations. It would be no easy matter to point out the complete conditions fulfilled in the combinations,—

$$\begin{array}{l} ACe \\ aBCe \\ aBcdE \\ abCe \\ abcE. \end{array}$$

After some trouble the reader may discover that the principal laws are $C = e$, and $A = Ae$; but he would hardly discover the remaining law, namely that $BD = BDe$" (*Principles of Science*, 1st ed., vol. I., p. 144; 2nd ed., p. 125).

"The inverse problem is always tentative, and consists in inventing laws, and trying whether their results agree with those before us" (*Studies in Deductive Logic*, p. 252).

I should myself rather prefer to state the problem as follows:—

Given a single proposition of the form,—

Everything is P_1P_2...*or* Q_1Q_2...*or*...,

to find a set of propositions involving as simple relations as possible which shall be equivalent to it.

It is strictly true that the inverse problem is indeterminate, for since we may find a number of sets of propositions

which are precisely equivalent in logical force, any inverse problem may admit of a number of solutions. But I do not think that it is necessary in order to solve any inverse problem to have recourse to a series of guesses, or that the method of solution need be described as wholly tentative. In the following sections are described three distinct methods for finding a fairly satisfactory solution of any inverse problem. Since, however, a number of solutions are possible, some of which are simpler than others, the process may to a certain extent be regarded as tentative in so far as we seek to obtain the most satisfactory solution.

It is difficult to lay down any absolute standard of simplicity. What we desire to obtain however is a set of propositions the terms of which may involve conjunctive combination or disjunctive combination but *not both*[1]; and comparing two equivalent sets of such propositions, we may generally speaking regard that one as the simpler which contains the smaller number of propositions. If the number of propositions is equal, then we may count the number of terms involved in their subjects and predicates taken together, and regard that one as the simpler which involves the fewer terms.

391. A General Solution of the Inverse Problem, —*i.e.*, Given a proposition limiting us to a number of complex alternatives, to find a set of propositions involving as simple relations as possible which shall be equivalent to it.

[1] When Professor Jevons speaks of the extreme difficulty of the inverse process, he apparently has in view a resolution into a small number of propositions of this kind; and at such a resolution I have accordingly aimed in my solutions of inverse problems. I have also endeavoured as far as possible to avoid redundancies in the solutions.

The data may be written in the form,—

Everything is P or Q or S or T or &c.,

where *P*, *Q*, &c., are complex terms[1].

By contraposition[2] we may bring over one or more of these complex terms from the predicate into the subject, so that we have,—

What is not either P or S or &c. is Q or T or &c.

The selection of certain terms for transposition in this way is arbitrary (and it is here that the indeterminateness of the problem becomes apparent); but it will generally be found best to take two or three which have as much in common as possible.

What is not either P or S or &c. is Q or T or &c.

will, when the subject is written in the affirmative form, immediately resolve itself into a series of propositions, which taken together give all the information originally given[3]. If any of these are themselves very complex we may proceed with them in the same way. We may then suppose ourselves left with a series of fairly simple propositions; but it will probably be found that some of these merely repeat information given by others, so that they may be omitted.

We may find to what extent this is the case, by adopting any one of the three following methods:—

First, by leaving out each proposition in turn, and determining (by the ordinary rules) what the remainder by combination give concerning its subject. If we find that

[1] The proposition in its original form may admit of simplification in accordance with the rules laid down in chapter I. It will generally speaking be found advantageous to have recourse to such simplification before proceeding further with the solution.

[2] See section 318.

[3] See section 302.

it adds nothing to the information that they give it may be omitted. Or if a portion of the information which it gives is also given independently, then part of the statement may be dropped[1].

Secondly, by bringing each proposition to the form,—

Nothing is X_1 *or* X_2............*or* X_n,

and then comparing it with the combination of the remainder.

Thirdly, by writing down all possible combinations after Jevons's plan, (*Pure Logic*, p. 46; *Principles of Science*, chapter VI; *Studies*, p. 181), and noting which are excluded by each proposition in turn. If certain combinations are excluded by more than one proposition, then one or other of such propositions may be modified accordingly.

We are now left with a series of propositions which are mutually independent. All that is further necessary is to make sure that each of these propositions is itself expressed in its simplest form[2]; and to observe whether any two or more of the propositions admit of a simple recombination[3]. When these simplifications have been carried as far as possible we have our final solution.

The solution may, if we wish, be verified by recombining the propositions that have been obtained, an operation by which we arrive again at a series of alternatives identical with those originally given us. Such verification is however not essential to the validity of our process, which, if it has been correctly performed, contains no possible source of error.

The following examples worked out in full detail will serve to illustrate the above method.

[1] If, for example, other propositions contain the information that *All A is D*, then for *All AB is CD* we may substitute *All AB is C*.

[2] For example, *X is AB or aBC or aBc* may be reduced to *X is B*.

[3] For example, *ac is d* and *Bc is d* may be combined into *cD is Ab*.

I. For our first example we may take one of those chosen by Jevons in the extract quoted in the preceding section.

Given the proposition that *Everything is either ABC or Abc or aBC or abC*, find a set of propositions involving as simple relations as possible which shall be equivalent to it.

By reduction of dual terms and contraposition, *What is neither ABC nor Abc is aC*; therefore, *What is a or Bc or bC is aC*;

$$i.e.,\quad \begin{cases} a \text{ is } C, \\ Bc \text{ is non-existent}, \\ bC \text{ is } a. \end{cases}$$

Bc is non-existent is reducible to *B is C*; and this proposition and *a is C* may be combined into *c is Ab.*

Our solution therefore is,—

$$\begin{cases} c \text{ is } Ab, \\ bC \text{ is } a. \end{cases}$$

By combining these propositions it will be found that we regain the original proposition.

II. We may next take the more complex example contained in the same extract from Jevons.

The given alternatives are,—*ACe*, *aBCe*, *aBcdE*, *abCe*, *abcE*; and by the reduction of dual terms, they become *aBcdE*, *abcE*, *Ce.*

Therefore, *What is not aBcdE or abcE is Ce*;

$$\text{therefore,}\quad \begin{cases} A \text{ is } Ce; & (1) \\ C \text{ is } e; & (2) \\ e \text{ is } C; & (3) \\ BD \text{ is } Ce. & (4) \end{cases}$$

But since by (2) *C is e*, (1) may be reduced to *A is C*;

and this proposition may be combined with (3) yielding *c is aE*. Also by (2), (4) may be reduced to *BD is C*.

Hence our solution becomes,—

$$\begin{cases} C \textit{ is } e; \\ c \textit{ is } aE; \\ BD \textit{ is } C. \end{cases}$$

III. The following problem is from Jevons, *Principles of Science*, 2nd ed., p. 127 (Problem v.).

The given alternatives are,—*ABCD*, *ABCd*, *ABcd*, *AbCD*, *AbcD*, *aBCD*, *aBcD*, *aBcd*, *abCd*.

By the reduction of duals these alternatives may be written: *ABC or ABcd or AbD or aBCD or aBc or abCd*.

Therefore by contraposition, *What is not ABC or AbD or aBc is ABcd or aBCD or abCd*.

But *What is not ABC or AbD or aBc* is equivalent to *What is ABc or aBC or ab or bd*. Hence we have for our solution the following set of propositions:

(1) *ABc is d*,

(2) *aBC is D*,

(3) *ab is Cd*,

(4) *bd is a*[1].

This is equivalent to the solution given in Jevons, *Studies*, p. 256.

If we wish to verify our solution we may proceed as follows:

By (3), *Everything is A or B or Cd*;

[1] We first obtain *bd is aC*; but since by (3), *abd is C*, this may be reduced to *bd is a*.

By (4), *Everything is a or B or D*;
therefore, *Everything is AD or aCd or B.*

By (1), *Everything is a or b or C or d*;
therefore, *Everything is AbD or ACD or aB or aCd or BC or Bd*;

By (2), *Everything is A or b or c or D*;
therefore, *Everything is ABC or ABd or AbD or ACD or aBc or aBD or abCd or BCD or Bcd*;

But, *AbD is AbCD or AbcD*; and expanding all the terms similarly, we have,—*Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcD or aBcd or abCd.* These are precisely the alternatives originally given.

IV. The following example is also from Jevons, *Principles of Science*, 2nd Edition, p. 127 (Problem viii). In his *Studies*, p. 256, he speaks of the solution as *unknown*. A fairly simple solution, however, may be obtained by the application of the general rule formulated in this section.

The given alternatives are,—*ABCDE, ABCDe, ABCde, ABcde, AbCDE, AbcdE, Abcde, aBCDe, aBCde, aBcDe, abCDe, abCdE, abcDe, abcdE.*

By the reduction of duals these alternatives may be written: *ABCe or ABcde or Abcd or ACDE or aBCde or abdE or aDe.*

Therefore by contraposition, *Whatever is not either ABCe or ABcde or Abcd or abdE or aDe is ACDE or aBCde.*

But it will be found that, by the application of the ordinary rule for obtaining the contradictory of a given term, *Whatever is not either ABCe or ABcde or Abcd or abdE or aDe* is equivalent to *Whatever is AbC or ade or BE or AcD or DE.*

Hence our proposition is resolvable into the following:

(i) *AbC is DE*;
(ii) *ade is BC*;
(iii) *BE is ACD*;
(iv) *AcD is non-existent*;
(v) *DE is AC.*

But by (v), *BE is AC or d*; therefore, (iii) may be reduced to *BE is D.*

Hence we have the following as our final solution:—

(1) *AbC is DE*;
(2) *ade is BC*;
(3) *BE is D*;
(4) *cD is a.*
(5) *DE is AC.*

392. Another Method of Solution of the Inverse Problem.

Another method of solving the Inverse Problem, suggested to me (in a slightly different form) by Dr Venn, is to write down the original complex proposition in the negative form, *i.e.*, to obvert it, before resolving it. It has already been shewn that a negative proposition with a disjunctive predicate may be immediately broken up into a set of simpler propositions.

In some cases, especially where the number of destroyed combinations as compared with those that are saved is small, this plan is of easier application than that given in the preceding section.

To illustrate this method we may take two or three of the examples already discussed.

I. *Everything is ABC or Abc or aBC or abC*;

therefore, by obversion, *Nothing is AbC or Bc or ac*; and this proposition is at once resolvable into,—

$$\begin{cases} Ab \text{ is } c. \\ c \text{ is } Ab.[1] \end{cases}$$

II. *Everything is ACe or aBCe or aBcdE or abCe or abcE*;

therefore, by obversion, *Nothing is CE or Ac or BcD or ce.*

This proposition may be successively resolved as follows:

$$\begin{cases} \text{No } E \text{ is } C; \\ \text{No } c \text{ is } A \text{ or } e; \\ \text{No } BD \text{ is } c. \end{cases}$$

$$\begin{cases} E \text{ is } c; \\ c \text{ is } aE; \\ BD \text{ is } C. \end{cases}$$

This again is equivalent to our former solution.

III. *Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcD or aBcd or abCd*;

therefore, by obversion, *Nothing is Abd or ABcD or abc or abD or aBCd*;

and this proposition may be successively resolved as follows:

$$\begin{cases} \text{No } bd \text{ is } A; \\ \text{No } ABc \text{ is } D; \\ \text{No } ab \text{ is } c \text{ or } D; \\ \text{No } aBC \text{ is } d. \end{cases}$$

[1] The equivalence between this and our former solution is immediately obvious. Equationally it would be written $Ab = c$.

$$\begin{cases} bd \textit{ is } a; \\ ABc \textit{ is } d; \\ ab \textit{ is } Cd; \\ aBC \textit{ is } D. \end{cases}$$

This again repeats our original solution. It is rather interesting to find that notwithstanding the indeterminateness of the problem we obtain by independent methods the same result in each of the above cases.

393. Another Method of Solution of the Inverse Problem.

The following is a third independent method of solution of the Inverse Problem, and it is in some cases easier of application than either of the two preceding methods.

Any proposition of the form

Everything is ......

may be resolved into the two propositions:

$$\begin{cases} \textit{All } A \textit{ is } \ldots\ldots \\ \textit{All } a \textit{ is } \ldots\ldots \end{cases}$$

which taken together are equivalent to it; similarly *All A is ......* may be resolved into the two *All AB is ......*, *All Ab is ......*; and it is clear that by taking pairs of contradictories in this way we may resolve any given proposition into a set of propositions which contain no disjunctive terms[1].

To illustrate this method we may again take the first three examples given in section 391.

I. *Everything is ABC or Abc or aBC or abC* may be resolved successively as follows:

[1] Redundancies must of course as before be as far as possible avoided.

$$\begin{cases} C \text{ is } AB \text{ or } aB \text{ or } ab; \\ c \text{ is } Ab. \end{cases}$$

$$\begin{cases} bC \text{ is } a^{1}; \\ c \text{ is } Ab. \end{cases}$$

II. *Everything is ACe or aBCe or aBcdE or abCe or abcE* may be resolved successively as follows:

$$\begin{cases} C \text{ is } Ae \text{ or } aBe \text{ or } abe; \\ c \text{ is } aBdE \text{ or } abE. \end{cases}$$

$$\begin{cases} C \text{ is } e; \\ c \text{ is } aE; \\ c \text{ is } Bd \text{ or } b. \end{cases}$$

$$\begin{cases} C \text{ is } e; \\ c \text{ is } aE; \\ Bc \text{ is } d. \end{cases}$$

III. *Everything is ABCD or ABCd or ABcd or AbCD or AbcD or aBCD or aBcD or aBcd or abCd* may be resolved successively as follows:

$$\begin{cases} B \text{ is } ACD \text{ or } ACd \text{ or } Acd \text{ or } aCD \text{ or } acD \text{ or } acd \\ b \text{ is } ACD \text{ or } AcD \text{ or } aCd. \end{cases}$$

$$\begin{cases} B \text{ is } AC \text{ or } aD \text{ or } cd; \\ b \text{ is } AD \text{ or } aCd. \end{cases}$$

$$\begin{cases} BC \text{ is } A \text{ or } aD; \\ Bc \text{ is } aD \text{ or } d; \\ Ab \text{ is } D; \\ ab \text{ is } Cd. \end{cases}$$

$$\begin{cases} BCd \text{ is } A; \\ ABc \text{ is } d; \\ Ab \text{ is } D; \\ ab \text{ is } Cd. \end{cases}$$

[1] Taking *BC* for our subject we have *BC is A or a*, which is a merely identical proposition, and which may therefore be omitted.

The above solutions are practically the same as those obtained in the two preceding sections.

394. Find propositions that leave only the following combinations,—*ABCD*, *ABcD*, *AbCd*, *aBCd*, *abcd*.
[Jevons, *Studies*, p. 254.]

Jevons gives this as the most difficult of his series of inverse problems involving four terms. It may be solved as follows:—

Everything is ABCD or ABcD or AbCd or aBCd or abcd; therefore, by contraposition and reduction of dual terms, *Whatever is not either AbCd or aBCd is ABD or abcd.*

Therefore, *Whatever is AB or ab or c or D is ABD or abcd*; and this is resolvable into the four propositions,—

$$\begin{cases} AB \textit{ is } D, & (1) \\ ab \textit{ is } cd, & (2) \\ c \textit{ is } ABD \textit{ or } abd, & (3) \\ D \textit{ is } AB. & (4). \end{cases}$$

Since by (4), *D is AB*, and by (2), *ab is d*, (3) may be reduced to *c is D or ab*, *i.e.*, *cd is ab.*

Our set of propositions may therefore be reduced to,—

$$\begin{cases} AB \textit{ is } D, \\ ab \textit{ is } cd, \\ cd \textit{ is } ab, \\ D \textit{ is } AB.^{1} \end{cases}$$

[1] Written equationally, this solution would appear still simpler; namely,—

$$AB = D,$$
$$ab = cd.$$

395. Resolve the proposition *Everything is ABCDeF or ABcDEf or AbCDEF or AbCDeF or AbcDeF or aBCDEf or aBcDEf or abCDeF or abCdeF or abcDef or abcdef* into a series of simple propositions.

[Jevons, *Principles of Science*, 2nd ed., p. 127, (Problem x.).]

The following is a solution :—

(1) *ABE* is *cDf*;
(2) *AcDF* is *be*;
(3) *aF* is *bCe*;
(4) *bf* is *ace*;
(5) *d* is *ae*;
(6) *ef* is *abc*.

This is rather less complex than the solution by Dr John Hopkinson given in Jevons, *Studies*, p. 256, namely :—

(i) *d* is *ab*;
(ii) *b* is *AF* or *ae*;
(iii) *Af* is *BcDE*;
(iv) *E* is *Bf* or *AbCDF*;
(v) *Be* is *ACDF*;
(vi) *abc* is *ef*;
(vii) *abef* is *c*.

396. How many and what non-disjunctive propositions are equivalent to the statement that "What is either *Ab* or *bC* is *Cd* or *cD*, and *vice versa*"?

[Jevons, *Studies*, p. 246.]

We have given—

$$\left\{\begin{array}{ll} Ab \textit{ is } Cd \textit{ or } cD, & \text{(i)} \\ bC \textit{ is } Cd \textit{ or } cD, & \text{(ii)} \\ Cd \textit{ is } Ab \textit{ or } bC, & \text{(iii)} \\ cD \textit{ is } Ab \textit{ or } bC. & \text{(iv)} \end{array}\right.$$

(i) may be resolved into,—

$$\begin{cases} \textit{Abc is D}, & \text{(v)} \\ \textit{AbD is c}. & \text{(vi)} \end{cases}$$

But (vi) is inferable from (ii); and observing some other obvious simplifications we obtain immediately the following solution:

(1) *Abc is D*;
(2) *bC is d*;
(3) *Cd is b*;
(4) *cD is Ab.*

EXERCISES.

397. Shew the equivalence between the two sets of propositions given in section 395.

398. Resolve each of the following complex propositions into a series of relatively simple propositions:

(1) *Everything is ABCD or AbCd or aBcD or abcd*;

(2) *Everything is AbCD or AbCd or Abcd or aBcd or abCD or abCd or abcd*;

(3) *Everything is AbcDE or aBCd or aBCE or aBcd or aBde or abCe or abce or abDe or abde or BcdE or bCDe*;

(4) *Everything is ABCE or ABcd or ABcE or ABde or Abcd or abCE or abcE or abdE or abde or BCde*;

(5) *Everything is ABCDE or ABCdE or ABcDE or ABcDe or ABcde or AbCdE or Abcde or aBCDE or aBCde or abCDE or abcDe*;

(6) *Everything is ABDe or ABDF or AcDe or Acef or aBDe or aBDF or abCD or abCd or abcD or abcd or aCDE or aCDe or aCdE or aCde or acDe or aDEF or aDEf or aDeF or aDef or BcDF or bceF or bcef*;

(7) *Everything is AbdE or Abef or AbF or Acdef or aBDF or abCF or aCdE or ade or bCDe or bCdf or bDEF*;

(8) *Everything is ABCEf or Abe or aBCdf or aBcdE or aBcdeF or abef or bceF.*

399. Solve the fourth problem given in section 391, (α) by the method described in section 392, (β) by that described in section 393.

400. Shew that any universal complex proposition may be resolved into a set of propositions in which no conjunctive combination of terms occurs.

INDEX.

CAMBRIDGE: PRINTED BY C. J. CLAY, M.A. AND SONS, AT THE UNIVERSITY PRESS.

4